Bundling in Vertically Differentiated Communication Markets

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Abstract

We look at the competition and the welfare effects of bundling in the context of vertically differentiated communication services (e.g. Television, Telephone and Internet). We consider a two-stage game with two asymmetric firms (e.g. Telecom and Cable Operator). In the first stage firms simultaneously commit to adopt bundling or component pricing. These decisions give four possible configurations: (i) a configuration where both firms use component pricing; (ii) a configuration where both firms use bundling; and finally (iii) the two configurations where one firm use bundling and the other firm does not. In the second stage firms set simultaneously prices. We show that bundling is a dominant strategy equilibrium for both firms. The reason is that bundling increases the differentiation of services and reduces the intensity of price competition. We also find that although the bundling-bundling equilibrium reduces consumers' surplus, total economic welfare is higher than when both firms use component pricing.

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1. Introduction

Today consumers are offered telephone, high speed Internet and television services by cable operators and telecom companies. Cable operators supply broadband Internet access and voice telephony in addition to their "traditional" video services. Similarly, telecom companies supply telephone, video images and high speed Internet[1]. Typically cable operators and telecoms require subscribers to take their traditional service and they offer add-on services for an extra payment that is lower than the stand alone price of these services.

The purchase of all services from a single supplier is said to be convenient for buyers. It is also said to be a deterrent to churn [2] because disappointment with one service can be compensated by satisfaction with another service[3].

It is believed that the traditional telephone operator provides better telephone service than the cable operator, whereas the latter provides better television. Both offer a similar quality of high speed Internet[4]. Consumers therefore have to choose, between lower quality telephone combined

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[1] In Quebec the dominant cable operator, Videotron provides digital television, telephone and high-speed Internet services with the coaxial cable technology while the dominant telecommunication company, Bell Canada provides the same services with satellite transmission and twisted pair. Coaxial cable is the kind of cable used by cable TV companies between the community antenna and the user homes and businesses. It carries broadband services for a great distance. To offer high speed Internet services, a cable operator creates a data network that operates over its hybrid fiber/ coax (HFC) plant. A twisted pair is an ordinary copper wire that connects home and business computers to the telephone company. DSL (Digital Subscriber line) Internet access provided by the local telephone company convert existing twisted-pair telephone lines into access paths for multimedia and high-speed data communications. So with satellite and twisted pair technologies, a local telecommunication company can also supply same kind of services as a local cable operator.

[2] The rate at which customer discontinues service (in order to shift to competitor) - among high usage customers, at the expense of profit margins: Keith Damsell "Telecom bundling seem luring customers. Grouping services together for lower prices builds loyalty, turn "churn" low. Study says "The Globe and Mail, 29 September 2003, at p.138, citing Convergence Consulting Group ltd study: The Battle for the North American Couch Potatoes, and referring to Cox Communications, extremely low churn rate with the triple play services of digital television, high speed internet access, and local telephone services.

[3] On the other hand consumers can drop all services when they are disappointed with one of them.

[4] Indeed cable telephony has some limitations: e.g. it doesn't work when there is power failure and drop out when broadband3.5 demand (the ability of the user to view content across the internet that includes large files, such as video, audio and 3D) is high. Also not all areas are served by the POC since hybrid fiber/ coax (HFC) plants are expensive to install. Consequently additional costs of providing services to additional customers are higher for co-ax (HFC) technology than twisted pair technology. On the other hand the television service of telecoms has also severe

with high quality television offered by the cable operator, and higher quality telephone with lower quality television offered by the telephone company.

This paper addresses the following questions: (i) under what conditions do suppliers bundle? that is under what conditions does it sell two or more services as a package only? ; (ii) how does bundling compare to component selling in terms of welfare? ; and (iii) what attitude should competition authorities adopt toward such bundling?

There are no clear-cut results pertaining to the profitability and welfare effect of bundling as opposed to separate selling of components. Adams and Yellen (1976), Schmalensee (1982, 1984), Mc Afee et al (1989), and Whinston (1990) show that under monopoly bundling raises profits when variable costs are zero. However, the vast majority of consumer services are supplied in non-monopolistic environments. Only few papers [(Matutes and Regibeau, 1989; Economides, 1993; Anderson and Leruth, 1993; Kopalle and al, 1999)] examine the non-monopolistic case where firms have the option of bundling. Theses papers assume horizontal differentiation of services and their conclusions are numerical.

Economides (1993) considers a two-stage game and shows that the Nash equilibrium is mixed bundling[5] rather than component selling. Because competition is more intense under mixed bundling, a prisoner's dilemma arises, that is firms would be better off if they could commit not to bundle. Anderson and Leruth (1993) show in a two-stage model that the Nash equilibrium is both firms offer components selling. The reason is that firms fear the extra degree of competition intrinsic to mixed bundling. Kopalle and al (1999) reconcile the result of Economides (1993) and Anderson and Leruth (1993) by incorporating the role of market expansion on equilibrium bundling strategies. They show that for complementary components mixed bundling dominates component selling only when it creates a new market for the bundle.

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limitations in competing with cable TV. It encounters some constraints due to broadband transmission, to cities' architectures, to weather conditions and it needs an installation of a non esthetical device, the dish.

[5] Mixed bundling means that the packages as well as the individual components of the package are available.

Matutes and Regibeau (1992) consider a game where in the first stage there is a choice between compatibility versus incompatibility[6]. In the second and the third stage of the game firms choose the selling strategy and prices respectively. Matutes and Regibeau ask whether firms would choose to make their products compatible and whether they would sell their products as a bundle. For compatible components, they find that, depending on consumer's reservation price there can be two kinds of equilibria. In the first, one firm bundles and one firm does not. In the second, both firms bundle[7].

None of the aforementioned papers (i) is concerned with vertically differentiated services; (ii) They do not give clear results about the welfare effect of bundling. The underlying motivation of this paper is to analyze the competition and the welfare effect of bundling in the communication market within the context of vertically differentiated services. We consider a two-stage game with two asymmetric firms. In the first stage firms simultaneously commit to use bundling or component pricing. These decisions give four possible configurations: (i) a configuration where both firms use component pricing; (ii) a configuration where both firms use bundling; and finally (iii) the two configurations where one firm use bundling and the other firm does not. In the second stage firms set prices simultaneously.

We show that bundling is a dominant strategy equilibrium for both firms. The reason is that bundling increases the differentiation of services and reduces the intensity of price competition[8]. We also find that although the bundling-bundling equilibrium reduces consumers' surplus, the total economic welfare is higher than when both firms use component pricing.

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[6] A component is incompatible with components sold by other firms', if it cannot be assembled with them to form a usable system. The economic consequence of compatibility versus incompatibility have been examined by Matutes and Regibeau (1988), Economides (1989, 1991), and Einhorn (1992). They have looked at the case where each firm supplied all the necessary goods.

[7] The first equilibrium occurs when consumer's reservation price is low, while the second one occurs when it is high.

[8] Chen (1997) also analyzes bundling as differentiation tool. He studies the case where two sellers compete in a first market, and both also sell another product in a second competitive market. Absent bundling, Bertrand competition drives both sellers' profits in the first market to zero. If one seller uses bundling and the other does not, however, both can earn positive profits since the bundle and the individual first-market product are effectively differentiated products.

The paper is structured as follows. In section 2, we present a duopoly model, where each firm has the choice to sell its services either separately or as a bundle. In section 3, we analyze the game. In section and 4, we derive the equilibrium selling strategy of each firm. In section 5, we analyze the welfare consequences of bundling. In section 6, we provide an application of the model to the communication market and we conclude.

2. The Model

There are two firms, denoted *h* and *l*. E.g. firm *h* is a telecom company and firm *l* is a cable operator. Each firm sells two services, denoted *A* and *B*. E.g. service *A* is a telephone service and service *B* is an Internet service. The service *A* comes in two qualities, a_h and a_l supplied respectively by firm *h* and firm *l*, $a_h > a_l > 0$. The quality b of service *B* is the same for both firms. Both the variable cost and the fixed cost are zero for each service. Every consumer demands one or zero unit of service of *A* and/or *B*.

A consumer with a parameter θ derives a utility θa_i from quality a_i of service *A*, $i = h,l$. Similarly, a consumer with a parameter γ derives a utility γb from quality *b* of service *B*. If the consumer chooses not to buy a service, she receives her reference utility which is normalized to zero. A consumer with preference indices (θ, γ) who buys one unit of *A* of quality a_i at price p_i and one unit of *B* at price p_B receives a net surplus:

$$U = (\theta a_i - p_i) + (\gamma b - p_B)\,, i = h,l$$

Each consumer makes her purchase decision to maximize her consumer surplus. Consumer preference indices θ and γ are independently and uniformly distributed on $[0,1]\times[0,1]$.

We model the competition as a two-stage game. In the first stage, firms decide whether to bundle, or not to bundle; in the second stage[9] they set prices. There are four possible subgames at stage 2: (i) (C_h, C_l) denotes the game where both firms sell components separately; (ii) (B_h, B_l) denotes

[9] Firms observe the choices made in the first stage.

the game where both firms bundle; (iii) (B_h, C_l) denotes the game where firm *h* bundles, and firm *l* sells its components separately; and (iv) (C_h, B_l) denotes the game where firm *h* sells its components separately, and firm *l* bundles. We will examine under what conditions each of the subgame is an equilibrium.

3. Price determination

3.1 Case (i): (C_h, C_l), Pure Components by both Firms

Since both firms produce the same quality of service *B*, Bertrand competition insures that its price is driven down to marginal cost, which is zero. With regard to service *A*, we know[10] that an equilibrium with two active firms requires[11]: $\frac{p_h}{a_h} \geq \frac{p_l}{a_l}$. We designate by $\bar{\theta}$ the consumer indifferent between not purchasing and purchasing one unit of A_l, and by $\tilde{\theta}$ the consumer who is indifferent between purchasing A_l and A_h. Figure 1 displays market shares as $(1-\tilde{\theta})$ for firm *h* and $(\tilde{\theta}-\bar{\theta})$ for firm *l* when firms compete in prices.

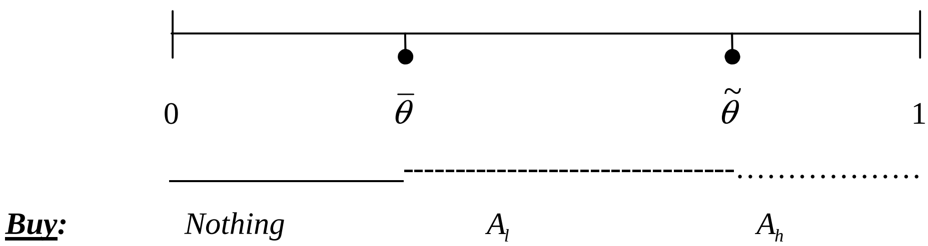

Figure 1. Market areas under the regime (C_h, C_l)

[10] See Tirole (1988)

[11] The condition states that the *price per unit of quality* is higher for a_h than for a_l. This means that low quality is not dominated by high quality. If low quality is dominated by high quality then the firm with the low quality exits the market.

The firms' profits are:

$$\pi_h^{C_h,C_l} = p_h(1-\tilde{\theta}) \text{ for firm } h,$$

$$\pi_l^{C_h,C_l} = p_l(\tilde{\theta}-\bar{\theta}) \text{ for firm } l,$$

where $\bar{\theta} = \frac{p_l}{a_l}$ and $\tilde{\theta} = \frac{p_h - p_l}{a_h - a_l}$. Prices are chosen optimally when they satisfy the conditions below

$$p_h = \frac{2(a_h^2 - a_h a_l)}{4a_h - a_l}, \text{ and } p_l = \frac{(a_h - a_l)a_l}{4a_h - a_l}.$$

Then:

$$\pi_h^C = \frac{4a_h^2(a_h - a_l)}{(4a_h - a_l)^2}, \text{ and } \pi_l^C = \frac{a_h a_l(a_h - a_l)}{(4a_h - a_l)^2}.$$

3.2 Case (ii): (B_h, B_l), Bundling by both Firms:

Denote by p_{Gh} and p_{Gl} the prices of bundles $A_h B$ and $A_l B$ respectively. The individual-rationality constraints are

for consumers of $A_h B$: $\theta a_h + \gamma b - p_{Gh} \geq 0$, (R_h)

for consumers of $A_l B$: $\theta a_l + \gamma b - p_{Gl} \geq 0$. (R_l)

Self-selection constraints are

for consumers of $A_h B$: $\theta a_h + \gamma b - p_{Gh} \geq \theta a_l + \gamma b - p_{Gl}$, (S_h)

for consumers of $A_l B$: $\theta a_l + \gamma b - p_{Gl} \geq \theta a_h + \gamma b - p_{Gh}$. (S_l)

The condition $\theta a_h + \gamma b - p_{Gh} \geq Max(0, \theta a_l + \gamma b - p_{Gl})$ must be satisfied by buyers of $A_h B$. The condition $\theta a_l + \gamma b - p_{Gl} \geq Max(0, \theta a_h + \gamma b - p_{Gh})$ must be satisfied by buyers of $A_l B$.

We find again that market areas depend on the ranking of *price per unit of quality* of service *A*. To see how, we define the preference parameter of the consumer indifferent between the bundles $A_h B$ and $A_l B$ by $\theta^* \equiv \frac{p_{Gh} - p_{Gl}}{a_h - a_l}$. We distinguish three cases.

Case 1: $\frac{p_{Gh}}{a_h} \leq \frac{p_{Gl}}{a_l}$.

Case 2: $\frac{p_{Gh}}{a_h} \geq \frac{p_{Gl}}{a_l}$ and $\theta^* < 1$.

Case 3: $\frac{p_{Gh}}{a_h} \geq \frac{p_{Gl}}{a_l}$ and $\theta^* > 1$

In case 1 the *price per unit of quality* of $A_h B$ is lower than *price per unit of quality* of $A_l B$. In case 2 and case 3 *the price per unit of quality* of $A_h B$ is higher than the *price per unit of quality* of $A_l B$. The difference between case 2 and case 3 is that in the latter there is no consumer indifferent between $A_h B$ and $A_l B$.

Case 1: $\frac{p_{Gh}}{a_h} \leq \frac{p_{Gl}}{a_l}$.

The price of bundle $A_h B$ per unit of quality of service *A* is lower than the price of bundle $A_l B$ per unit of quality of service *A*. The lines labelled R_h and R_l in Figure 2 are the individual-rationality constraints of high and low quality buyers.

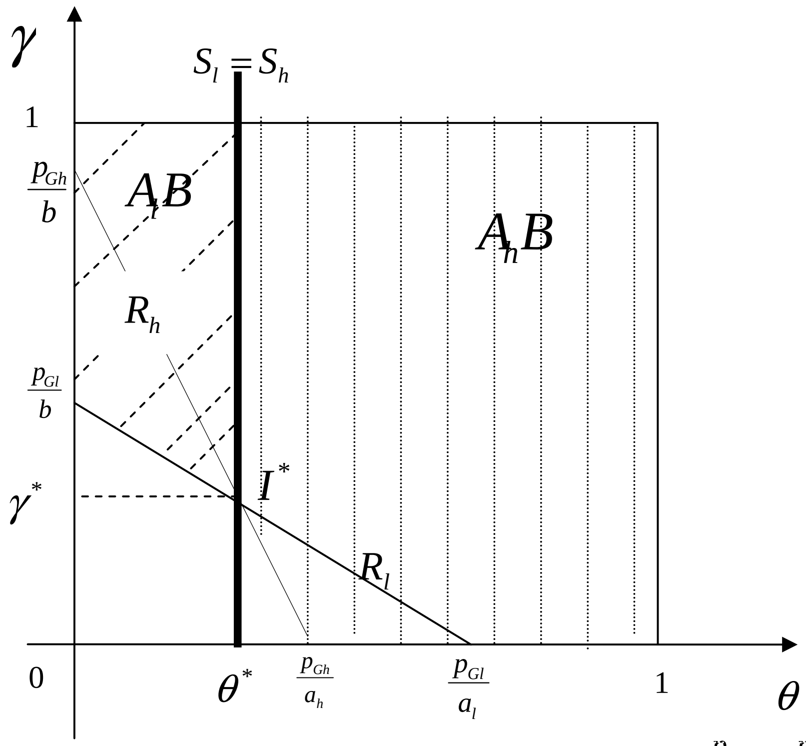

Figure 2: Market areas under the regime (B_h, B_l) when $\frac{p_{Gh}}{a_h} \leq \frac{p_{Gl}}{a_l}$

The lines S_h and S_l represent[12] the self-selection constraint faced by consumers. Consumers with preference parameters above R_h derive positive utility from $A_h B$. Consumers with preference parameters above R_l derive positive utility from $A_l B$. We note that S_h, S_l, R_h and R_l intersect at $I^* \equiv \left(\theta^* \equiv \frac{p_{Gh} - p_{Gl}}{a_h - a_l}; \gamma^* \equiv \frac{a_h p_{Gl} - a_l p_{Gh}}{a_h - a_l} \right)$.

[12] The constraints S_h and S_l yield the same line.

The high quality firm serves consumers with preference parameters $\theta \in [\theta^*,1]$ and above R_h ; the low quality firm serves consumers with preference parameters $\theta \in [0,\theta^*]$ and above R_l. The size of market served by the firm *h* is $D_{Gh} = 1 - \theta^* - \frac{(\gamma^*)^2}{2a_h}$ and the size of market served by the firm *l* *is* $D_{Gl} = \theta^*(1 - \frac{p_{Gl}}{2b} - \frac{\gamma^*}{2})$. Firm *h* and firm *l* profits' are respectively:

$$\pi_h^{PB} = p_{Gh} D_{Gh} \text{ and } \pi_l^{PB} = p_{Gl} D_{Gl} .$$

In contrast to the standard model of single differentiated good, we find that there can be two active firms even when $\frac{p_{Gh}}{a_h} \leq \frac{p_{Gl}}{a_l}$ [13]. The Difference is as follows: in standard models of a single differentiated good, consumers make the comparison on a service by service basis. If A_l is dominated by A_h, all consumers obtain more surplus from A_h than from A_l. Nobody purchases A_l. We also know that for service *B*, Bertrand competition and zero marginal cost imply that consumers obtain *B* for free from both firms. Thus firm *l* is excluded from market *A*, but remains in market *B*. In the regime of (B_h, B_l) there is a competition for vertically differentiated system goods. Therefore, the best available alternative for consumers who wish to purchase only service *B* is to purchase the low quality bundle $A_l B$. In that case, if A_l is dominated by A_h the low quality firm can survive in both markets because it serves the bundle $A_l B$ to consumers who care very little about service *A*, while the high quality firm serves the bundle $A_h B$ to consumers who care for service *A* and for service *B*.

[13] That is the low quality system is dominated by the high quality system.

Case 2: $\frac{p_{Gh}}{a_h} \geq \frac{p_{Gl}}{a_l}$ and $\theta^* < 1$.

The price of bundle $A_h B$ per unit of quality of service *A* is higher than the price of bundle $A_l B$ per unit of quality of service *A* and there exist a consumer who is indifferent between the bundles. Market areas are shown in Figure 3.

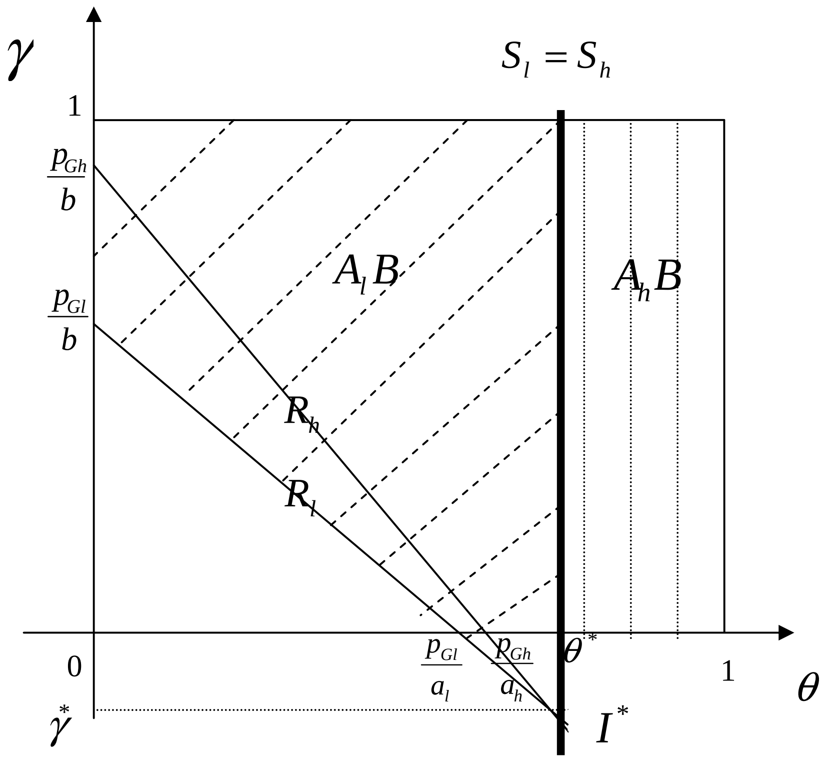

Figure 3: Market areas under the regime (B_h, B_l) when $\frac{p_{Gh}}{a_h} \geq \frac{p_{Gl}}{a_l}$ and $\theta^* < 1$.

The high quality firm serves consumers with preference parameters $\theta \in [\theta^*, 1]$ and above R_h. The low quality firm serves consumers with preference parameters $\theta \in [0, \theta^*]$ and above R_l. Consumers with preference parameters below R_l do not purchase at all. The market areas for $A_h B$ and $A_l B$ are respectively: $D_{Gh} = 1 - \theta^*$ and $D_{Gl} = \theta^* \left[1 - \frac{(p_{Gl})^2}{2ba_l} \right]$.

Case 3: $\frac{p_{Gh}}{a_h} \geq \frac{p_{Gl}}{a_l}$ and $\theta^* \geq 1$.

The price of bundle $A_h B$ per unit of quality of service A is higher than the price of bundle $A_l B$ per unit of quality of service A and nobody is indifferent between the bundles. Market areas are displayed in Figure 4.

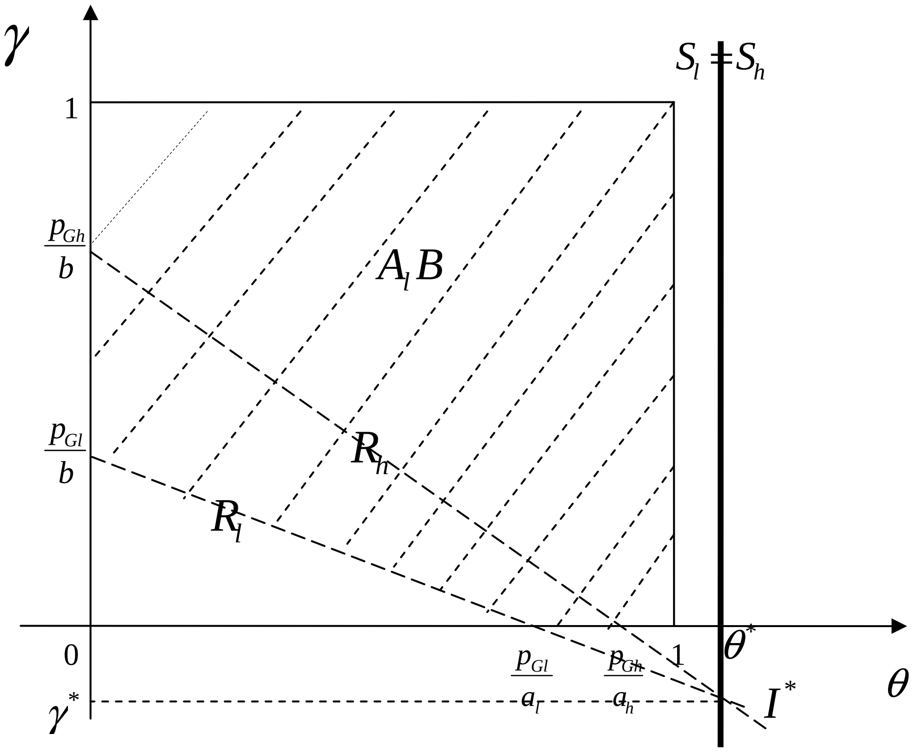

Figure 4: Market areas under the regime (B_h, B_l) when $\frac{p_{Gh}}{a_h} \geq \frac{p_{Gl}}{a_l}$ and $\theta^* \geq 1$.

Because $\theta^* \geq 1$, the demand for the bundle $A_h B$ is zero. The low quality firm serves consumers with preference parameters $\theta \in [0, \theta^*]$ and above R_l [14]. Consumers with preference parameters below R_l do not purchase at all. The market areas for $A_h B$ and $A_l B$ are respectively: $D_{Gh} = 0$ and $D_{Gl} = 1 - \frac{(p_{Gl})^2}{2ba_l}$

[14] In that case , firm *h* makes zero profit and it easy to see that this is not an equilibrium because firm *h* is always better off (makes positive profits) by choosing its price such that $\theta^* \leq 1$.

3.2.1 Determination of the equilibrium prices in (B_h, B_l)

In case 1 the first order conditions are:

$$\frac{\partial \pi_h^B}{\partial p_{Gh}} = 1 - \frac{2p_{Gh}}{a_h - a_l} - \frac{3}{2a_h}\left(\frac{a_l p_{Gh}}{a_h - a_l}\right)^2 + \left(\frac{a_l}{a_h - a_l} + \frac{2a_l p_{Gh}}{(a_h - a_l)^2}\right)p_{Gl} - \frac{a_h}{2}\left(\frac{p_{Gl}}{a_h - a_l}\right)^2 = 0$$

$$\frac{\partial \pi_l^B}{\partial p_{Gl}} = p_{Gh} + \frac{a_l (p_{Gh})^2}{2(a_h - a_l)} - 2p_{Gl} - \left(\frac{1}{b} + \frac{a_h + a_l}{a_h - a_l}\right)(p_{Gh} p_{Gl}) + \left(\frac{3}{2b} + \frac{3a_h}{2(a_h - a_l)}\right)(p_{Gl})^2 = 0$$

We can see that the first order conditions are quite complex. We obtain similar complicated first order conditions for all others cases. For this reason, we search the price equilibria numerically for a range of values of a_h and a_l.

In case 1, the equilibrium prices that we obtain always satisfy $\frac{p_{Gh}}{a_h} \le \frac{p_{Gl}}{a_l}$. Figure 5 shows the equilibrium values of $\frac{p_{Gh}}{a_h}$ and $\frac{p_{Gl}}{a_l}$ for different values of a_h.

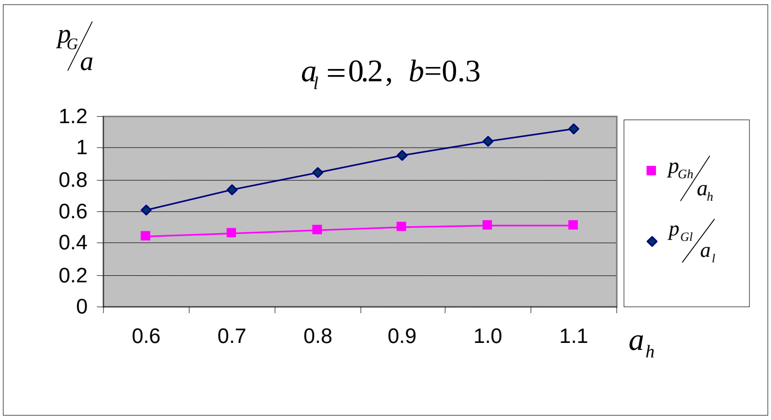

Figure 5: Comparison of prices per unit of quality under regime (B_h, B_l)

In case 2 and 3, the equilibrium prices that we obtain do not satisfy $\frac{p_{Gh}}{a_h} > \frac{p_{Gl}}{a_l}$. Therefore, we will only look at case 1.

It is interesting to compare profits in (B_h, B_l) to profits in (C_h, C_l). The profits under the regime (C_h, C_l) and the regime (B_h, B_l) for different values of a_h are shown in Figure 6. $\pi_h^{B_h, B_l}$ is always higher than $\pi_h^{C_h, C_l}$. Similarly, $\pi_l^{B_h, B_l}$ is always higher than $\pi_l^{C_h, C_l}$. We obtain similar results for various values of a_l.

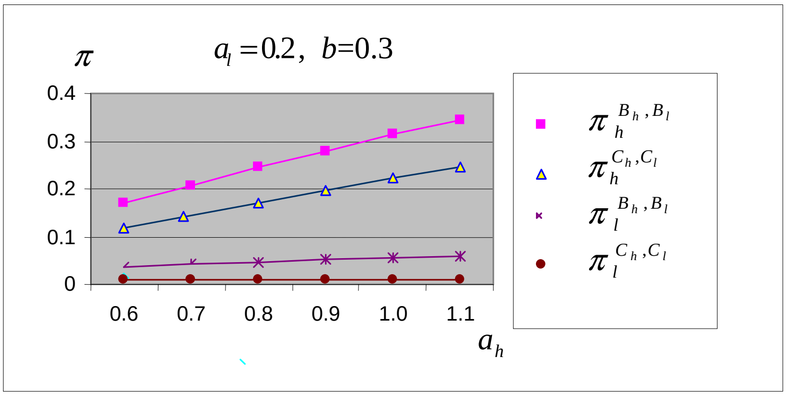

Figure 6: Comparison of firms' profits under the regimes (C_h, C_l) and (B_h, B_l).

We see that both firms are better when they bundle. The reason is that bundling affects the intensity of competition via two channels: (i) it reduces the intensity of the competition for service B by increasing differentiation; and (ii) it increases the intensity of the competition for service A by reducing differentiation. The net effect of bundling is a decrease of competition between the two firms because the competition for B under component pricing is extreme (Bertrand competition). Therefore each makes more profit in the subgame (B_h, B_l) than in the subgame (C_h, C_l).

3.3 Case (iii): (B_h, C_l), Bundling by firm *h*, Pure Component Selling by firm *l*

The individual-rationality constraints are now

for consumers of $A_h B$: $\quad \theta a_h + \gamma b - p_{Gh} \geq 0, \quad (R_h)$

for consumers of A_l : $\quad \theta a_l - p_l \geq 0 \quad , \quad (R_l)$

for consumers of B : $\quad \gamma b - p_B \geq 0 \quad , \quad (R_b)$

for consumers of $A_l + B$ [15] : $\theta a_l + \gamma b - p_l - p_B \geq 0$. (R_m)

The self-selection constraints are

for consumers of $A_h B$: $\theta a_h + \gamma b - p_{Gh} \geq Max(\theta a_l + \gamma b - p_l - p_B, \theta a_l - p_l, \gamma b - p_B)$, (S_h)

for consumers of A_l : $\theta a_l - p_l \geq Max(\theta a_h + \gamma b - p_{Gh}; \theta a_l + \gamma b - p_l - p_B; \gamma b - p_B)$, (S_l)

for consumers of B : $\gamma b - p_B \geq Max(\theta a_h + \gamma b - p_{Gh}; \theta a_l + \gamma b - p_l - p_B; \theta a_l - p_l)$, (S_b)

for consumers of $A_l + B$: $\theta a_l + \gamma b - p_l - p_B \geq Max(\theta a_h + \gamma b - p_{Gh}; \gamma b - p_B; \theta a_l - p_l)$. (S_m)

The condition $\theta a_h + \gamma b - p_{Gh} \geq Max(0, \theta a_l + \gamma b - p_l - p_B, \theta a_l - p_l, \gamma b - p_B)$ must be satisfied by buyers of $A_h B$.

The condition $\theta a_l - p_l \geq Max(0, \theta a_h + \gamma b - p_{Gh}; \theta a_l + \gamma b - p_l - p_B; \gamma b - p_B)$ must be satisfied by buyers of A_l alone.

The condition $\gamma b - p_B \geq Max(0, \theta a_h + \gamma b - p_{Gh}; \theta a_l + \gamma b - p_l - p_B; \theta a_l - p_l)$ must be satisfied by buyers of B alone.

And finally the condition $\theta a_l + \gamma b - p_l - p_B \geq Max(0, \theta a_h + \gamma b - p_{Gh})$ must be satisfied by buyers of both A_l and B.

From S_h, we derive that the preference index of the consumer indifferent between purchasing $A_h B$ and purchasing A_l and B separately is $\hat{\theta} \equiv \dfrac{p_{Gh} - p_l - p_B}{a_h - a_l}$. Note that S_l, R_h, and R_l intersect at $\hat{I} \equiv \left(\dfrac{p_l}{a_l}; \hat{\gamma} \equiv \dfrac{1}{b}(p_{Gh} - \dfrac{a_h}{a_l} p_l) \right)$. $\hat{\gamma}$ can be understood as the implicit *price per unit of quality of B*, when the quality of *A* is valued at the price set by firm *l*. Note also that R_h , R_b, and

[15] Consumers of $A_l + B$ means consumers of both A_l and B but each component is purchased separately.

S_b intersect at $\left(\frac{p_{Gh}-p_B}{a_h};\frac{p_B}{b}\right)$[16]. The market areas depend on whether $\hat{\gamma}$, the *implicit price per unit of quality* of B is greater, lower or equal to $\frac{p_B}{b}$, the *explicit price per unit of quality* of *B* set by firm *l*. We now distinguish three cases.

Case 1: $p_{Gh} > p_l + p_B$ and $\hat{\gamma} > \frac{p_B}{b}$.

Case 2: $p_{Gh} > p_l + p_B$ and $\hat{\gamma} \leq \frac{p_B}{b}$.

Case 3: $p_{Gh} \leq p_l + p_B$.

In case 1 and case 2 the price of the bundle $A_h B$ is higher than the price of both A_l and B. The difference between the two cases is that in case 1 the *implicit price per unit of quality* of *B is* greater than the *explicit price per unit of quality* of *B* set by firm *l*. While in case 2 the *implicit price per unit of quality* of *B is* lower than the *explicit price per unit of quality* of *B* set by firm *l*. In case 3 the price of the bundle $A_h B$ is lower than the price of both A_l and B.

[16] Service B is sold as part of the bundle $A_h B$. Remark that $\frac{p_B}{b} < \frac{1}{b}(p_{Gh} - \frac{a_h}{a_l} p_l) \equiv \hat{\gamma}$ can be written as: $\frac{p_{Gh}-p_B}{a_h} > \frac{p_l}{a_l}$. The ratio $\frac{p_{Gh}-p_B}{a_h}$ represents the *implicit price per unit of quality* of A_h when it is sold as part of the bundle $A_h B$. Thus market areas depends on whether this *implicit price per unit of quality* is greater, lower or equal to the explicit *price per unit of quality* of A_l .

Case 1: $p_{Gh} > p_l + p_b$ and $\hat{\gamma} > \frac{p_B}{b}$

It is the case where the implicit *price per unit of quality of B* is larger than the *explicit price per unit of quality* of *B*. Figure 7 shows that parameter space divides into five segments[17]

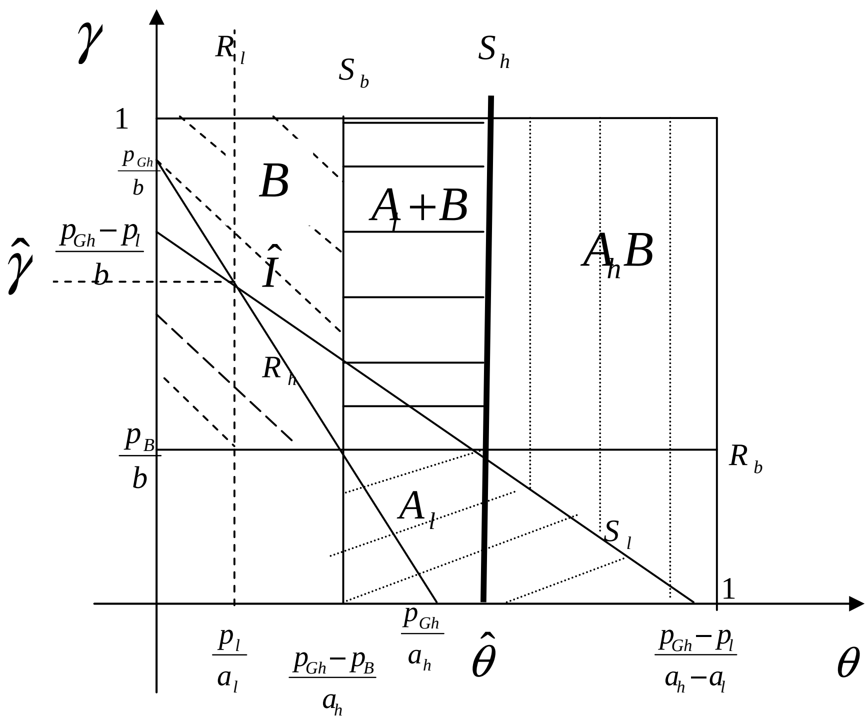

Figure 7: Market areas under the regime (B_h, C_l) when $p_{Gh} > p_l + p_b$ and $\hat{\gamma} > \frac{p_B}{b}$

Consumers with preference parameters to the right of S_h and above S_l purchase the bundle A_hB. The market area of those consumers is $D_{Gh} = 1 - \hat{\theta} - \frac{(p_B)^2}{2b(a_h - a_l)}$. Consumers with preference parameters between S_b and S_l and below R_b purchase A_l alone. The market area of those consumers is $D_{A_l} = \frac{p_B}{b}\left[\hat{\theta} - \frac{p_{Gh} - p_B}{a_h} + \frac{p_B}{2(a_h - a_l)}\right]$. Consumers with preference parameters to the left of S_h, to the right of S_b, and above R_b purchase A_l and *B*. The market area of those

[17] We recall that $\hat{\theta} \equiv \frac{p_{Gh} - p_l - p_B}{a_h - a_l}$ and $\hat{\gamma} \equiv \frac{1}{b}(p_{Gh} - \frac{a_h}{a_l} p_l)$.

consumers is $D_{A_l+B} = \left(\hat{\theta} - \frac{p_{Gh} - p_B}{a_h}\right)\left(1 - \frac{p_B}{b}\right)$. Finally consumers with preference parameter to the left of S_b and above R_b purchase B alone. The market area of those consumers is $D_B = \left(\frac{p_{Gh} - p_B}{a_h}\right)\left(1 - \frac{p_B}{b}\right)$. Others don't purchase.

Case 2: $p_{Gh} > p_l + p_b$ and $\hat{\gamma} \le \frac{p_b}{b}$

It is the case where the implicit *price per unit of quality of B* is lower than the explicit *price per unit of quality* of *B*. Figure 8 shows that parameter space divides into four segments.

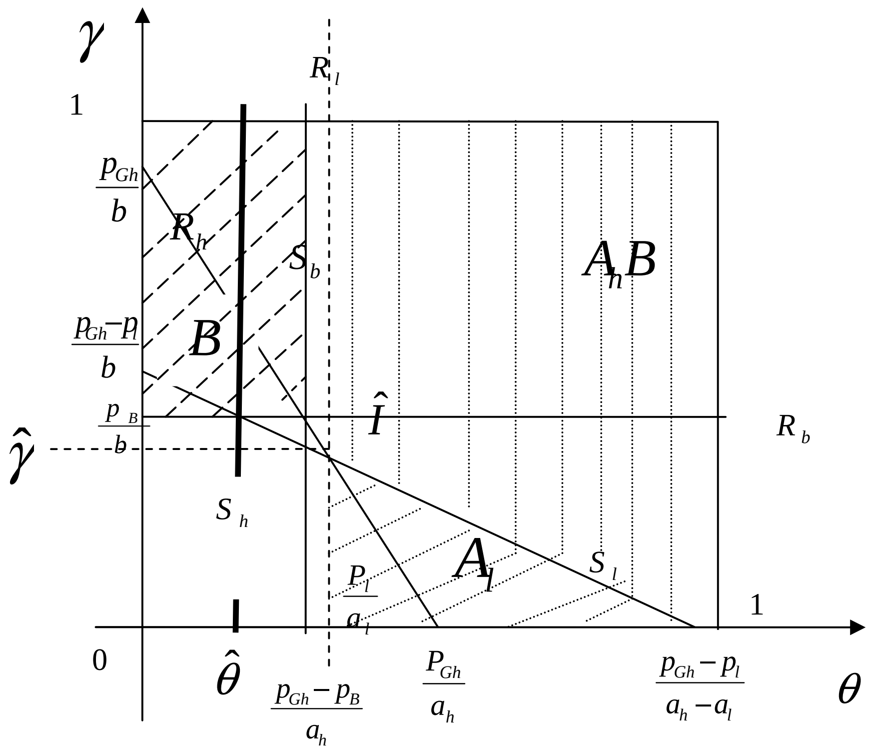

Figure 8: Market areas under the regime (B_h, C_l) when $p_{Gh} > p_l + p_B$ and $\hat{\gamma} \le \frac{p_B}{b}$

The allocation of consumers is as follows: consumers with preference parameters to the right of S_b, above R_h and above S_l purchase $A_h B$. Those with preference parameters to the left of S_b

and above R_b purchase B alone. Finally consumers with preference parameter to the right of R_l and below S_l purchase A_l alone. Others don't purchase.

We see in Figure 8 that $\hat{\theta} < \frac{p_{Gh} - p_B}{a_h}$ and S_b is to the right of S_h. Therefore nobody buys $A_l + B$, that is no consumer purchases B separately when she also purchases A_l [18]. The reason is that the explicit price of B is higher than the implicit price of B when purchased as part in the bundle $A_h B$.

For the particular case where $\hat{\gamma} \equiv \frac{p_B}{b}$, we have $S_h \equiv S_l \equiv R_l$ and the allocation of consumers in the parameter space is the same as above.

[18] Recall that buy $A_l + B$ means buy both A_l and B.

Case 3: $p_{Gh} \leq p_l + p_B$

It is the case where consumers who purchase B separately, also purchase A_l and pay for both services a price higher than the price of the bundle $A_h B$. In this case the demand for both A_l and B is zero since consumers can purchase $A_h B$, that is they can get a better bundle at a lower price. But consumers purchase A_l alone and B alone. Market areas are shown in Figure 9.

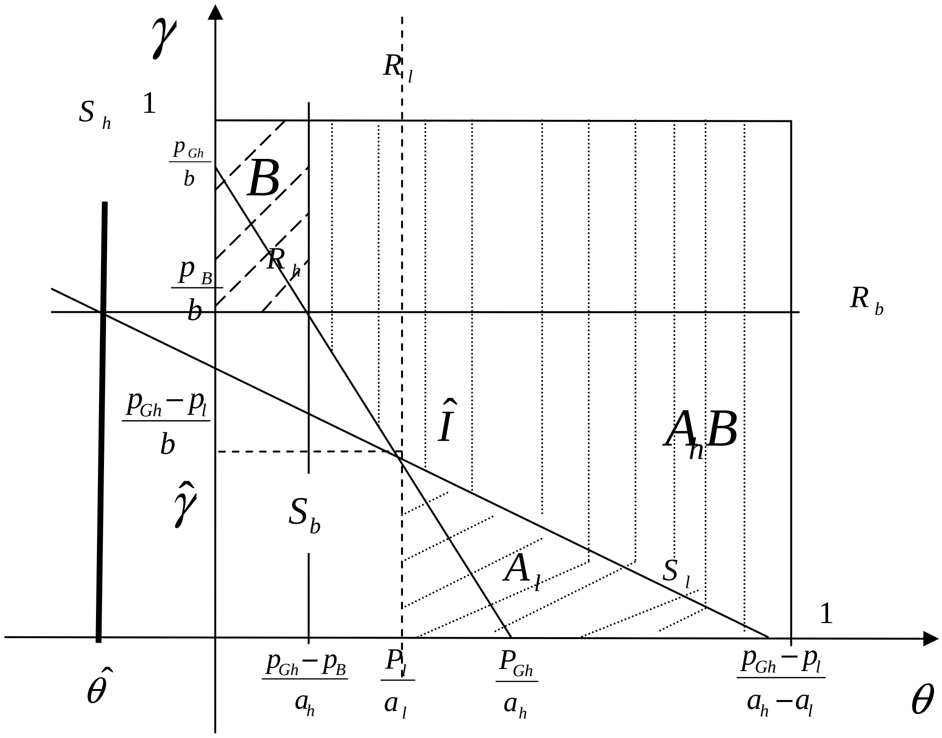

Figure 9: Market areas under the regime (B_h, C_l) when $p_{Gh} \leq p_l + p_B$

Figure 9 depicts a similar pattern as for Figure 3.8. Therefore case 2 and case 3 give the same allocation of consumers in the parameter space.

3.3.1 Determination of the equilibrium prices in (B_h, C_l)

Here also the first order conditions are quite complex. We search the price equilibria numerically for a range of values of a_h and a_l. Figure 10 shows the equilibrium values of $\hat{\gamma}$ and $\frac{p_B}{b}$ for a range of values of a_h[19] in case 1. The equilibrium prices that we obtain always satisfy $\hat{\gamma} > \frac{p_B}{b}$. In case 2 and 3, the equilibrium prices that we obtain do not satisfy $\hat{\gamma} \leq \frac{p_B}{b}$. Therefore, we will only look at case 1.

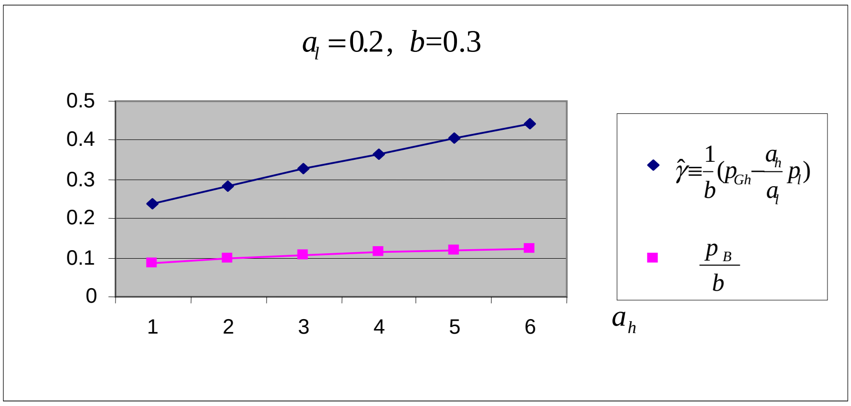

Figure 10: Comparison of prices per unit of quality under the regime of (B_h, C_l)

Note that gap between the explicit price of service *B* and the implicit price of service *B* becomes larger when a_h becomes larger.

[19] The equilibrium condition $\frac{p_B}{b} \geq \hat{\gamma}$ is not satisfied for the parameter values that we have chooses for simulation.

3.4 Pure Components by Firm *h*, Bundling by Firm *l*: (C_h, B_l)

The participation constraints are

for consumers of $A_l B$: $\theta a_l + \gamma b - p_{Gl} \geq 0$, (R_l)

for consumers of A_h : $\theta a_h - p_h \geq 0$, (R_h)

for consumers of B : $\gamma b - p_b \geq 0$, (R_b)

for consumers of $A_h + B$[20] : $\theta a_h + \gamma b - p_h - p_B \geq 0$. (R_m)

The self-selection constraints are

for consumers of $A_l B$: $\theta a_l + \gamma b - p_{Gl} \geq Max(0, \theta a_h + \gamma b - p_h - p_B, \theta a_h - p_h, \gamma b - p_B)$, (S_h)

for consumers of A_h : $\theta a_h - p_h \geq Max(\theta a_l + \gamma b - p_{Gl}; \theta a_h + \gamma b - p_h - p_B; \gamma b - p_B)$, (S_l)

for consumers of B : $\gamma b - p_B \geq Max(\theta a_l + \gamma b - p_{Gl}; \theta a_h + \gamma b - p_h - p_B; \theta a_h - p_h)$, (S_b)

for consumers of $A_h + B$: $\theta a_h + \gamma b - p_h - p_B \geq Max(\theta a_l + \gamma b - p_{Gl}; \gamma b - p_B; \theta a_h - p_h)$. (S_m)

The condition $\theta a_l + \gamma b - p_{Gl} \geq Max(0; \theta a_h + \gamma b - p_h - p_B; \theta a_h - p_h; \gamma b - p_B)$ must be satisfied by buyers of $A_l B$.

The condition $\theta a_h - p_h \geq Max(0; \theta a_l + \gamma b - p_{Gl}; \theta a_h + \gamma b - p_h - p_B; \gamma b - p_B)$ must be satisfied by buyers of A_h alone.

The condition $\gamma b - p_B \geq Max(0; \theta a_l + \gamma b - p_{Gl}; \theta a_h + \gamma b - p_h - p_B; \theta a_h - p_h)$ must be satisfied by buyers of B alone.

And finally $\theta a_h + \gamma b - p_h - p_B \geq Max(0; \theta a_l + \gamma b - p_{Gl}; \theta a_h - p_h; \gamma b - p_B)$ must be satisfied by buyers of both A_h and B.

[20] Consumers of $A_h + B$ means consumers of both A_h and B but each component is purchased separately.

From S_h and R_b, we derive that the preference index of the consumer indifferent between purchasing $A_l B$ and purchasing A_h and B separately is $\overset{\leftrightarrow}{\theta} \equiv \dfrac{p_{Gl} - p_h - p_B}{a_h - a_l}$. Now S_l, R_h, and R_l intersect at $\vec{I} = \left(\dfrac{p_h}{a_h}; \overset{\leftrightarrow}{\gamma} \equiv \dfrac{1}{b}(p_{Gl} - \dfrac{a_l}{a_h} p_h) \right)$. $\overset{\leftrightarrow}{\gamma}$ can be understood as the implicit *price per unit of quality of B*, when the quality of *A* is valued at the price set by firm *h*. Note also that R_l, R_b, and S_b intersect at $\left(\dfrac{p_{Gl} - p_B}{a_l}; \dfrac{p_B}{b} \right)$[21]. The market areas depend on whether this *implicit price* is greater, lower or equal to the explicit *price per unit of quality* of *B* set by firm *h*. There are now four cases to consider that correspond to number of cells in Table 1.[22]

Table 1: Different cases under the regime of (C_h, B_l)

	$\overset{\leftrightarrow}{\gamma} < \dfrac{p_B}{b}$	$\overset{\leftrightarrow}{\gamma} \geq \dfrac{p_B}{b}$
$p_h \leq p_{Gl} < p_h + p_B$	Case 1	Case 2
$p_{Gl} < p_h$	Case 3	Case 4

In all cells except cell 3 we have $\overset{\leftrightarrow}{\theta} < 1$, i.e. the preference parameter of the consumer indifferent between $A_l B$ and both A_h and *B* is lower than one. For cell 3, we consider separately the case where $\overset{\leftrightarrow}{\theta} < 1$ and the case where $\overset{\leftrightarrow}{\theta} \geq 1$.

[21] Also remark that $\dfrac{p_B}{b} < \dfrac{1}{b}(p_{Gl} - \dfrac{a_l}{a_h} p_h) \equiv \overset{\leftrightarrow}{\gamma}$ can be written as: $\dfrac{p_{Gl} - p_B}{a_l} > \dfrac{p_h}{a_h}$. The ratio $\dfrac{p_{Gl} - p_B}{a_l}$ represents the *implicit price per unit of quality* of A_l when it is sold as part of the bundle $A_l B$. Thus market areas depends on whether this *implicit price per unit of quality* is greater, lower or equal to the explicit *price per unit of quality* of A_h.

[22] There is no equilibrium with $p_{Gl} > p_h + p_B$ because nobody buys $A_l B$.

Case 1: $\quad p_h \le p_{Gl} < p_h + p_B$ and $\overset{\leftrightarrow}{\gamma} < \dfrac{p_B}{b}$

Figure 11 shows a segmentation of the space of preference parameters into five segments. $\overset{\leftrightarrow}{\theta} = \dfrac{p_h + p_B - p_{Gl}}{a_h - a_l}$ is derived from S_h.

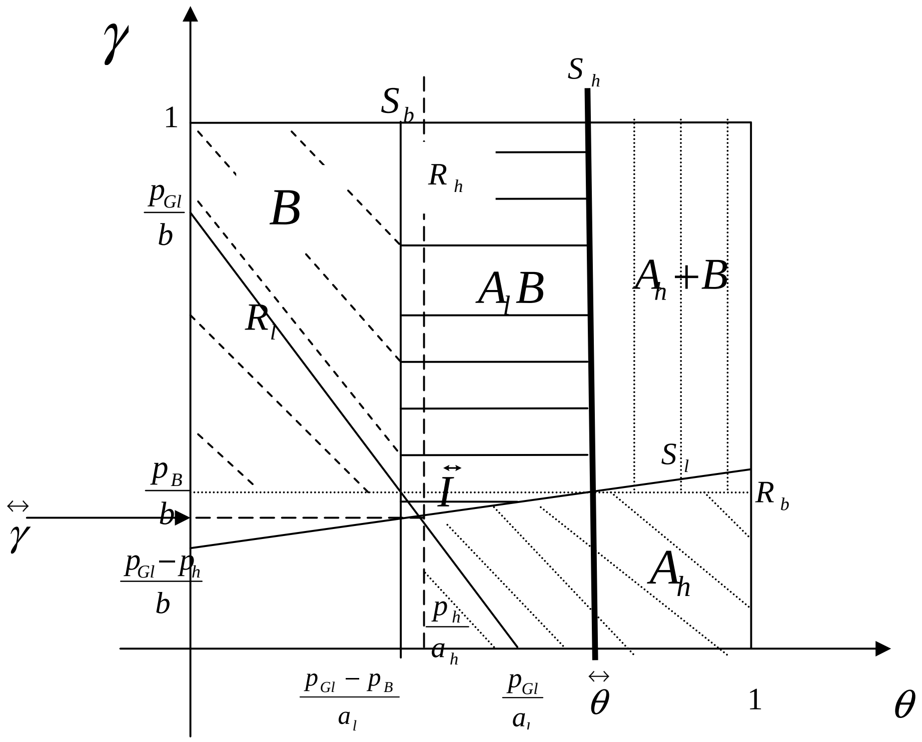

Figure 11: Market areas under the regime (C_h, B_l) when $p_h \le p_{Gl} < p_h + p_B$ and

$$\overset{\leftrightarrow}{\gamma} < \frac{p_B}{b}$$

Consumers with preference parameters to the left of S_h, to the right S_b and above S_l and R_l purchase $A_l B$. The market area of those consumers is:

$$D_{Gl} = (1 - \frac{p_B}{b})(\overset{\leftrightarrow}{\theta} - \frac{p_{Gl} - p_B}{a_l}) + \frac{1}{2}(\frac{p_B}{b} - \overset{\leftrightarrow}{\gamma})(\overset{\leftrightarrow}{\theta} - \frac{p_{Gl} - p_B}{a_l}),\ D_{Gl} = (\overset{\leftrightarrow}{\theta} - \frac{p_{Gl} - p_B}{a_l})(1 - \frac{p_B}{2b} - \frac{\overset{\leftrightarrow}{\gamma}}{2}).$$

Consumers with preference parameters to the right of S_h and above S_b purchase both A_h and B. Consumers with preference parameters to the left of S_b and above R_b, purchase B alone.

Finally consumers with preference parameters to the right of R_h and below S_l purchase A_h only. Demands D_{A_h} for A_h and D_B for B are respectively:

$$D_{A_h} = 1 - \overset{\leftrightarrow}{\theta} + \frac{1}{2}(\overset{\leftrightarrow}{\theta} - \frac{p_h}{a_h})(\frac{p_B}{b} + \overset{\leftrightarrow}{\gamma}), \text{ and } D_B = (1 - \frac{p_b}{b})(1 - \overset{\leftrightarrow}{\theta} + \frac{p_{Gh} - p_b}{a_l}).$$

The others consumers do not purchase.

Case 2: $p_{Gl} < p_h$ and $\overset{\leftrightarrow}{\gamma} < \frac{p_B}{b}$

When $\overset{\leftrightarrow}{\theta} < 1$

Figure 12 displays also a segmentation of the space of preference parameters into five segments. We obtain the same segmentation of preference parameters space than in case 1. Therefore demands in case 2 when $\overset{\leftrightarrow}{\theta} < 1$ are similar to demands in case 1.

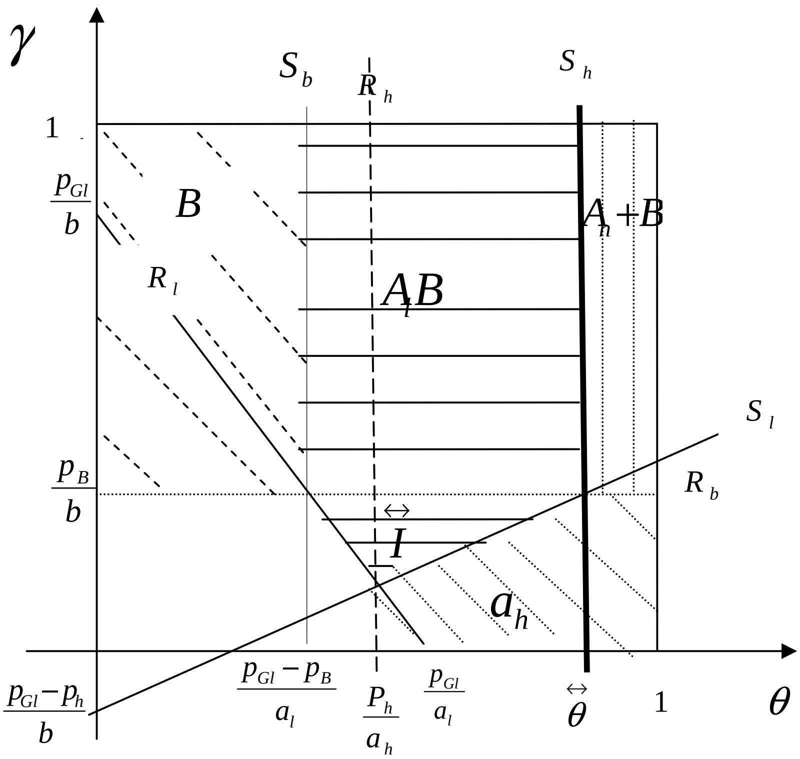

Figure 12: Market areas under the regime (C_h, B_i) when $p_{Gl} < p_h$, $\overset{\leftrightarrow}{\gamma} < \frac{p_B}{b}$ and $\overset{\leftrightarrow}{\theta} < 1$.

When $\overset{\leftrightarrow}{\theta} \geq 1$

Figure 13 shows a segmentation of the space of preference parameters into three segments.

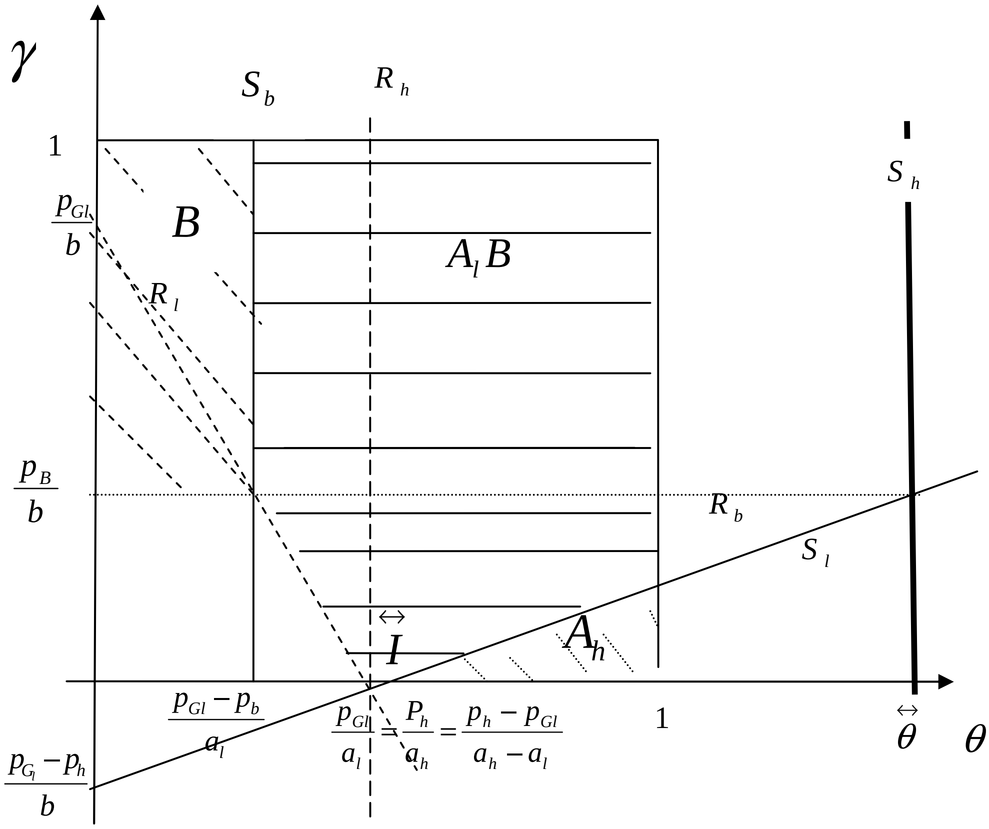

Figure 13: Market areas under the regime (C_h, B_l) when $p_{Gl} < p_h$, $\overset{\leftrightarrow}{\gamma} < \frac{p_B}{b}$ and $\overset{\leftrightarrow}{\theta} \geq 1$

Consumers with preference parameters to the right of S_b, above R_l and above S_l purchase $A_l B$. Those with preference parameters to the left of S_b and above R_b , purchase B alone. Consumers with preference parameters below S_l purchase A_h only, to the right S_b and above S_l and R_l purchase $A_l B$. Others don't purchase. No one purchases both A_h and B.

Case 3: $p_h \leq p_{Gl} < p_h + p_B$ and $\overset{\leftrightarrow}{\gamma} \geq \frac{p_B}{b}$

Figure 14 shows segmentation of the space of preference parameters into four segments.

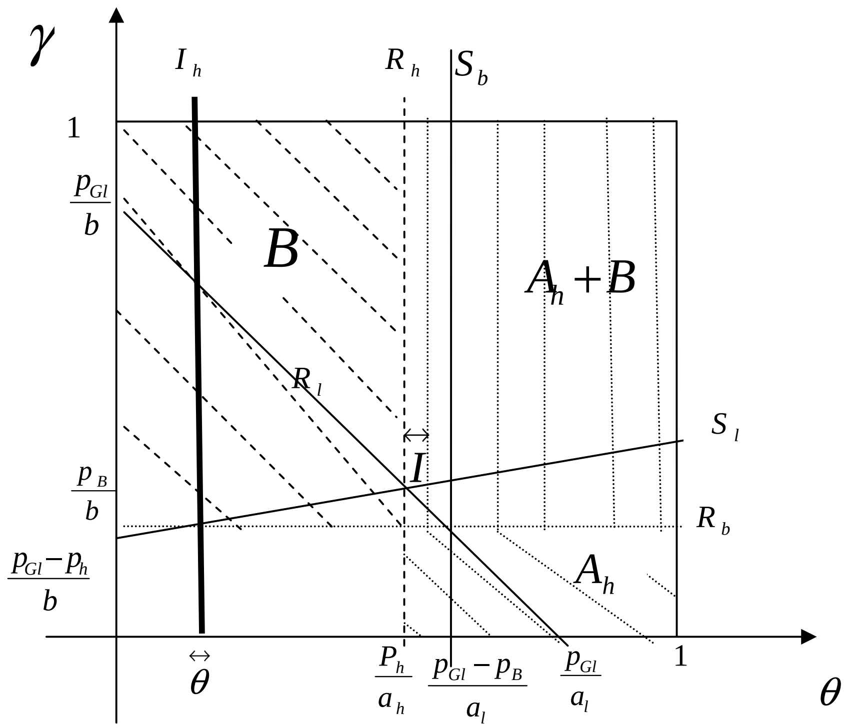

Figure 14: Market areas under the regime (C_h, B_l) when $p_{Gl} < p_h + p_B$, $\overset{\leftrightarrow}{\gamma} \geq \frac{p_B}{b}$ and $p_{Gl} > p_h$

Consumers with preference parameters to the right R_h and above R_b purchase both A_h and B. Those with preference parameters to the left of R_h and above R_b , purchase B alone. Consumers with preference parameters to the right R_h and below R_b purchase A_h only. Others don't purchase. Demand for $A_l B$ is zero.

Case 4: $p_{Gl} < p_h$ and $\overset{\leftrightarrow}{\gamma} \geq \frac{p_B}{b}$

Figure 15 depicts the same segmentation of preference parameters space than in case 3. Therefore demands in cases 4 are similar to demands in case 3.

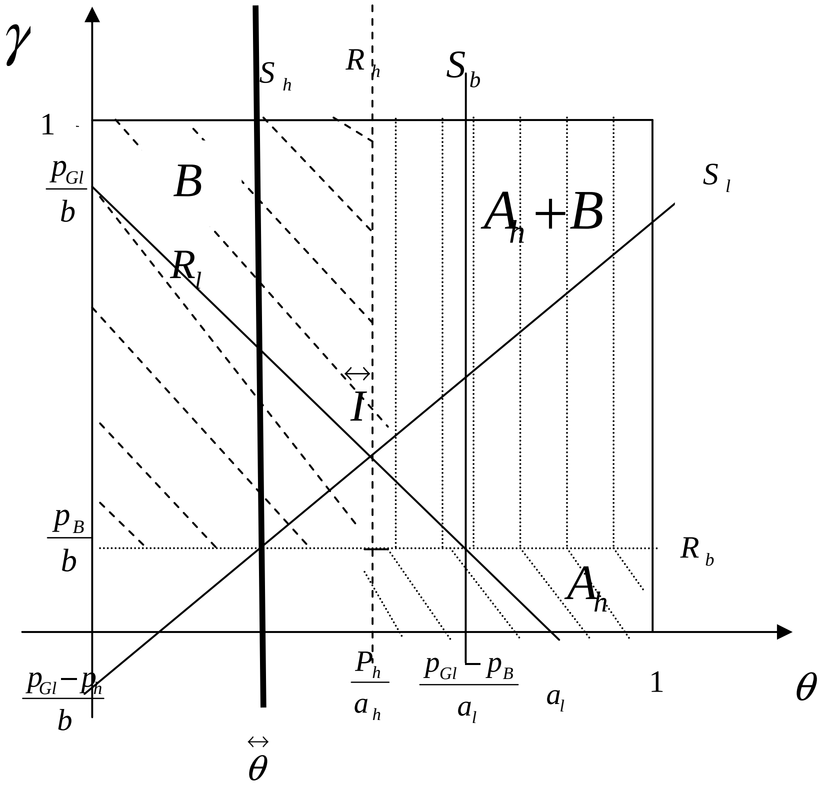

Figure 15: Market areas under the regime (C_h, B_l) when $p_{Gl} < p_h$ and $\overset{\leftrightarrow}{\gamma} \geq \frac{p_B}{b}$

3.4.1 Determination of the equilibrium prices in (C_h, B_l)

We note that there is no duopoly equilibrium with $\overset{\leftrightarrow}{\gamma} \geq \frac{p_B}{b}$ because this condition entails zero sales by the low quality firm. Therefore, the remaining case to analyze is the case where $\overset{\leftrightarrow}{\gamma} < \frac{p_B}{b}$ and $\overset{\leftrightarrow}{\theta} < 1$ or $\overset{\leftrightarrow}{\theta} \geq 1$. So the question is what subcase constitutes a Nash equilibrium in prices?

As in the previous sections, analytical difficulties lead us to search the price equilibria numerically for a range of values of a_h and a_l. Figure 16 displays the value of $\overset{\leftrightarrow}{\gamma}$ and $\frac{p_B}{b}$ for a range of values of a_h[23]. We find that the only equilibrium prices that we ever get always are consistent with $\overset{\leftrightarrow}{\gamma} < \frac{p_B}{b}$. Also at equilibrium $\overset{\leftrightarrow}{\theta} < 1$. Therefore we look only at case 1 when $\overset{\leftrightarrow}{\theta} < 1$.

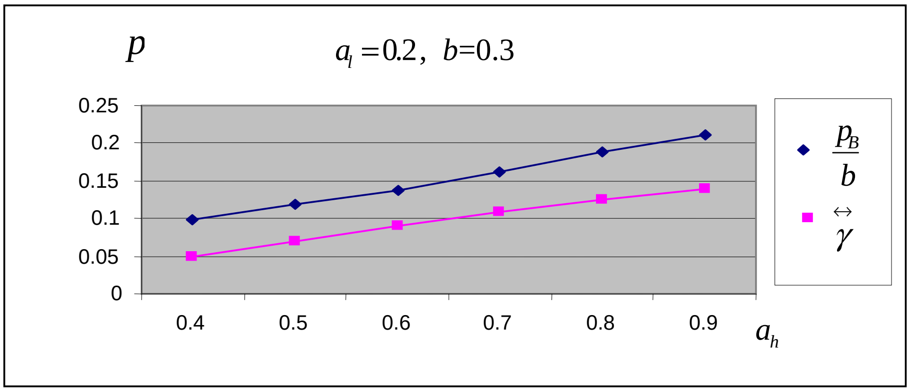

Figure 16: Equilibrium characteristic of the subgame (C_h, B_l)

Here also the gap between the explicit price of service *B* and the implicit price of service *B* becomes larger when a_h becomes larger.

[23] The equilibrium condition $\overset{\leftrightarrow}{\gamma} \geq \frac{p_B}{b}$ is not satisfied for the parameter values that we have chooses for simulation.

4. Equilibrium Strategy of the Game

We now compare profit under each of the four possibilities. We let a_h and a_l vary in the interval $[0.2,+\infty[$. Figure 17 shows firms' profit in each possibility. We always find that:

$$\pi_h^{B_h,B_l} \geq \pi_h^{C_h,B_l} \geq \pi_h^{B_h,C_l} \geq \pi_h^{C_h,C_l}$$

$$\pi_l^{B_h,B_l} \geq \pi_l^{B_h,C_l} \geq \pi_l^{C_h,B_l} \geq \pi_l^{C_h,C_l}$$

We conclude that (B_h, B_l) is an equilibrium in dominant strategy.

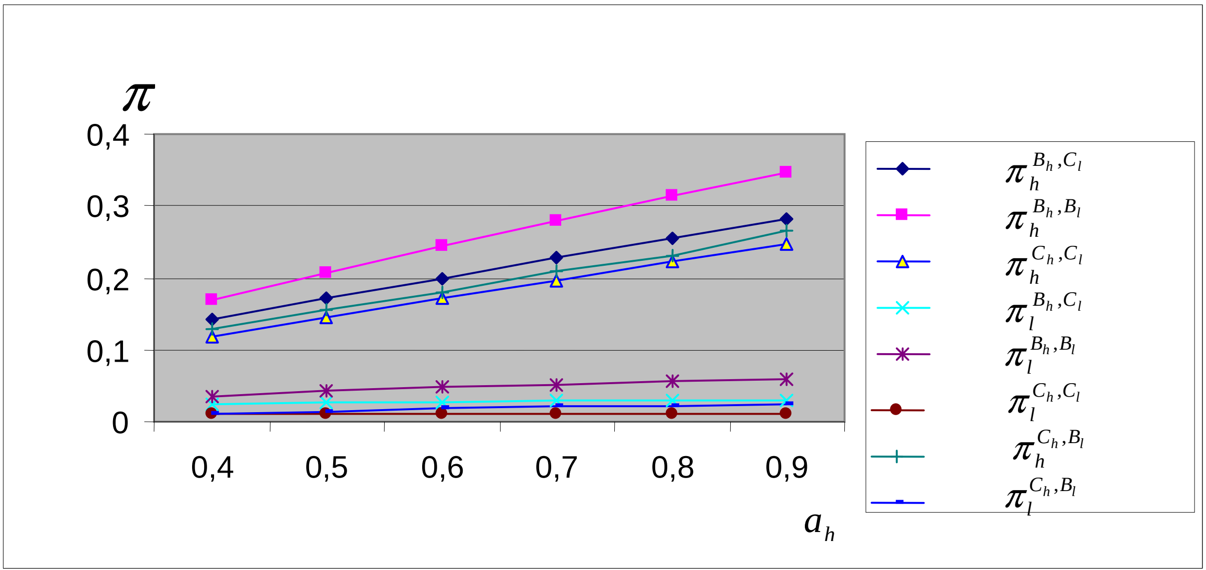

Figure 17: Equilibrium Strategy of the Game

Thus we state the following result:

(i) *Pure bundling is a dominant strategy equilibrium for both firms.*

Bundling is a dominant strategy for both firms because it reduces the intensity of the competition between the two firms by increasing the differentiation of services. Therefore firms' profits are higher under (B_h, B_l) than under the other subgames where one of two firms at least sells its services separately.

5. Welfare Implications

Now let us see how consumers' surplus and social welfare are affected when the regime shifts from (C_h, C_l) to (B_h, B_l).

For (C_h, C_l), consumers' surplus denoted by CS^S is:

$$CS^C = \int_{\tilde{\theta}}^{1} (\theta a_h - P_h) d\theta + \int_{\bar{\theta}}^{\tilde{\theta}} (\theta a_l - P_l) d\theta + \int_{0}^{1} (\gamma - 0) d\gamma .$$

The first, second and third expression of CS^S are respectively the surplus of purchasers of A_h, A_l and B. We obtain:

$$CS^C = a_h (\frac{1-\tilde{\theta}^2}{2}) - p_h (1-\tilde{\theta}) + a_l (\frac{\tilde{\theta}^2 - \bar{\theta}^2}{2}) - p_l (\tilde{\theta} - \bar{\theta}) + \frac{1}{2}$$

The social welfare denoted by SW^C gives:

$$SW^C = CS^C + \pi_h^{C_h, C_l} + \pi_l^{C_h, C} .$$

For (B_h, B_l), the surplus of consumers of $A_h B$ denoted by CS_h^B is:

$$CS_h^B = \int_{\theta^*}^{\frac{p_{Gh}}{a_h}} \int_{p_{Gh} - \theta a_h}^{1} (\theta a_h + \gamma - p_{Gh}) d\gamma d\theta + \int_{\frac{p_{Gh}}{a_h}}^{1} \int_{0}^{1} (\theta a_h + \gamma - p_{Gh}) d\gamma d\theta$$

$$CS_h^B = \frac{a_h}{2} + (1 + \theta^* - 2\frac{p_{Gh}}{a_h})(\frac{1}{2} - p_{Gh}) - a_h \frac{(\theta^*)^2}{2} + \frac{(a_h \theta^* - p_{Gh})^3}{6 a_h}$$

While the surplus of consumers of $A_l B$ is:

$$CS_l^B = \int_{\gamma^*}^{p_{Gl}} \int_{\frac{p_{Gl} - \gamma}{a_l}}^{\theta^*} (\theta a_l + \gamma - p_{Gl}) d\theta d\gamma + \int_{p_{Gl}}^{1} \int_{0}^{\theta^*} (\theta a_l + \gamma - p_{Gl}) d\theta d\gamma$$

$$CS_l^B = \frac{\theta^*}{2} \left[1 + (p_{Gl})^2 - 2\gamma^* + a_l (1 - \gamma^*) \right] - \frac{(\gamma^* - p_{Gl})^3}{6 a_l}$$

Thus, consumers' surplus under (B_h, B_l) is:

$$CS^B = CS_h^B + C_l^B$$

The social welfare under (B_h, B_l) denoted by SW^B is:

$$SW^B = CS^B + \pi_h^{B_h,B_l} + \pi_l^{B_h,B_l}$$

We now compare social welfare under (C_h, C_l) and (B_h, B_l) for a_h and a_l varying in the interval $[0.2,+\infty[$. Figure 18 displays the total economic welfare under (C_h, C_l) and (B_h, B_l). It shows that:

$$SW^B > SW^C.$$

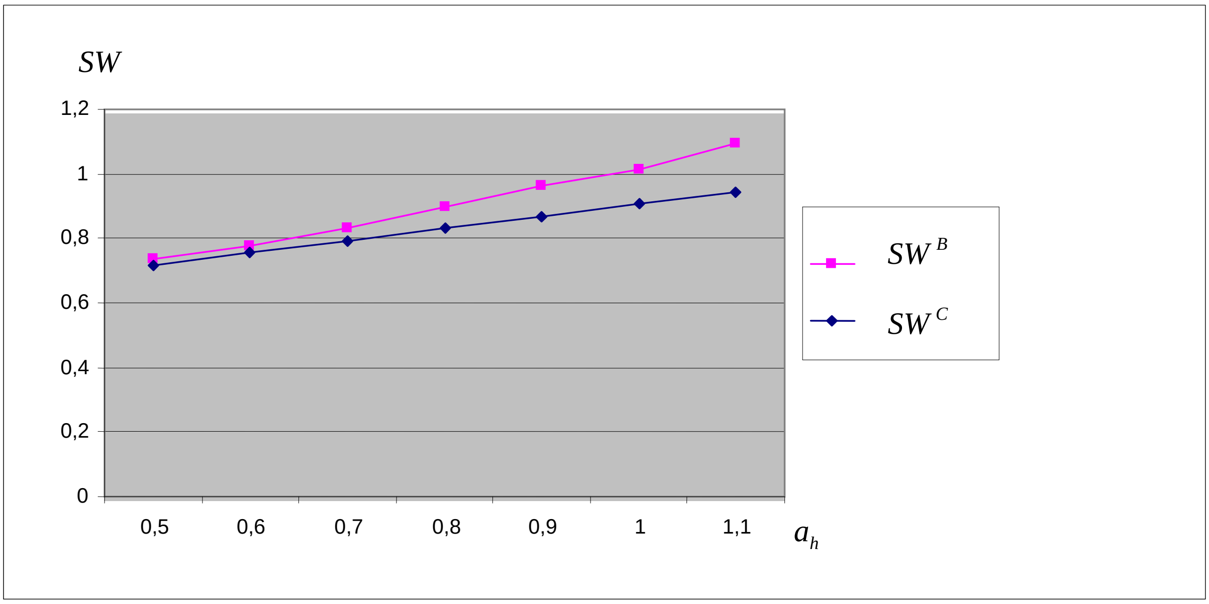

Figure 18: Total welfare under the regimes of (C_h, C_l) and (B_h, B_l)

Thus we state the following result:

(ii) *A shift from (C_h, C_l) to (B_h, B_l) results in a decrease in social welfare when there is a small differentiation between services.*

It is obvious that bundling in this context reduces consumers'surplus. Indeed bundling increases prices and there are more constraints under the regime of (B_h, B_l) because to obtain *B* consumers must purchase $A_l B$ even though they don't want *A*. So for welfare to increase, aggregate profits must raise enough to offset the reduction of aggregate consumer surplus and then result in a potential efficiency gain. For all the parameters we have chooses we find that total welfare increase.

6. Concluding Remarks

In the traditional literature on bundling by duopolists, the conclusion is bundling is a dominant strategy equilibrium for both firms but it is not a profitable strategy for both firms. We find also that bundling is a dominant strategy equilibrium. But contrary to other studies, we find that bundling is a profitable strategy for both firms. The reason is that in the context of vertically differentiated services, bundling can be used as differentiation tool. It then reduces the intensity of competition between the firms and then they make more profits under bundling than they would under component selling. We also find that bundling increases total welfare.

Cable operators and telecommunication companies offer the combination of telephone, television, and Internet as a bundled service. Our results suggest that they would compete more vigorously and would realize less profit if there were restriction on bundling. If buyers care for the quality of services their surplus is reduced under bundling. However, the fact that a single supplier offers all services makes bundling convenient for buyers. So the benefit of the convenience must be balanced with the reduction of consumers' surplus to obtain the effects of bundling on buyers' welfare. Also authorities must decide what weight they give to the buyers' surplus and to the suppliers' profits to obtain the net welfare effects of bundling.

Our result should be interpreted under the assumptions that both the variable cost and the fixed cost are zero for each service and services are vertically differentiated. It would be interesting to analyze the case where the fixed costs are positive. Also since we know that firms in communication markets are both horizontally and vertically differentiated, we can study both differentiations to see how the results of this paper are robust to these assumptions.

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