Schumpeterian Growth, Unemployment and Tax/Benefit system in European Countries

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Introduction

The observed high unemployment in continental Europe and the slowdown in economic growth in lasts decades naturally raise the question of whether this two phenomena are related. On the empirical side, there is no consensus regarding the sign of the correlation between growth and unemployment, either across countries or across longer periods of time in the same country.[1] The same is true on the theoretical side.[2] Perhaps this is due to the fact that the two rates are simultaneously determined in the economy. If so, the rigidities on the labor market could affect not just unemployment but economic growth, inducing even uncorrelated comovements on average. However, whereas the institutions causing elevate labor costs are widely accepted as the primary cause for high unemployment in continental European countries[3], the statistical relationship between these variables and long run economic growth is a moot point.[4]

Despite the divergences, the empirical results of Daveri *et al.* (2000), and Adjemian and Langot (2006) about the explicative role of labor-market variables on the bad performance of European countries constitutes an interesting point to be explored deeply. With this aim, in this paper we construct a theoretical economy to analyse the effects of labor market institutions on growth and unemployment. Moreover, we conduct a welfare analysis to shed light on the way to atteint the optimal outcomes. The main hypotheses of our model are the following: *(i)* Innovations are the engine of growth. This implies a "creative destruction" process generating jobs reallocation. *(ii)* Agents are heterogeneous: the skilled arbitre between be employed in production or be engaged in R&D activities, while the unskilled cannot be researchers, just be employed in production; and *(iii)*there is no full employment because the

[1]See Mortensen (2004) for a wide review of the empirical literature, which shows the diversity of results about the correlation between growth and unemployment.

[2]This is due to the offsetting nature of two main effects: a higher rate of growth in productivity will reduce unemployment trough a positive "capitalization" effect on investment in job creation; whereas the "creative destruction effect", inherent to the growth process, leads to a faster obsolescence of technologies and so to a faster rate of job destruction.

[3]See, among others, Blanchard and Wolfers (1999) and Daveri, Tabellini, Bentolila, and Huizinga (2000)

[4]For instance, Layard and Nickell (1999) show that the relationship between unemployment-causing variables and TFP growth is weak or nonexistent. Conversely, Daveri *et al.* (2000) find positive correlation of labor taxes and growth, while Alonso, Echeverria, and Tran (2004) report a negative significant impact of these variables on growth. Lingens (2003) shows that unions cause unemployment, but has an ambiguous effect on growth. Mortensen (2004) that payroll tax subsidies increase (decrease) the growth rate (the unemployment rate), whereas firing costs decrease unemployment but also the growth rate.

trade unions representing each class of workers set the wage rates above the competitive level.

We show that: *(i)* An increase in the unemployment compensation, or in the payroll taxes payed by employers, or in the taxes payed by workers, leads to bigger unemployment levels of both skills and to a slowdown of the economic growth. *(ii)* An increase in the cost of employment protection increases just the unemployment of skilled, and decreases the rate of economic growth. *(iii)* A coordinated bargain process increases employment of both skills. *(iv)* There exists an optimal taxation rule such that the equilibrium growth rate equal the optimal growth rate. The paper is organized as follows. Section 1 describes the model. Section 2 presents the analysis of the impact of labor market institutions on growth and unemployment. Finally, section 3 shows the welfare analysis.

1 The model

1.1 Preferences

We suppose that the economy is populated by two categories of agents: skilled workers, in a fixed quantity L^s, and unskilled workers, in a fixed quantity L^{us}. Each agent is endowed with one unit flow of labor, so L^s and L^{us} are also equal to the aggregate flow of labor supply of each category. Unskilled workers can be employed in production (x^{us}) or unemployed (u^{us}): $L^{us} = x^{us} + u^{us}$; while skilled workers may be employed in production (x^s), engaged in research and development activities (n) or unemployed (u^s): $L^s = x^s + n + u^s$. When employed, workers pay a tax t^{us} on their wage income if unskilled, and t^s if skilled.

All individuals share the same linear preferences over lifetime consumption C of the single final good:

$$U(C) = E_0 \int_0^\infty C_t e^{-\rho t} dt \tag{1}$$

where $\rho > 0$ is the subjective rate of time preference and C_t is the individual's consumption of the single final good at time t. Each household is free to borrow and lend at interest rate r_t. However, given linear preferences, the optimal household's behavior implies $\rho = r_t \ \forall t$. Hence, the level of consumption is undefined. One particular and popular solution to this problem is to assume that households consume all their wage income without saving. In this case we can analyze the impact of the unemployment benefit system.

1.2 Final-good sector

Final good is produced using the latest vintage of a unit continuum of intermediate inputs,[5]

$$C = \int_0^1 A_j x_j^{\alpha} dj, \quad 0 < \alpha < 1, \quad j \in [0,1] \tag{2}$$

A_j represents the productivity in the production of the latest vintage of intermediate j and is determined by the number of technical improvements made up to date t, knowing that between two innovations the gain in productivity is equal to $q > 1$ (step size).

Taking the final good as *numéraire* the profits flow is equal to

$$C - \int_0^1 p(x_j) x_j dj$$

1.3 Intermediate-goods sector

Production of each unit of intermediate good j requires one unit of each class of labor as input:

$$x_j = (x_j^{us})^a (x_j^s)^{1-a}, \quad a \in (0,1) \tag{3}$$

Since the final goods sector is perfectly competitive, the price of each intermediate good, $p(x_j)$, is equal to the value of its marginal product:

$$p(x_j) = \frac{\partial C}{\partial x_j} = \alpha A_j x_j^{\alpha - 1} \quad \forall j \tag{4}$$

1.4 R&D sector

For each intermediate good j, the technological spillovers lead to good-specific public knowledge allowing to the potential innovators to begin their efforts to improve upon the current "state of the art". But there are no spillovers between sectors. Then, when an amount n_j of skilled labor is used in R&D on good j, innovations arrive randomly at a Poisson rate hn_j, with $h > 0$ a parameter indicating the productivity of the research technology: a potential entrant obtains ideas for new products at frequency h per period. Albeit the innovation frequencies are independent across goods, the expected gains are the same everywhere, hence $\forall j \; n_j = n \quad \Rightarrow \quad A_j = A$.

[5]We omit the time index since between two innovations all is constant.

1.5 Arbitrage condition for R&D

At the "state of the art" τ, the aggregate number of potential innovators is given by the arbitrage condition faced by skilled workers:

$$\frac{(1-t^s)W^s_{j,\tau}}{h} = V_{j,\tau+1} \tag{5}$$

The cost of $R\&D$ can be thought off as an opportunity cost: the income that the individual loses $(1-t^s)W^s_{j,\tau}$ times the expected duration of the innovation process $1/h$. On the other hand, $V_{j,\tau+1}$ is the discounted expected payoff of next innovation on sector j,[6] and is determined by the asset equation:

$$rV_{j,\tau+1} = \Pi_{j,\tau+1} - hn_{\tau+1}(V_{j,\tau+1} + E_{\tau+1}) \tag{6}$$

$\Pi_{j,\tau+1}$ are the monopolistic profits earned by the successful innovator, which gets a patent on her innovation, from the sales to the final goods sector until the arrival of next innovation. We assume that the employment protection laws imply a cost E of shutting down a firm, and that the monopolist pays a proportional payroll tax τ^{us} (τ^s) over unskilled (skilled) employment. Then,

$$\Pi_{j,\tau+1} = \alpha A_{\tau+1}x^{\alpha}_{j,\tau+1} - W^{us}_{j,\tau+1}(1+\tau^{us})x^{us}_{j,\tau+1} - W^{s}_{j,\tau+1}(1+\tau^{s})x^{s}_{j,\tau+1} \tag{7}$$

so, the expected income generated by a licence on an innovation is equal to the instantaneous profit minus the expected capital loss that will occur when the current innovator is replaced by a new innovator (the flow probability of the profits loss is the arrival rate $hn_{\tau+1}$ which is the same for all j). By normalizing lasts expressions by the productivity level associated to the $(\tau+1)^{th}$ innovation we obtain:

$$\pi_{j,\tau+1} = \alpha x^{\alpha}_{j,\tau+1} - w^{us}_{j}(1+\tau^{us})x^{us}_{j,\tau+1} - w^{s}_{j}(1+\tau^{s})x^{s}_{j,\tau+1} \tag{8}$$

and

$$v_{j,\tau+1} = \frac{\pi_{j,\tau+1} - hn_{\tau+1}e}{r + hn_{\tau+1}} \tag{9}$$

for $\pi \equiv \frac{\Pi}{A}$, $w^{us} \equiv \frac{W^{us}}{A}$, $w^s \equiv \frac{W^s}{A}$, $e \equiv \frac{E}{A}$ and $v \equiv \frac{V}{A}$.

1.6 Wage bargaining processes

For each skill and intermediate good sector there is a trade union representing the workers' interests. So the wage rates are the solution to a bargain problem

[6] Equivalently, the entry condition also reflects the fact that skilled labor can be freely allocated between production and research: w^s_j is the value of an hour in production while $qhv_{j,\tau+1}$ is the expected value of an hour in research.

between the monopolist and each trade union. Given the bargain outcomes the monopolist choose the labor demands that maximize her profit flow. We assume that all jobs for the same skill are equally productive and that all workers have the same unemployment benefits so that the wage fixed for each type of job is the same everywhere. But the firm and the trade unions in sector j are too small to influence the market, so the wage rates are fixed taking everything else constant.

Taking as exogenous both the demand and the wage for skilled workers ($x_j^s = \bar{x}_j^s$, $w_j^s = \bar{w}^s$) the union anticipates the unskilled labor demand as

$$x_{j,\tau+1}^{us}(w_j^{us}) = \arg\max\{\pi_{j,\tau+1}(x_{j,\tau+1}^{us}, w_j^{us})\} = \left(\frac{a\alpha^2(\bar{x}_{j,\tau+1}^s)^{\alpha(1-a)}}{w_j^{us}}\right)^{\frac{1}{1-\alpha a}}$$

Then, for $0 \leq \beta^{us} \leq 1$, the bargained unskilled wage is:

$$\begin{aligned} w_j^{us} &= \arg\max\left\{[((1-t^{us})w_j^{us} - b)x_{j,\tau+1}^{us}]^{\beta^{us}}(\pi_{j,\tau+1} - hn_{\tau+1}e - \bar{\pi}_{\tau+1}^{us})^{1-\beta^{us}}\right\} \\ &= \left(1 + \frac{\beta^{us}(1-\alpha a)}{\alpha a}\right)\left(\frac{b}{1-t^{us}}\right) \end{aligned}$$

$\bar{\pi}_{\tau+1}^{us} \equiv -w^s x_{j,\tau+1}^s - hn_{\tau+1}e$ denotes the firm's disagreement point and $b \equiv \frac{B}{A}$ the adjusted unemployment compensation.

Similarly, taking both the demand and the wage for unskilled workers as given ($x_j^{us} = \bar{x}_j^{us}$, $w^{us} = \bar{w}^{us}$) the union anticipates the unskilled labor demand as

$$x_{j,\tau+1}^s(w_j^s) = \arg\max\{\pi_{j,\tau+1}(x_{j,\tau+1}^s, w_j^s)\} = \left(\frac{(1-a)\alpha^2(\bar{x}_{j,\tau+1}^{us})^{\alpha a}}{w_j^s}\right)^{\frac{1}{1-\alpha(1-a)}}$$

Then, for $0 \leq \beta^s \leq 1$ the bargained skilled wage is:

$$\begin{aligned} w_j^s &= \arg\max\left\{[(1-t^s)w_j^s - b)x_{j,\tau+1}^s]^{\beta^s}(\pi_{j,\tau+1} - hn_{\tau+1}e - \bar{\pi}_{\tau+1}^s)^{1-\beta^s}\right\} \\ &= \left(1 + \frac{\beta^s(1-\alpha(1-a))}{\alpha(1-a)}\right)\left(\frac{b}{1-t^s}\right) \end{aligned}$$

where $\bar{\pi}_{\tau+1}^s \equiv -w^{us}x_{j,\tau+1}^{us} - hn_{\tau+1}e$ is the firm's disagreement point.

1.7 Equilibrium

Given $r > 0$, for all intermediate good sector j and for all "state of the art" τ a ***steady-state (or balanced growth path) equilibrium*** is defined as follows:

(i) **Wage rules:**

$$w^{us} = \frac{\beta_1 b}{1-t^{us}}, \quad \beta_1 \equiv 1 + \frac{\beta^{us}(1-\alpha a)}{\alpha a} \tag{10}$$

$$w^{s} = \frac{\beta_2 b}{1-t^{s}}, \quad \beta_2 \equiv 1 + \frac{\beta^{s}(1-\alpha(1-a))}{\alpha(1-a)} \tag{11}$$

(ii) **Labor demands:**

$$x^{us} = \left(\frac{a^{1-\alpha(1-a)}(1-a)^{\alpha(1-a)}\alpha^2}{((1+\tau^{us})w^{us})^{1-\alpha(1-a)}((1+\tau^{s})w^{s})^{\alpha(1-a)}} \right)^{\frac{1}{1-\alpha}} \tag{12}$$

$$x^{s} = \left(\frac{a^{\alpha a}(1-a)^{1-\alpha a}\alpha^2}{((1+\tau^{us})w^{us})^{\alpha a}((1+\tau^{s})w^{s})^{1-\alpha a}} \right)^{\frac{1}{1-\alpha}} \tag{13}$$

Moreover, demands are related as follows:

$$x^s = \frac{(1-a)(1-t^s)(1+\tau^{us})\beta_1}{a(1-t^{us})(1+\tau^s)\beta_2} x^{us}$$

(iii) **Unemployment:**
Unemployment of unskilled workers is deduced from the employment identity given the endowment of unskilled labor L^{us} and the demand for unskilled labor x^{us}:

$$u^{us} = L^{us} - x^{us} \tag{14}$$

while unemployment of skilled workers is deduced from the employment identity given the endowment of skilled labor L^s, the demand for skilled labor x^s and the aggregate number of researchers $\frac{\delta}{h}$:

$$u^s = L^s - x^s - n \tag{15}$$

(iv) **Economic growth:** The rate of growth in aggregate consumption is given by (see the appendix A):

$$g_t = hn\ln(q) \tag{16}$$

From the free entry condition (5) we deduce:

$$hn = \frac{qh\pi - r\beta_2 b}{\beta_2 b + qhe} \tag{17}$$

where

$$\pi = \frac{(1-\alpha)(1+\tau^{us})\beta_1 b}{\alpha a(1-t^{us})} x^{us} \tag{18}$$

Thus, given the labor demand for unskilled workers, the unemployment levels and the economic growth are well determined.

2 The impact of labor market institutions on growth and unemployment

2.1 Passive labor market policies

In this section we analyze the consequences for growth and unemployment of a more generous unemployment insurance, *(ii)* bigger taxes on labor incomes, and the *(iii)* employment protection.

Proposition. 1 a. *An increase in the unemployment compensation (b), or in the payroll taxes payed by employers (τ^{us}, τ^{s}), or in the taxes payed by workers (t^{us}, t^{s}), leads to (i) an increase in the unemployment levels of both skills and (ii) to an decrease in the economic growth.*

b. *An increase in the cost of employment protection e (i) increases the unemployment of skilled, while unemployment of unskilled is unaffected, and (ii)decreases the rate of economic growth.*

Proof. a. We know that $\frac{du^{us}}{dx^{us}} = -1$, $\frac{du^{s}}{dx^{us}} < 0$ and $\frac{dg}{dx^{us}} > 0$, and it is easy to show that: $\frac{\partial x^{us}}{\partial x}|_{x=b,\tau^{us},\tau^{s},t^{us},t^{s}} < 0$ and $\frac{\partial \pi}{\partial x}|_{x=b,\tau^{us},\tau^{s},t^{us},t^{s}} < 0$. So, $\frac{\partial g}{\partial x}|_{x=b,\tau^{us},\tau^{s},t^{us},t^{s}} = \frac{qh\ln(q)}{\beta_2 b+qhe}\frac{\partial \pi}{\partial x}|_{x=b,\tau^{us},\tau^{s},t^{us},t^{s}} < 0$.

This result is very intuitive: a bigger labor cost implies a higher wage (equations (10) and (11)) and so a decline in the labor demand for production (equations (12) and (13)). The total outcome is a contraction of the monopolistic profits with the subsequent reduction in the expected value of an innovation. This, together with the fact that the bigger wages make more attractive production than R&D, tends to reduce the number of researchers. Thus, the economic growth rate falls too.

b. $\frac{\partial x^{us}}{\partial e} = \frac{\partial x^{s}}{\partial e} = 0 \Rightarrow \frac{\partial u^{us}}{\partial e} = \frac{\partial u^{s}}{\partial e} = 0$.

Since the wage rates does not change, nor the labor demands, the solely effect is a contraction of the profits. This discourages skilled workers to engage in *R&D* activities, and then the growth rate falls.

2.2 The wage bargaining processes

The impact of the unions can be analyzed in two steps. First, for an uncoordinated wage bargaining process, one can derive the implications of a high bargaining power. Second, we propose to compare the outcome of an efficient bargaining process to the usual inefficient outcome computed above.

2.2.1 The bargaining powers

Proposition. 2 *An increase in the workers' bargaining power (in one or in both of them) leads to an increase in the unemployment levels of both skills and to an decrease in the economic growth.*

Proof. Analogous to the proof of proposition 1a: $\frac{\partial x^{us}}{\partial(\beta^{us},\beta^{s})} < 0$ and $\frac{\partial \pi}{\partial(\beta^{us},\beta^{s})} < 0$. So, $\frac{\partial g}{\partial \beta^{us}} = \frac{qh\ln(q)}{\beta_2 b+qhe}\frac{\partial \pi}{\partial \beta^{us}} < 0$ and $\frac{\partial g}{\partial \beta^{s}} = -\frac{(1-\alpha(1-a))\ln(q)}{\beta_2 b+qhe}\left(\frac{\pi}{(1-\alpha)\beta_2}+\frac{(r+hn)b}{\alpha(1-a)}\right)$.

2.2.2 Inefficient v.s. efficient bargain

If in each sector the monopolistic firm and each trade union bargain over both the labor demand and the wage rate, the outcome is the efficient one (E). That is, the wage and the firm size pairs are the solution to the following problems:

$$(w^{us,E}_{j,\tau+1}, x^{us,E}_{j,\tau+1}) = \arg\max\Big\{[((1-t^{us})w^{us,E}_{j,\tau+1}-b)x^{us,E}_{j,\tau+1}]^{\beta^{us}} \\ (\pi^{E}_{j,\tau+1}-hn^{E}_{\tau+1}e-\bar{\pi}^{us,E}_{\tau+1})^{1-\beta^{us}}\Big\}$$

$$(w^{s,E}_{j,\tau+1}, x^{s,E}_{j,\tau+1}) = \arg\max\Big\{[((1-t^{s})w^{s,E}_{j,\tau+1}-b)x^{s,E}_{j,\tau+1}]^{\beta^{s}} \\ (\pi^{E}_{j,\tau+1}-hn^{E}_{\tau+1}e-\bar{\pi}^{s,E}_{\tau+1})^{1-\beta^{s}}\Big\}$$

The firm's disagreement points and the instantaneous profit flow are respectively:

$$\begin{aligned}
\bar{\pi}^{us}_{\tau+1} &\equiv -w^{s}x^{s}_{j,\tau+1}-hn_{\tau+1}e \\
\bar{\pi}^{s}_{\tau+1} &\equiv -w^{us}x^{us}_{j,\tau+1}-hn_{\tau+1}e \\
\pi^{E}_{j,\tau+1} &= \alpha(x^{E}_{\tau+1})^{\alpha}-w^{us,E}_{j,\tau+1}(1+\tau^{us})x^{us,E}_{j,\tau+1}-w^{s,E}_{j,\tau+1}(1+\tau^{s})x^{s,E}_{j,\tau+1}
\end{aligned}$$

Then at equilibrium, for all j and for all τ:

$$w^{us}_{E} = w^{us} \tag{19}$$
$$w^{s}_{E} = w^{s} \tag{20}$$
$$x^{us}_{E} = x^{us}\beta_1^{\frac{1-\alpha(1-a)}{1-\alpha}}\beta_2^{\frac{\alpha(1-a)}{1-\alpha}} \tag{21}$$
$$x^{s}_{E} = x^{s}\beta_1^{\frac{\alpha a}{1-\alpha}}\beta_2^{\frac{1-\alpha a}{1-\alpha}} \tag{22}$$
$$\pi_{E} = \kappa\pi, \quad \kappa = \frac{1-\alpha(a\beta_1+(1-a)\beta_2)}{1-\alpha}\beta_1^{\frac{\alpha a}{1-\alpha}}\beta_2^{\frac{\alpha(1-a)}{1-\alpha}} \tag{23}$$

Proposition. 3 *The efficient economy has bigger employment levels and smaller rate of economic growth than the uncoordinated economy. As consequence, the unskilled unemployment is lower in the efficient economy; however the relative position of both the skilled unemployment and the economic growth rates is unclear.*

Proof. Since $\beta_1 \geq 1$ and $\beta_2 \geq 1$, then $x^{us} \leq x_E^{us}$ and $x^s \leq x_E^s$. Consequently, unskilled unemployment is smaller too: $u^{us} \leq u_E^{us}$. However, the signe of κ is unclear.

3 Welfare Analysis

3.1 The impact of growth on welfare

This section concentrates on the impact on welfare, measured by the level of the agents' utility, of changes in the rate of economic growth. From equation (1) we know that utility is increasing in consumption so we shall first determine the effects of variations in the growth rate on consumption. At equilibrium $C = Ax^{\alpha}$, where $A = q^{hn}$. Then,

$$\begin{aligned}\frac{\partial C}{\partial h} &= x^{\alpha} h \ln(q) q^{hn} > 0 \\ \frac{\partial C}{\partial n} &= x^{\alpha} n \ln(q) q^{hn} > 0\end{aligned}$$

Similarly, from the expression for the growth of rate at equilibrium:

$$\begin{aligned}\frac{\partial g}{\partial h} &= n \ln(q) > 0 \\ \frac{\partial g}{\partial n} &= h \ln(q) > 0\end{aligned}$$

Then, the economic growth and welfare vary in the same way.

3.2 The optimal economic growth

In order to discuss deeply the impact of labor market institutions on welfare, in this section we derive the average growth rate that would be chosen by a social planner whose objective was to maximize the expected present value of consumption, given by (see appendix B for details)

$$E(U) = \frac{A_0 x^{\alpha}}{r - hn(q-1)} \tag{24}$$

Then, the social planner will choose (x^{us}, x^s, n) to maximize (24) subject to the labor constraints $L^{us} = x^{us}$ and $L^s = x^s + n$, and to the technological constraint $x = (x^{us})^a (x^s)^{1-a}$. This problem is reduced to find the optimal level of research $n*$, such that

$$n^* = \arg\max \left\{ \frac{A_0 (L^{us})^{\alpha a} (L^s - n)^{\alpha(1-a)}}{r - hn(q-1)} \right\} \tag{25}$$

The first order condition implies that

$$(L^s - n^*) h(q-1) = \alpha(1-a)(r - hn(q-1)) \tag{26}$$

From the technological constraint we have that

$$\frac{\partial x / \partial x^s}{\partial x / \partial x^{us}} = \frac{(1-a) x^{us}}{a x^s} = 1 \quad \Leftrightarrow \quad x^s = \frac{(1-a) x^{us}}{a} \tag{27}$$

Using (27) and the labor constraints the optimality condition (26) becomes:

$$\frac{(1-a) L^{us} h(q-1)}{a} = \alpha(1-a)(r - hn(q-1)) \tag{28}$$

From this we deduce the optimal level of research:

$$n^* = \frac{r}{h(q-1)} - \frac{L^{us}}{\alpha a} \tag{29}$$

3.3 Equilibrium growth rate v.s. optimal growth rate

Given that the average growth rate is proportional to the number of researchers, it is sufficient to compare the optimal level of research with the equilibrium level of our economy, which is given by equations (17) and (18):

$$\hat{n} = \frac{qh \left(\frac{1-\alpha}{\alpha a}\right) \left(\frac{1+\tau^{us}}{1-t^{us}}\right) \beta_1 b (L^{us} - u^{us}) - r\beta_2 b}{h(\beta_2 b + qhe)} \tag{30}$$

In order to simplify the comparison between n^* and $\hat{n}$ we rewrite (29) and (30) respectively as:

$$1 = \frac{h(q-1) \left(\frac{1}{\alpha a}\right) L^{us}}{r - hn^*(q-1)} \tag{31}$$

$$1 = \frac{qh \left(\frac{1-\alpha}{\alpha a}\right) \left(\frac{1+\tau^{us}}{1-t^{us}}\right) \left(\frac{\beta_1}{\beta_2}\right) (L^{us} - u^{us}) - \frac{qh^2 \hat{n} e}{\beta_2 b}}{r + h\hat{n}} \tag{32}$$

Remark that when there is just one kind of agents ($a = 1$) and there are no labor market rigidities, n^* and $\hat{n}$ are reduced to expressions in the Aghion and Howitt (1998)' canonical model. Thus, as in their model, we find the following basic differences between n^* and $\hat{n}$:

D1 The social discount rate $r - hn^*(q-1)$ is less than the market discount rate $r + h\hat{n}$.

D2 The private monopolist in unable to appropriate the whole output flow, but just a fraction $(1-\alpha)$.

D3 The factor $(q-1)$ corresponds to the so-called "business-stealing" effect: the private researcher does not internalize the loss to the previous monopolist caused by an innovation.

In Aghion and Howitt (1998) distortions $D1$ and $D2$ tend to make the average growth rate less than optimal, whereas $D3$ tends to make it greater. Due to the offsetting nature of these effects, the market average growth rate may be more or less than optimal. Indeed, in their model $D1$ and $D2$ dominate when the size of innovations q is large. At the opposite, when there is much monopoly power (α close to zero) and innovations are not too large, $D3$ dominates.

However, we have some other differences due to the rigidities and institutions working in the labor market, say:

D4 There is no full employment because the trade unions fix wages above the competitive level: $L^{us} - u^{us} < L^{us}$.

D5 The effect of the relative strength of trade unions, captured by the term $\frac{\beta_1}{\beta_2}$, is growth enhancing if $\beta_1 > \beta_2 \Leftrightarrow \frac{(1-\alpha a)\beta^{us}}{\alpha a} < \frac{(1-\alpha(1-a))\beta^{s}}{\alpha(1-a)}$. Else, the relative strength diminishes economic growth.

D6 The cost of the employment protection implies a reduction of $\frac{qh^2ne}{\beta_2 b}$ in the numerator of (30).

D7 The taxes on labor imply an increase in the numerator of (30): $\frac{1+\tau^{us}}{1-t^{us}} > 1$

Clearly, $D4$ and $D6$ tend to make the average growth rate less than optimal, whereas $D7$ tends to make it greater. Even if once again effects are offsetting, the stark difference with $D1$ to $D3$ is that the effects of $D4$ to $D7$ depend on policy variables, and thus, at least theoretically, may be controlled. This naturally suggest the question of wether the labor market variables can reduce the gap between the optimal growth rate $g^* = g(n^*)$ and the market growth rate $\hat{g} = g(\hat{n})$. This issue is discussed in next section.

3.4 Labor market institutions and convergence to optimal growth

As in Aghion and Howitt (1998)' model, because the effects discussed above are conflicting, the equilibrium average growth rate may be more or less than optimal. So we are interested on the possibility of reaching the optimal growth rate by controlling key labor-market policy variables. From the analysis in section 2 we deduce that the impact of labor market variables on the level of research $\hat{n}$ is negative, *i.e.* $\frac{\partial \hat{n}}{\partial x}|_{x=\beta^{us},\beta^{s},\tau^{us},\tau^{s},t^{us},t^{s},e} < 0$. Thus, if the structural parameters are such that the number of researchers is above the optimal one, a positive adjustment of policy variables may be appealing, and viceversa.

To conclude this welfare analysis we conduct two simple experiences of economic policy to illustrate how the optimal outcome can be reached. With this aim, let us consider the symmetric setting in which $a = 1 - a$, $\beta^{us} = \beta^{s}$, $\tau^{us} = \tau^{s} \equiv \tau$, and $t^{us} = t^{s} \equiv t$. This imply $\beta_1 = \beta_2 \equiv \beta$ and $L^{us} - u^{us} = x^{us} = \left(\frac{a\alpha^2}{(1+T)\beta b}\right)^{\frac{1}{1-\alpha}}$, where $1 + T \equiv \frac{1+\tau}{1-t}$. Then, expression (30) becomes:

$$\hat{n} = \frac{qh(1-\alpha)(1+T)^{-\frac{\alpha}{1-\alpha}}(\beta b)^{-\frac{\alpha}{1-\alpha}}\alpha^{\frac{1+\alpha}{1-\alpha}}a^{\frac{\alpha}{1-\alpha}} - r\beta b}{h(\beta b + qhe)} \tag{33}$$

Proposition. 4 *Given $e = 0$ and $n^* > 0$, there exists T such that $\hat{n} = n^*$.*

Proof.

$$\begin{aligned} \hat{n} &= n^* \quad \Leftrightarrow \\ \frac{q(1-\alpha)(1+T)^{-\frac{\alpha}{1-\alpha}}\alpha^{\frac{1+\alpha}{1-\alpha}}a^{\frac{\alpha}{1-\alpha}}}{(\beta b)^{\frac{1}{1-\alpha}}} - \frac{r}{h} &= \frac{r}{h(q-1)} - \frac{L^{us}}{\alpha a} \quad \Leftrightarrow \\ 1 + T = \left(\frac{q(1-\alpha)}{\frac{rq}{h(q-1)} - \frac{L^{us}}{\alpha a}}\right)^{\frac{1-\alpha}{\alpha}} \frac{\alpha^{\frac{1+\alpha}{\alpha}}a}{(\beta b)^{\frac{1}{\alpha}}} &> 0 \end{aligned}$$

Inequality comes from the fact that $n^* > 0 \Rightarrow \frac{r}{h(q-1)} > \frac{L^{us}}{\alpha a}$; since $q > 1$, then $\frac{rq}{h(q-1)} > \frac{L^{us}}{\alpha a}$.

Proposition. 5 *Given $T = 0 (\Leftrightarrow \tau = -t)$ and $n^* > 0$, there exists e such that $\hat{n} = n^*$.*

Proof.

$$\hat{n} = n^* \quad \Leftrightarrow$$

$$\frac{qh(1-\alpha)(\beta b)^{-\frac{\alpha}{1-\alpha}}\alpha^{\frac{1+\alpha}{1-\alpha}}a^{\frac{\alpha}{1-\alpha}}-r\beta b}{h(\beta b+qhe)} = \frac{r}{h(q-1)}-\frac{L^{us}}{\alpha a} \Leftrightarrow$$

$$e=\frac{\beta b}{\frac{r}{q-1}-\frac{hL^{us}}{\alpha a}}\left[\frac{(1-\alpha)\alpha^{\frac{1-\alpha}{1-\alpha}}a^{\frac{\alpha}{1-\alpha}}}{(\beta b)^{\frac{1}{1-\alpha}}}+\frac{1}{q}\left(\frac{hL^{us}}{\alpha a}-\frac{rq}{q-1}\right)\right]$$

Then, $e>0 \Leftrightarrow \frac{hL^{us}}{\alpha a}>\frac{rq}{q-1}$; else, $e\leq 0$.

Concluding remarks

We have constructed a general equilibrium model in which economic growth and unemployment are endogenously determined by the number of innovations made in the economy, which in turn is determined by the skilled-workers' incentive to engage in R&D activities. We have shown that: *(i)* High labor cost or powerful trade unions leads to bigger unemployment levels of both skills and to a slowdown of the economic growth. *(ii)* An efficient bargaining allows to have lower unemployment for both skills, at the price of a lower growth rate, if the other labor market institutions still unchanged. Finally, we have suggested a way to atteint the optimal rate of growth.

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A The Rate of Growth

The average growth rate of consumption good (or final output) is deduced as follows: we know that between two consecutive innovations, say τ and $\tau+1$, final output is augmented a fixed amount q, $C_{\tau+1} = qC_{\tau}$. Hence, between date t and date $t+1$ expected output is given by the following relationship

$$E[C_{t+1}] = q^{\int_0^1 hn_t dt} C_t \tag{34}$$

since from the law of large numbers, the expected value of the number of innovations (the aggregate arrival rate hn) is the same across sectors. Then, by taking logarithms and arranging terms we have that

$$g_t \equiv E[\ln C_{t+1} - \ln C_t] = hn_t \ln(q) \tag{35}$$

B Expected welfare

The expected welfare $E(U)$ is defined as the expected present value of lifetime consumption, but consumption is a random variable that takes the values $\left\{A_0 x^{\alpha}, A_0 q x^{\alpha}, A_0 q^2 x^{\alpha}, \ldots, A_0 q^k x^{\alpha}, \ldots\right\}_{k \in N}$. Then,

$$\begin{aligned} E(U) &= \int_0^{\infty} e^{-rt} E(C_t) dt \\ &= \int_0^{\infty} e^{-rt} \left(\sum_{k=0}^{\infty} p(k,t) A_t x_t^{\alpha} \right) dt \end{aligned}$$

where $p(k,t) = \frac{(hnt)^k e^{-hnt}}{k!}$ is the probability to have exactly k innovations up to time t. So,

$$E(U) = \left(\sum_{k=0}^{\infty} \frac{(hnt)^k q^k}{k!} \right) \int_0^{\infty} e^{-(r+hn)t} A_0 x^{\alpha} dt$$

Note that $\sum_{k=0}^{\infty} \frac{(hnt)^k q^k}{k!} = \sum_{k=0}^{\infty} \frac{(hnt)^k q^k e^{-hntq}}{k!} e^{hntq} = e^{hntq}$. Then,

$$\begin{aligned} E(U) &= A_0 x^{\alpha} \int_0^{\infty} e^{-(r-hn(q-1))t} dt \\ &= A_0 x^{\alpha} \left[-\frac{e^{-(r-hn(q-1))t}}{r - hn(q-1)} \right]_0^{\infty} \\ &= \frac{A_0 x^{\alpha}}{r - hn(q-1)} \end{aligned}$$