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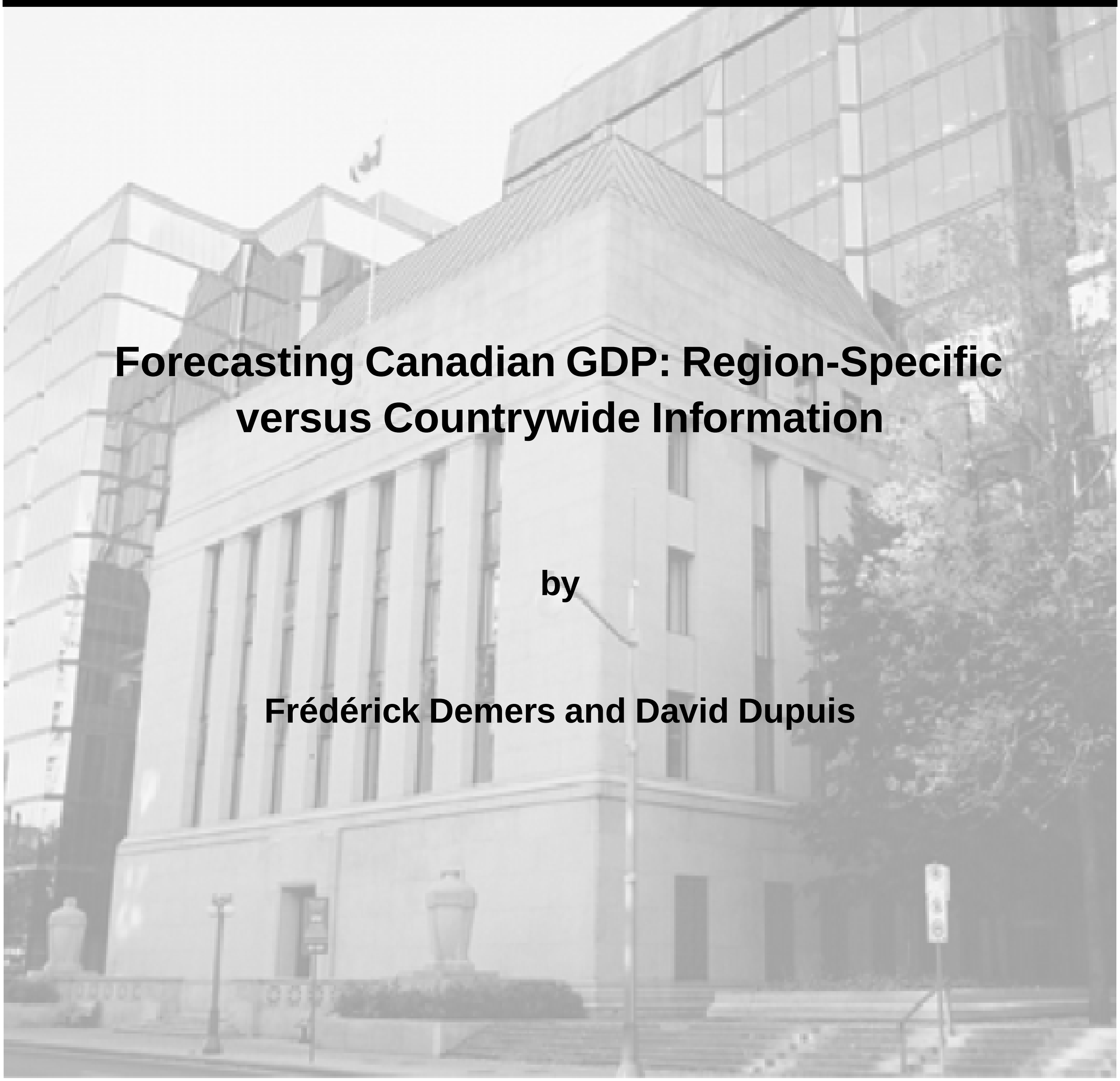

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Abstract

The authors investigate whether the aggregation of region-specific forecasts improves upon the direct forecasting of Canadian GDP growth. They follow Marcellino, Stock, and Watson (2003) and use disaggregate information to predict aggregate GDP growth. An array of multivariate forecasting models are considered for five Canadian regions, and single-equation models are considered for direct forecasting of Canadian GDP. The authors focus on forecasts at 1-, 2-, 4-, and 8-quarter horizons, which best represent the monetary policy transmission framework of long and variable lags. Region-specific forecasts are aggregated to the country level and tested against aggregate country-level forecasts. The empirical results show that Canadian GDP growth forecasts can be improved by indirectly forecasting the GDP growth of the Canadian economic regions using a multivariate approach, namely a vector autoregression and moving average with exogenous regressors (VARMAX) model.

JEL classification: E17, C32, C53
Bank classification: Econometric and statistical methods

Résumé

Les auteurs cherchent à déterminer si l'agrégation de prévisions régionales permet d'améliorer la prévision de la croissance du PIB canadien. Suivant l'exemple de Marcellino, Stock et Watson (2003), ils utilisent des données désagrégées pour prédire la croissance globale du PIB. Ils appliquent une série de modèles multivariés à la prévision du PIB de cinq régions canadiennes et des modèles à une équation à celle du PIB national. Les horizons de un, deux, quatre et huit trimestres retenus par les auteurs correspondent bien aux décalages longs et variables qui caractérisent le mécanisme de transmission de la politique monétaire. Les prévisions régionales sont regroupées à l'échelle nationale, puis comparées à celles obtenues pour l'ensemble du pays au moyen des données globales. D'après les résultats empiriques, il est possible d'améliorer les prévisions de l'évolution du PIB canadien en déterminant indirectement la progression future du PIB des régions économiques du Canada à l'aide d'un modèle vectoriel autorégressif multivarié à moyenne mobile et à régresseurs exogènes (VARMAX).

Classification JEL : E17, C32, C53
Classification de la Banque : Méthodes économétriques et statistiques

1 Introduction

A body of literature has developed to investigate whether the aggregation of component-specific forecasts improves upon the direct forecast of an aggregate variable. In general, a disaggregated approach is of interest when it is suspected that different components of the aggregate respond asymmetrically to shocks. From a forecasting point of view, the pooling of disaggregated forecasts should help reduce the variance of the forecast errors, because the potential heterogeneity can be better captured. In the case of the Canadian economy, one could think that heterogeneity exists across regions, especially in the event of shocks. An unexpected increase in oil prices is a good example of where different responses might arise. In such instances, the various Canadian regions are expected to fare differently: the Prairies would be expected to benefit from Alberta's position as a petroleum net exporter, whereas Ontario and Quebec, which rely on external suppliers for oil, should, all things equal, experience a slowdown in economic activity. Examples like this in a number of studies suggest that both supply and demand shocks have important asymmetric effects on output across the Canadian regions. For example, Poloz (1990), Bayoumi and Eichengreen (1993), DeSerres and Lalonde (1994), Dupasquier, Lalonde, and St-Amant (1997), and Beine and Coulombe (2003) demonstrate that the regional diversity in the structure of the Canadian economy has led to a relatively asymmetrical transmission of external shocks.[1]

Marcellino, Stock, and Watson (2003) argue that forecasts of the Economic and Monetary Union's (EMU's) output and prices series can be improved by indirectly forecasting country-specific subaggregates rather than by forecasting directly the EMU aggregates.[2] Zellner and Tobias (2000) also report improvements in forecast accuracy; they compare aggregate and disaggregate approaches to forecast the median growth rate for the GDP of 18 industrialized countries. When examining core inflation in the context of forecasting under contemporal aggregation, Demers and De Champlain (2005) and Hubrich (2005) find that forecast accuracy can be improved when indirectly forecasting aggregate inflation by means of its subindices. While some studies find some improvement in accuracy, the practice of disaggregation has limits, as Granger (1990) points out. In particular, if the quality of the data deteriorates with disaggregation, or if proper modelling of the data-generating process becomes tedious, the pooled forecasts can become less accurate than forecasts derived from an aggregate model. Taking all this into account, an approach that would pool forecasts from region-specific components of GDP should appear to have some potential benefits in

[1]Nevertheless, asymmetrical responses to external shocks seem to be less important among Canadian regions than between Canada and the United States.

[2]For more on inflation forecasting, see Albacete, Espasa, and Senra (2002).

the Canadian context, at least for short horizons.[3]

Given the heterogeneous nature of the various regional responses to macroeconomic shocks, we investigate in the Canadian context whether the use of pooled region-specific forecasts improves upon the forecasting performance of countrywide Canadian GDP.[4] For this purpose, we consider an array of forecasting models that explicitly exploit the dynamic correlation of five Canadian economic regions.[5] Our models build both on the usual time-series concept by using multivariate specifications and, more importantly, on spatiotemporal principles by explicitly modelling the inter-regional dependencies that reflect the fact that Canadian regions have strong economic links. The possibility that innovations from the Ontario equation are causal to the other regions is also considered, thereby leading to the general specification of the vector autoregression and moving average with exogenous regressors (VARMAX) class.

We focus on forecasts at 1-, 2-, 4-, and 8-quarter horizons, which best represent the monetary policy transmission framework of long and variable lags. Comparable forecasting models are constructed for each of the five regions and the resulting forecasts are aggregated to the country level. Then, after selecting the most accurate specification to forecast aggregate GDP growth indirectly, we compare the forecast accuracy of the best disaggregate models against the forecast accuracy of the best aggregate model.

The empirical results show that the variance of the one-step-ahead forecast error can be significantly reduced by forecasting GDP growth indirectly via the five Canadian economic regions. For longer horizons, the variance of the aggregate forecast errors is not significantly different than the variance of the disaggregated forecast errors; nevertheless, the variance of the forecast errors for the disaggregated models is systematically less than that of the best aggregate models.[6] Our results show that the most accurate forecasts for Canadian GDP growth are obtained using a VARMAX process.

The remainder of this paper is organized as follows. Section 2 discusses the concept of spatial aggregation of time series. Section 3 describes some practical data issues. Section 4 develops the empirical models. Section 5 presents the pseudo out-of-sample exercise and discusses the empirical results. Section 6 tests for the equality of the forecast accuracy. Section 7 offers some conclusions.

[3] Of course, as the forecast horizon increases, most macroeconomic time series become unpredictable (see Galbraith 2003).

[4] In using the term GDP, we refer to real GDP at all times.

[5] The five Canadian economic regions comprise the Atlantic provinces; Quebec; Ontario; the Prairie provinces, Nunavut, and Northwest Territories; and British Columbia and Yukon.

[6] In the longest considered horizon, the variance of the eight-step-ahead forecast error for the best disaggregate model is at least 7 per cent smaller than that of the best aggregate model.

2 Regional Analysis and Aggregation: A Brief Overview of Issues

In the econometrics of regional analysis, spatially dependent time series are generally involved.[7] Over the years, the analysis of spatial time series has produced a considerable body of literature; Anselin (1988), Anselin and Florax (1995), Elhorst (2001), and, more recently, Giacomini and Granger (2004) provide an excellent review of the fundamental concepts and some recent developments.

While time-series analysis is usually concerned with the unique dimension of time, many economic data display a spatial dimension as well; examples are the Canadian and American GDP measures, which are the sum of provinces and states' GDP, respectively. Of course, economic units, when defined at the regional level, whether they are parts of units such as, say, countries or continents, are virtually never orthogonal to each other. Economic regions today are often highly integrated and enjoy relatively important trade flows. Not only do they share explicit structural common factors—e.g., geographical proximity, and a common industrial and agricultural base—but shocks often propagate across regions due to various diffusion mechanisms peculiar to modern economies—e.g., financial markets integration and rule of law.

The approach adopted in this paper draws on the idea of forecasting aggregate time processes developed in Lütkepohl (1987) and, to a lesser extent, on the results related to space-time relations summarized in Giacomini and Granger (2004). As the latter explain, space-time processes can be expressed as usual unrestricted vector autoregression (VAR) processes or, equivalently, they can be estimated in the context of panel-data methods such as the seemingly unrelated regressions (SURE). Of course, as the number of cross-sectional units grows and, more importantly, as the structure of the temporal and spatial dependence involves long lags, the estimation of the system becomes cumbersome and the "curse of dimensionality" renders estimation results, at best, sensitive and inefficient. In the worst situation, estimation is simply not feasible either by least squares or maximum likelihood, because of identification problems.

Traditionally, most econometricians have directly forecast the aggregate time series itself, thereby ignoring part of the available information. It has been shown in the literature, however, that making use of the disaggregate information, under general conditions, is theoretically preferable, since it can lead to more efficiency (Lütkepohl 1987; Granger 1990; Giacomini and Granger 2004). In particular, Giacomini and Granger (2004) show that ig-

[7] Principles of spatial analysis were introduced in econometric theory during the early 1950s; the first formal comprehensive reviews of spatiotemporal theoretical results are provided in Cliff and Ord (1973) and Cliff et al. (1975).

noring even relatively weak spatial dependence can be costly. Obviously, if the quality of the data deteriorates with disaggregation, or if proper modelling of the data-generating process becomes tedious and/or time consuming,[8] forecasts from disaggregate models can become less accurate than forecasts derived from an aggregate model.

Practitioners in the regional field of economics, however, seldom account for spatial dependency, because of the computational burden that is involved in manipulating large information sets. To overcome the dimensionality problem and to keep the estimation of the models feasible, most studies involving spatial econometric analysis have used a weighting scheme based on geographical distance in order to reduce the number of interrelations to estimate (Elhorst 2001). The weights, W, are treated as known and are often set in an arbitrary fashion, thereby leading to important consequences on inference, because the structure of the spatial dependence is then a nuisance (Hallin, Lu, and Tran 2005). In this paper, we do not directly estimate weights; rather, we reduce the dimensionality of the system by first creating homogeneous regional subaggregates and then conditioning a particular region's output growth on a measure of GDP growth for the rest of the country, which we denote as $y_t^{x,i}$.[9] Although a comparison of various schemes for W in an unrestricted version could help uncover more precise inter-regional dependencies, the practical restriction we advocate has tangible merits: it reduces the number of free parameters to estimate and does not impose constraints related to W. It also drops the need to explicitly specify the weight matrix, because it is part of a composite parameter. Meanwhile, we are still accounting for inter-regional correlations by conditioning on $y_t^{x,i}$. We can thus say that our approach lies somewhere between a complete specification of a regional model (Lütkepohl 1987) and a formal spatial model (Giacomini and Granger 2004).

3 Overview of the Data

In Canada, GDP estimates for the ten provinces are produced by the Conference Board of Canada.[10] The data, which are based on Statistics Canada estimates, are reported in the Conference Board's provincial outlook (Conference Board 2005). Output per provinces varies substantially, with Ontario representing more than 40 per cent of Canada's GDP but

[8] For example, if the linearity hypothesis is rejected by the disaggregate data, estimating non-linear models for the micro relations is not trivial.

[9] The subaggregation scheme has the advantage that it fits the regional representation framework put in place by the Bank of Canada, thereby providing another analytical tool for the Bank's regional offices.

[10] The Conference Board of Canada is the only body that produces consistent and comparable quarterly provincial data sets. The Institut de la Statistique du Québec and the Ontario Ministry of Finance produce, on a more timely basis, estimates for their respective regions that are compatible with Statistics Canada's national accounts (Statistics Canada 2005), but they do not provide comparable data for other regions.

Prince Edward Island less than 1 per cent. Researchers who study Canadian regions have often followed a practical subaggregation scheme by grouping the four Atlantic provinces (New Brunswick, Nova Scotia, Newfoundland and Labrador, and Prince Edward Island) together, to create a *homogeneous* regional subaggregate referred to as the Atlantic region. Similarly, the three provinces located between Ontario and British Columbia (Manitoba, Saskatchewan, and Alberta, along with Nunavut and the Northwest Territories) are grouped together and are referred to as the Prairies region. In the end, we have five *homogeneous* regional subaggregates, which, to a certain extent, share many similar economic endowments.

Figure 1 plots quarterly GDP growth for Canada and its five regions. Selected statistics are also provided in Table 1. It would appear that the calculated means and variances differ from one region to another. For example, the fastest-growing region is British Columbia, with an average growth rate of 4.4 per cent (Q/Q per cent), over the period 1961 to 2004. The growth profile, however, is also the most volatile over the period, with a variance of about 65. Conversely, the weakest growth, on average, is observed in Quebec, with 3.2 per cent. Quebec displays the smoothest profile for growth, having a variance of about 16, which is a quarter of the variance observed in British Columbia.

The Conference Board's data set is reviewed every quarter, to keep it in line with the most recent national accounts data. Also included in the set of explanatory variables are the U.S. GDP; the interest rate spread, as defined by the difference between the 3-month prime corporate paper rate and the Government of Canada long-bond rate;[11] the real Canada-U.S. bilateral exchange rate; the Bank of Canada's real commodity price index; the Bank's real energy price index; the Bank's real non-energy commodity price index; and the Canadian GDP growth, excluding the GDP growth of the region under investigation.

A unit root was tested for by means of the augmented unit root test of Said and Fuller (1984). Results show that all variables, except the yield spread, can be considered as I(1) processes. All variables (except the yield spread) are therefore expressed in the first-difference of the log (multiplied by 100).

The variables used in this study are defined as follows (see Appendix A for details):

[11] As defined by bonds with maturity in excess of 10 years.

Y_t :	Canadian GDP level
Y_{it} :	Regional GDP level
y_t^C :	Canadian GDP growth
y_t^{US} :	U.S. GDP growth
y_t^{BC} :	British Columbia GDP growth
y_t^{PR} :	Prairies GDP growth
y_t^{ON} :	Ontario GDP growth
y_t^{QC} :	Quebec GDP growth
y_t^{AT} :	Atlantic GDP growth
s_t :	Interest rate spread
e_t :	Real exchange rate growth
p_t^c :	Real commodity price (BCPI) growth
p_t^e :	Real price of energy products (BCENER) growth
p_t^n :	Real price of non-energy products (BCNE) growth
$y_t^{x,i}$:	Canadian GDP growth excluding the ith region

Figure 2 plots the exogenous variables used in our specifications.

4 Forecast Methods

4.1 Aggregate Canadian GDP models

Typically, GDP growth is modelled within an IS-curve framework and, compared with simple autoregressive moving-average (ARMA) models, provides more accurate forecasts.[12] Canada's output growth is set within this framework to be dependent upon the past stance of monetary policy, foreign demand, foreign exchange, and commodity price conditions. Of course, the IS-curve relation does not hold exactly, being a mere approximation of the data-generating process, and forecasters and policy-makers are always faced with tremendous uncertainty regarding the future economic growth of a country.[13]

To forecast Canadian GDP growth, we consider very standard specifications based on the theory of the IS curve. The aggregate specification has the following general form:

$$y_t^C = \mu + \beta(L)x_t + \theta(L)v_t, \tag{1}$$

[12] See Duguay (1994) and Demers (2004) for reviews.

[13] The performance of the IS-curve framework would also appear to vary across regions.

where μ is simply the intercept; x_t is the information set, which includes lagged values of the scalar process, y_t^C, and some set of exogenous variables, with coefficient vector $\beta(L)$, such that $\beta(L) = \beta_1 L^1 + ... + \beta_p L^p$, with p being the lag length.[14] The Gaussian innovations, v_t, are also allowed to be described by an MA(q) process, such that $\theta(L)$ is a polynomial matrix in the lag operator, with $\theta(L) = 1 + \theta_1 L + ... + \theta_q L^q$. For the disaggregate models, a total of eight different information sets are designed to forecast y_t^C:

$$
\begin{aligned}
&A1\text{:}\ y_t^C = f(y_{t-j}^C, y_{t-j}^{US}, s_{t-j}),\\
&A2\text{:}\ y_t^C = f(y_{t-j}^C, y_{t-j}^{US}, s_{t-j}, e_{t-j}),\\
&A3\text{:}\ y_t^C = f(y_{t-j}^C, y_{t-j}^{US}, s_{t-j}, p_{t-j}^c),\\
&A4\text{:}\ y_t^C = f(y_{t-j}^C, y_{t-j}^{US}, s_{t-j}, e_{t-j}, p_{t-j}^c),\\
&A5\text{:}\ y_t^C = f(y_{t-j}^C, y_{t-j}^{US}, s_{t-j}, p_{t-j}^e),\\
&A6\text{:}\ y_t^C = f(y_{t-j}^C, y_{t-j}^{US}, s_{t-j}, e_{t-j}, p_{t-j}^e),\\
&A7\text{:}\ y_t^C = f(y_{t-j}^C, y_{t-j}^{US}, s_{t-j}, p_{t-j}^n),\\
&A8\text{:}\ y_t^C = f(y_{t-j}^C, y_{t-j}^{US}, s_{t-j}, e_{t-j}, p_{t-j}^n).
\end{aligned}
$$

Up to 3 lags of x_t are used.[15] Each of these models is also estimated with an MA(1) structure for the error process. A total of 72 forecasting models can then be obtained from (1). Models for which $\theta(L) = 1$ are denoted as Ai, for $i = 1, ...8$, whereas models for which $\theta(L) = 1 - \theta_1 L$ are denoted as Ai-MA. An MA process is used because it is well known that invertible MA(q) processes can be approximated by AR(p) processes, and vice versa. Such approximations tend, however, to induce a large amount of AR/MA lags in order to parameterize the complete structure of the dynamics embodied in the data, a feature often found in the VAR literature.

4.2 Disaggregate Canadian GDP models

For the regional models, we use an array of models inspired by (1) to forecast output growth. Canada's output growth is obtained from the following identities: $Y_t \equiv \sum_{i=1}^{5} Y_{i,t}$ and $Y_{i,t} \equiv Y_{i,t-1} + y_{i,t}$. The general N-dimensional VARMAX model used for disaggregate forecasting has the following form:

[14] The lag length is the same across all explanatory variables. This assumption could be relaxed, of course, but the computing cost would not be small, given the number of models we are comparing.

[15] To a certain extent, allowing for only 3 lags can be seen as restrictive, but in the context of estimating the system simultaneously, this restriction is imposed for practical purposes.

$$\mathbf{y}_t = \boldsymbol{\mu} + \boldsymbol{\beta}_p(L)\mathbf{x}_t + \boldsymbol{\theta}_q(L)\boldsymbol{\varepsilon}_t, \tag{2}$$

where $\mathbf{y}_t = \{y_t^{BC}, y_t^{PR}, y_t^{ON}, y_t^{QC}, y_t^{AT}\}'$; $\boldsymbol{\mu}$ is a vector of constant terms; $\boldsymbol{\beta}_p(L) = \boldsymbol{\beta}_1 L + ... + \boldsymbol{\beta}_p L^p$ and $\boldsymbol{\theta}_q(L) = I_k + \boldsymbol{\theta}_1 L + ... + \boldsymbol{\theta}_q L^q$, with coefficient matrices $\boldsymbol{\beta}_n$ and $\boldsymbol{\theta}_m$ for $n = 1, ..., p$ and $m = 0, ..., q$; and $\boldsymbol{\varepsilon}_t$ is an N-dimensional vector white noise with $E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'] = \Sigma_N$ (positive definite). This type of specification is equivalent to that of a SURE model estimated by direct (concentrated) maximum likelihood.

For the VARMAX models whose MA process is parameterized with a diagonal $\boldsymbol{\theta}_1$ matrix and the VARX models, we have $E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_{t-s}'] = 0$ for $s \neq t$. Compared with the usual VAR representation, the k-dimensional vector $\mathbf{x}_t$ in (2) is here defined as $\mathbf{x}_t = \{y_t^{x,i}, X_t'\}'$, where p is the lag length, $y_t^{x,i}$ is the growth rate of the rest of the country, and X_t includes the exogenous information set and lagged values of the dependent variable, equation by equation. For instance, the equation for British Columbia's output growth equation is based on $y_t^{x,i} = y_t^{xBC}$; namely, the growth rate of the Canadian economy excluding the contribution stemming from British Columbia. Every regional equation uses a similar information set and all explanatory variables, except $y_t^{x,i}$, are taken as exogenous. With respect to the information set, $\mathbf{x}_t$, (2) can, in general, be interpreted as a restricted VARMA process. Since the multivariate representation of the regional model involves the estimation of a large number of free parameters, properly parameterizing the MA part of the process should ensure a more parsimonious specification, and, all else equal, should provide us with more accurate forecasts.

To keep the model parsimonious, and maximum-likelihood estimation feasible, additional constraints are imposed on (2). First, the order of the MA process is limited to one. This constraint is necessary because it is often difficult to identify all the elements of the matrix $\boldsymbol{\theta}_q(L)$ when $q > 1$. This is true even when the MA process is of order one. Second, the MA process needs to be invertible; namely, $|\boldsymbol{\theta}_q(z)| \neq 0$ (i.e., no MA roots on the unit circle). This is achieved by optimizing the likelihood function under the constraint that the roots of the polynomial $\boldsymbol{\theta}_q(z)$ lie outside the unit circle. When we produce the pseudo out-of-sample forecasts, this constraint is sometimes found to be active. In such cases where the parameters lie on the boundary of their space, the algorithm reinitiates the maximization of the likelihood function from another set of starting values. If, after 10 attempts, the constraint is found to remain active, then the forecasts are produced using the last estimated set of parameters obtained in the previous recursion. Similarly, the AR roots of the AR(1) and AR(2) processes are constrained, but not the roots of the AR(3) processes. In most cases, the roots are found to lie well within the stationary region.

Two different structures for $\boldsymbol{\theta}_1(L)$ are compared. In one case, $\boldsymbol{\theta}_1(L)$ is simply a diagonal matrix, so that innovations from a particular region are not causal to other regions. In the second case, we relax this assumption by letting the innovations from the Ontario equation be causal to the output growth of other Canadian regions.[16] By using this particular parameterization, we can judge whether innovations from Ontario's economy directly affect other regions of Canada. This set-up is also consistent with the fact that Ontario is the single most important economic region in Canada, having more than 40 per cent of the national GDP.

As with the aggregate models described in section 4.1, forecasts based on different information sets are compared. In this case, forecasts from 13 different specifications are compared:

$$
\begin{aligned}
D1 &: \mathbf{y}_t = f(y_{t-j}, y^{US}_{t-j}, s_{t-j}), \\
D2 &: \mathbf{y}_t = f(y_{t-j}, y^{x,i}_{t-j}, s_{t-j}, e_{t-j}), \\
D3 &: \mathbf{y}_t = f(y_{t-j}, y^{US}_{t-j}, y^{xi}_{t}, s_{t-j}), \\
D4 &: \mathbf{y}_t = f(y_{t-j}, y^{US}_{t-j}, y^{xi}_{t}, s_{t-j}, e_{t-j}), \\
D5 &: \mathbf{y}_t = f(y_{t-j}, y^{x,i}_{t-j}, s_{t-j}, e_{t-j}), \\
D6 &: \mathbf{y}_t = f(y_{t-j}, y^{US}_{t-j}, s_{t-j}, e_{t-j}), \\
D7 &: \mathbf{y}_t = f(y_{t-j}, y^{US}_{t-j}, s_{t-j}, p^{c}_{t-j}), \\
D8 &: \mathbf{y}_t = f(y_{t-j}, y^{US}_{t-j}, s_{t-j}, p^{c}_{t-j}, e_{t-j}), \\
D9 &: \mathbf{y}_t = f(y_{t-j}, y^{US}_{t-j}, s_{t-j}, p^{e}_{t-j}, e_{t-j}), \\
D10 &: \mathbf{y}_t = f(y_{t-j}, y^{US}_{t-j}, s_{t-j}, p^{n}_{t-j}, e_{t-j}), \\
D11 &: \mathbf{y}_t = f(y_{t-j}, y^{x,i}_{t-j}, s_{t-j}, e_{t-j}, p^{n}_{t-j}), \\
D12 &: \mathbf{y}_t = f(y_{t-j}, y^{x,i}_{t-j}, s_{t-j}, e_{t-j}, p^{e}_{t-j}), \\
D13 &: \mathbf{y}_t = f(y_{t-j}, y^{x,i}_{t-j}, s_{t-j}, e_{t-j}, p^{c}_{t-j}).
\end{aligned}
$$

The VARX models are simply denoted as Di, for $i = 1, ..., 13$. For the case where the MA process is described by a diagonal matrix, the models are denoted as Di-MA; for the case where innovations from the Ontario equation are causal to the other regions, they are denoted as Di-MA-O. In total, the forecasts from 117 models are compared. The rationale for allowing for a causal link from Ontario's innovations to other economic units of Canada is simple: Ontario's economy makes over 40 per cent of Canada's GDP and therefore acts as an *attractor* in the national economy.

[16] For more details on causality in VARMA processes, see Boudjellaba, Dufour, and Roy (1992).

5 Out-of-Sample Forecasts

5.1 The forecasting experiment

To compare the forecast accuracy of direct forecasting versus indirect forecasting, we compute up to 50 pseudo out-of-sample forecasts. For models that involve MA terms, we need to restrict ourselves to $P = 50$ out-of-sample forecasts, because of the difficulties often encountered in estimating MA terms. Some of the VARMAX models cannot be estimated adequately, mainly because of the identification problem, which causes the likelihood function to be ill behaved. Hence, of the 117 specifications attempted, less than 10 of them are abandoned because of the behaviour of the likelihood function, which, at some point in the recursion, is ill behaved. All equations described under (1) and (2), which nest every specification considered in this paper, are estimated by maximum likelihood using Gauss.

Forecasts are obtained for horizons (h) of 1, 2, 4, and 8 quarters. The models are re-estimated at every step of the recursion, starting at time T^* up to time T, thereby yielding pseudo out-of-sample forecasts. Our approach is based on an *expanding* window, in contrast to a *rolling* window, so that we add one observation at each step of the recursion. For the purpose of comparing direct and indirect forecasts, the explanatory variables such as commodity prices, interest rates, or U.S. economic activity are considered to be exogenous to the models and are set to their historical values when forecasting. The only endogenous variables of the systems are therefore the regional output-growth series.

To evaluate the forecast errors and the relative accuracy of the best aggregate and disaggregate models, a number of statistics are computed. Denoting the vector of forecast errors as $\{e_t\}_{t=1}^{n}$, with $n = T - T^* + 1$ as the number of out-of-sample forecast errors, the mean squared forecast error (MSFE) is computed as a metric for forecast accuracy. The MSFE is simply obtained by $e_t' e_t (P - h)^{-1}$. Another important step in evaluating forecasts is to assess their unbiasedness, which is simply measured by the mean of the e_t. To evaluate the density of the forecast errors, we compute the coefficients of skewness and kurtosis, as well as the Jarque-Bera (JB) test statistics for the null hypothesis of normality. Lastly, we also report the p-value from a Lagrange Multiplier (LM) test—based on an autoregression of order two—for the null hypothesis that e_t is distributed as an independently, identically distributed process (*i.i.d.*) process.[17]

Tables 2 to 5 report some forecast statistics of the top three specifications for every horizon considered. A comparison of the forecasting performance amongst the aggregate

[17] Note, however, that e_t will, in general, be described by an MA$(h-1)$ process, for h-step-ahead forecasts. Hence, we report the test statistics only for the case where $h = 1$.

models shows that models that take into account the complete structure of the dependence of the innovations by adding an MA term dominate over dynamic models that are simply based on an AR structure to approximate the serial dependence. This is clear for $h = 1$ and 2. For longer horizons, forecasts from VARX and VARMAX models are virtually equal at predicting GDP growth. As for which information set is best, while various combinations of information can yield equivalent accuracy in forecasting, it seems that the second set of information performs well at all horizons by ranking systematically in our top three ranking of the forecasts. Although the forecast models that depend upon commodity prices occasionally rank in the top three forecasts, the inclusion of commodity prices does not further improve forecast accuracy, particularly for longer horizons, where key factors such as U.S. growth, monetary policy, and foreign exchange seem to provide the most accurate forecasts. Furthermore, the great volatility observed in commodity prices and the difficulties in predicting them could, in practice, induce greater uncertainty in our GDP forecasts.

Similarly, the accuracy of the disaggregated forecasts is improved when using a parameterization based on an MA polynomial matrix that has some non-zero elements in the off-diagonal; i.e., Ontario's innovations being causal to other regions. The best disaggregated VARX model would rank fourth with a calculated MSFE of 0.122 at one-step-ahead, with almost 15 per cent more variance than that of the best VARMAX model.

To assess the information content of a particular model, a useful metric is to compare how its MSFE performs relative to the MSFE of a simple benchmark, say the unconditional mean of the data (Galbraith 2003). The variance of the actual GDP growth data, calculated over the period where the forecasting exercise is performed, is 0.259. Compared with the calculated MSFEs of our best regional models, this variance implies that the regional models convey a sizeable amount of information about future GDP growth relative to the unconditional mean forecasts, over all forecast horizons.

The superior forecast ability of dynamic models with an MA process is strongest for $h = 1$ and 2. The forecast ability is relatively similar whether the MA process is parameterized or not when examining longer horizons. Interestingly, short-horizon forecasts can be improved when the interrelation amongst Canadian regions, proxied by $y_t^{x,i}$, is used in the specification. As for the aggregate specifications, commodity prices do not appear to be key in determining the forecast ability of the regional models. For longer horizon forecasts, using an ARMA structure, lags of y_t^{US} and s_t are enough to obtain relatively accurate out-of-sample forecasts. This is interesting because s_t never gets revised and, relatively speaking, exhibits very moderate volatility, making it an ideal real-time explanatory variable. To a certain extent, it is probably less uncertain to predict near-term U.S. GDP growth than commodity prices or the exchange rate, for which it is often suggested that the random walk is the best model (Meese and Rogoff 1983).

Another interesting feature of the disaggregate forecasts relative to the ones obtained by directly forecasting Canadian GDP growth is the empirical behaviour of the forecast errors. Figures 3 and 4, respectively, show the best aggregate and disaggregate forecast for 1- and 8-step-ahead over the 1991Q3–2003Q4 period. The difference between the two approaches is particularly evident when it comes to predicting major slowdowns and strong expansions. The fact that the forecasts from the regional approach capture the peaks and troughs of the GDP growth with greater accuracy is interesting: it suggests that the use of a regional model could help explain and uncover asymmetries in the business cycle, as discussed in Diebold and Rudebusch (1999) and Demers (2004). Good examples are the marked economic slowdowns observed in 1995–96 and 2001, and the 2002–03 soft patch, in which cases the disaggregated approach is actually able to predict growth rates very close to the actual data. The same cannot be obtained by directly forecasting aggregate GDP, however. In periods where growth hovers around potential (i.e., 3 per cent), both approaches seem to yield comparable point forecasts.

Figure 5 plots the densities of one-step-ahead forecast errors from the top aggregate and disaggregate models. Although the aggregate and disaggregate approaches yield approximately normal forecast errors, the disaggregated VARMAX model reduces the likelihood of observing large positive errors. The disaggregated model also has a much heavier mass in the ± 0.5 (or ± 2 percentage points when measured at annual rates) range. For the large negative forecast errors (the left-end tail of the distribution), the two approaches provide fairly comparable accuracy.

Overall, we can conclude that the minimization of the MSFE is achieved by aggregation of disaggregate forecasts, which are estimated in a multivariate VARMAX framework. As for which information set provides the most accurate forecasts, using U.S. information is certainly key in obtaining an accurate forecast of Canada's GDP growth, but the stance of monetary policy and variations in the real Canada-U.S. exchange rate also play important roles.

In section 6, we will conduct a statistical analysis of the best aggregate and best disaggregate forecasts, but first, in section 5.2, we must briefly examine the empirical results of the top three one-step-ahead forecasting models, to evaluate their economic implications.

5.2 Empirical results of selected models

For aggregate output, y_t^C, the maximum-likelihood estimates for the top three one-step-ahead forecast models are shown in Table 6. Because each of the top three specifications have an MA process for the innovations, Table 6 also reports the estimates for a simple benchmark ARX model, namely specification $A8$, in order to compare parameter estimates. In terms of out-of-sample performance (at one-step-ahead), specification $A8$ performs third within its class. The parameter estimates appear to be compatible with the standard IS-curve framework and empirical results. The explanatory power, as measured by the R^2, is quite appropriate for models expressed in growth rates; for instance, they are comparable with the results obtained by Demers (2004), who forecasts short-term GDP (chained Fischer, at market prices) growth for Canada. For the most part, the estimated coefficients are of the expected sign, the only exception being the exchange rate—although it is only moderately significant in most cases. This could be attributable to the restrictions on the lag length that we have imposed for computational purposes. It has been documented that the full effect of real exchange rate fluctuations on real GDP growth can take up to three years in Canada (Duguay 1994; and Demers 2004). It is also worth noting the significance of the MA parameter, θ, which suggests the importance of the MA(1) process in capturing the complete structure of the dynamics in the data-generating process. In particular, if we compare the estimated persistence of output growth, which is obtained from $A8$ and $A8$-MA, we see that the sum of the AR terms falls from about 0.35 to turn slightly negative at roughly -0.1. Furthermore, the dynamic is changed substantially: for MA models, the first AR lag is negative and is followed by two positive lags, whereas for the usual VAR model the first lag is positive, followed by a small and insignificant negative lag, while the third is positive but, again, insignificant. On the other hand, the estimated responses to variations in y_t^{US}, s_t, e_t, and p_t^n are also affected, depending on whether θ is restricted to zero or not. For instance, the sum of the coefficient associated with y_t^{US} drops by about a full half, the exchange rate's response remains positive and very low (having a p-value of only 0.086), and the effect of commodity prices, p_t^n, remains virtually unchanged. The sensitivity of the persistence estimates shows that we should be cautious when evaluating impulse responses obtained from simple VAR models that involve GDP growth and ignore the MA part of the process.

For the disaggregate models, the maximum-likelihood estimates are reported in Tables 7 to 9: in order of performance, models $D4$-MA-O, $D5$-MA-O, and $D3$-MA-O.[18] Because

[18] The best one-step-ahead disaggregate model is selected based on its forecast performance at the aggregate

the top out-of-sample forecast performers were all specified with an MA polynomial matrix that has some non-zero elements in the off-diagonal, Tables 10 and 11 report benchmark estimates for the VARMAX and VARX specifications that performed best within their own category at the one-step-ahead; namely, $D10$-MA, which has an MSFE of 0.130, and $D4$, which has an MSFE that is calculated to be 0.122.

The first obvious result associated with our selected disaggregate models is the disparity between regions in the IS-curve's explanatory power. While the framework appears to work best for Quebec and relatively well for Ontario, the results are generally poor for the Prairies. This could be attributed in part to stabilizing monetary policy. Since the Quebec economy's structure most closely resembles that of the Canadian economy as a whole, monetary policy, which is designed with country-specific shocks in mind (rather than region-specific), could be best suited on average for that province.[19] The disparity could also be attributed to the fact that the significance of U.S. growth in the specification is much more informative for the manufacturing/export-based economy of Central Canada than it would be for the agricultural/resource-based economy of the Prairies.[20]

Moreover, maximum-likelihood estimates for the disaggregate models show that only lagged output growth (the AR process, the growth rate of the rest of the country and that of the United States) appears to be statistically significant in explaining the behaviour of regional economic growth. The exchange rate variable and yield curve do not seem to add much to the explanatory power; they are almost never significant. Again, this could be attributed to the restrictions on the lag length.

For the MA part of the process, the parameter estimates vary significantly, depending on the information set used for the estimation. For the best out-of-sample forecasting specification, $D4$-MA-O, the equations for British Columbia, Ontario, and Quebec exhibit a strong negative MA process with a coefficient hovering at around -0.5, but this is much different than the aggregate estimates, which range between 0.8 and 0.9. As for whether Ontario's innovations are causal to other regions, it seems that the most important effects are found in British Columbia, in which case, according to the top out-of-sample model, the estimated impact is -0.42. This interesting result supports the view that British Columbia's economy tends to be out of synchronization with that of the rest of the country. This also seems to be the case for the Prairies, where natural resources, specifically oil, play a key role in driving

level. This does not imply that forecast performances are maximized for each region.

[19] This could also explain, in part, the lowest volatility observed for Quebec's GDP.

[20] Further investigation also leads us to conclude that, for British Columbia and the Prairies, the use of Asian GDP growth might be more appropriate than that for the United States in the specification of regional IS-curves.

the pace of economic activity. For Quebec and the Atlantic region, Ontario's innovation does not appear to be causal: we cannot reject the null hypothesis of non-causality for both regions.

The best forecasting model where $\theta^{ONT} = 0$ in all regions, D10-MA, which uses non-energy commodity prices but ignores inter-regional dependency, provides regional adjustments that are much less interesting, judging by the R^2's reported in Table 10. These estimates also show that it is important to account for inter-regional relations when one wants to disentangle the impact on growth from domestic versus U.S. momentum. In effect, the latter appears to be inflated when $y_t^{x,i}$ is ignored. In fact, while Canada-U.S. trade is important, trade within Canadian provinces is also important. For example, close to 35 per cent of Quebec's and 28 per cent of Ontario's exports go to other Canadian provinces, while 37 per cent of Quebec's and 24 per cent of Ontario's imports are from within Canada.

Comparing the parameter estimates of the best disaggregate forecasting model with those obtained from a benchmark VARX model, $D4$, the counterpart of $D4$-MA-O, our finding is the same as for the aggregate models: ignoring the MA part of the process greatly affects the parameter estimates.

6 Testing for Equality of Forecast Accuracy and for Forecast Encompassing

In this section, we evaluate the forecasting properties of the most accurate forecasting models for each horizon considered.

6.1 Testing for equality of forecast accuracy and forecast encompassing

To test for the hypothesis that the h-step-ahead forecasts from two competing non-nested models are equal, the test proposed by Diebold and Mariano (1995) is used. From two competing models, say A and B, and denoting the vector of forecast errors as $\{\hat{e}_t\}_{t=1}^n$, n being the number of out-of-sample forecast errors, and also denoting the loss differential of interest as $L_t(e_t)$, the MSFE loss differential is defined as $\hat{L}_{1t}(e_t) = \hat{e}_{At}^2 - \hat{e}_{Bt}^2$. Then, we can test the null hypothesis that the forecasts from models A and B are equivalent, in the mean squared error sense, using the following test statistics:

$$S = \frac{\bar{L}_t}{\sqrt{\hat{V}\left(\bar{L}_t\right)}}, \tag{3}$$

where $\bar{L}_t = n^{-1}\sum_{t=1}^n \hat{L}_t$, and where $\hat{V}\left(\bar{L}_t\right)$ is an estimate of the asymptotic variance (at frequency zero) of $\bar{L}_t$ and is approximated by $n^{-1}\left(\gamma_0 + 2\sum_{k=1}^{h-1}\gamma_k\right)$, combined with Andrews'

(1991) quadratic spectral kernel.

To test the hypothesis that a model's forecasts encompass those from a competing model, we use the metric $\hat{L}_t = \hat{e}_{At}^2 - \hat{e}_{At}\hat{e}_{Bt}$, where the null hypothesis states that $E\left(\hat{L}_t\right) = 0$, which means that Model A encompasses Model B.

6.2 Test results

According to Diebold and Mariano's test, the best disaggregate one-step-ahead forecasts are more accurate than the best aggregate forecast (having a p-value of 0.08), but the forecast-encompassing test is inconclusive, since it would appear that the two forecasts contain the same information about one-quarter-ahead GDP growth. For longer horizons, the forecasts are equally accurate, according to Diebold and Mariano's test, while the forecast-encompassing test finds that the two-step-ahead disaggregate forecast encompasses the aggregate forecasts (having a p-value of 0.07).

7 Conclusion

This study has proposed various models to forecast GDP growth from both the national and regional level. Pseudo out-of-sample forecasts were compared at horizons up to 8 quarters ahead. Based on a comparison of the forecast errors, we have shown that higher forecast accuracy is achieved when we indirectly forecast GDP growth via Canada's five economic regions, namely: the Atlantic provinces; Quebec; Ontario; the Prairie provinces, Nunavut, and the Northwest Territories; and British Columbia and Yukon. Accuracy is greater and is statistically significant for short-horizon forecasts (i.e., one-quarter ahead). For longer horizons, the difference is only marginal and is not statistically significant.

We have also shown that ignoring the moving-average part of the innovation process can have an important impact on parameter estimates. In particular, impulse responses obtained from simple VAR models of the Canadian economy should be interpreted with caution, because estimates of the output-growth persistence are sensitive to the modelling of the MA process.

Finally, our empirical results illustrate the importance of properly parameterizing the regional model by accounting for a rich yet parsimonious specification of the Canadian economy at the regional level.

Given the benefits of using a disaggregated approach, we could expect to obtain even greater forecast accuracy by building a regional system of equations that would be based on the best individual regional specification. That is left for future research.

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Table 1: **Selected Regional Growth Statistics (1961Q1–2004Q3)***

Region	Mean	Median	Variance
Atlantic	3.26	2.92	23.81
Quebec	3.24	3.04	16.73
Ontario	3.77	3.73	19.01
Prairies	4.01	3.69	21.62
B.C.	4.44	4.24	64.96

*Expressed at annual rates.

Table 2: **Selected Forecast Statistics: One-step-ahead**

Model	Lags	MSFE	Mean	Skewness	Kurtosis	JB*	LM-AR(2)*
Aggregate Models							
$A2$-MA	3	0.157	−0.061	0.196	2.611	0.726	0.098
$A6$-MA	3	0.170	−0.050	0.251	2.688	0.697	0.000
$A8$-MA	3	0.171	−0.059	0.077	2.651	0.857	0.000
Disaggregate Models							
$D4$-MA-O	2	0.106	0.038	−0.341	3.271	0.569	0.979
$D5$-MA-O	2	0.112	0.011	−0.156	2.509	0.702	0.230
$D3$-MA-O	1	0.121	0.019	0.027	2.466	0.747	0.369

*p-values are reported.

Table 3: Selected Forecast Statistics: 2-step-ahead

Aggregate Models						
Model	Lags	MSFE	Mean	Skewness	Kurtosis	JB*
A2-MA	3	0.169	−0.054	0.123	2.627	0.811
A3-MA	2	0.173	−0.029	0.427	2.688	0.452
A1-MA	3	0.174	−0.045	0.426	2.572	0.394
Disaggregate Models						
D10-MA	2	0.159	0.000	0.181	2.871	0.860
D4-MA	3	0.161	0.003	0.287	2.411	0.496
D6-MA	2	0.168	-0.014	0.014	2.604	0.846

*p-values are reported.

Table 4: Selected Forecast Statistics: 4-step-ahead

Aggregate Models						
Model	Lags	MSFE	Mean	Skewness	Kurtosis	JB*
$A3$	2	0.181	−0.039	−0.066	2.231	0.550
$A2$-MA	3	0.184	−0.036	−0.012	2.119	0.532
$A3$-MA	2	0.188	−0.009	0.076	2.269	0.576
Disaggregate Models						
$D1$-MA	3	0.167	0.030	0.135	2.238	0.531
$D6$-MA	3	0.169	0.027	−0.221	2.551	0.818
$D6$	2	0.170	0.009	−0.152	2.377	0.630

*p-values are reported.

Table 5: Selected Forecast Statistics: 8-step-ahead

Aggregate Models						
Model	Lags	MSFE	Mean	Skewness	Kurtosis	JB*
$A2$	2	0.178	−0.002	−0.208	2.342	0.580
$A2$-MA	3	0.181	−0.001	−0.179	2.251	0.541
$A1$-MA	3	0.188	−0.001	0.039	2.249	0.601
Disaggregate Models						
$D1$-MA	2	0.166	0.045	0.172	2.441	0.677
$D4$	3	0.168	0.055	0.104	2.391	0.687
$D5$	3	0.169	0.044	−0.291	2.360	0.509

*p-values are reported.

Table 6: Selected Empirical Results – Aggregate Model

Param./Model	$A2$-MA	$A6$-MA	$A8$-MA	$A8$
y_{t-1}	-0.393 (0.002)	-0.401 (0.000)	-0.414 (0.000)	0.356 (0.000)
y_{t-2}	0.235 (0.023)	0.261 (0.009)	0.233 (0.011)	-0.072 (0.412)
y_{t-3}	0.087 (0.294)	0.090 (0.272)	0.100 (0.120)	0.069 (0.397)
y_{t-1}^{US}	0.177 (0.008)	0.168 (0.012)	0.177 (0.006)	0.199 (0.003)
y_{t-2}^{US}	0.178 (0.040)	0.173 (0.046)	0.176 (0.035)	0.031(0.650)
y_{t-3}^{US}	0.069 (0.302)	0.058 (0.396)	0.051 (0.413)	0.032 (0.640)
s_{t-1}	-0.021 (0.920)	0.000 (0.997)	0.009 (0.991)	-0.078 (0.815)
s_{t-2}	-0.532 (0.271)	-0.551 (0.240)	-0.532 (0.098)	0.062 (0.867)
s_{t-3}	0.200 (0.683)	0.209 (0.648)	0.151 (0.720)	-0.340 (0.427)
e_{t-1}	0.002 (0.941)	0.004 (0.880)	0.013 (0.616)	0.008 (0.781)
e_{t-2}	0.039 (0.169)	0.045 (0.114)	0.034 (0.232)	0.031 (0.270)
e_{t-3}	0.055 (0.046)	0.049 (0.075)	0.066 (0.013)	0.046 (0.086)
p_{t-1}^{e}	–	-0.004 (0.446)	–	–
p_{t-2}^{e}	–	-0.005 (0.423)	–	–
p_{t-3}^{e}	–	0.005 (0.393)	–	–
p_{t-1}^{n}	–	–	0.012 (0.356)	0.009 (0.479)
p_{t-2}^{n}	–	–	-0.007 (0.655)	-0.012 (0.347)
p_{t-3}^{n}	–	–	0.012 (0.361)	0.024 (0.074)
θ	0.825 (0.000)	0.851 (0.000)	0.910 (0.000)	–
R^2	0.373	0.384	0.430	0.374

Notes: Estimates for the intercept are omitted. p-values are in parentheses.

Table 7: Selected Empirical Results – Disaggregated Model: D4-MA-O

Param./Region	B.C.	PR	ONT	QC	ATLA
y_{t-1}	0.063 (0.609)	−0.072 (0.663)	0.572 (0.001)	0.243 (0.312)	−0.028 (0.937)
y_{t-2}	−0.001 (0.798)	0.096 (0.244)	−0.082 (0.086)	0.147 (0.107)	−0.163 (0.050)
$y_{t-1}^{x.i}$	1.046 (0.000)	0.503 (0.013)	0.311 (0.003)	0.468 (0.000)	0.489 (0.018)
$y_{t-2}^{x.i}$	−0.506 (0.023)	−0.251 (0.037)	−0.123 (0.233)	−0.218 (0.138)	0.142 (0.519)
y_{t-1}^{US}	0.487 (0.008)	0.196 (0.077)	0.180 (0.059)	0.153 (0.066)	0.140 (0.210)
y_{t-2}^{US}	−0.127 (0.513)	0.057 (0.630)	−0.035 (0.669)	−0.023 (0.772)	−0.129 (0.453)
s_{t-1}	0.524 (0.647)	−0.189 (0.778)	0.244 (0.637)	−0.617 (0.295)	−0.108 (0.883)
s_{t-2}	−1.301 (0.265)	0.157 (0.833)	−0.305 (0.575)	0.573 (0.333)	0.194 (0.796)
e_{t-1}	−0.113 (0.129)	0.011 (0.784)	0.000 (0.999)	0.008 (0.856)	0.046 (0.336)
e_{t-2}	0.118 (0.125)	−0.009 (0.783)	0.085 (0.028)	0.056 (0.000)	0.013 (0.832)
θ	−0.548 (0.000)	0.004 (0.982)	−0.500 (0.001)	−0.492 (0.053)	−0.155 (0.663)
θ^{ONT}	−0.420 (0.022)	−0.261 (0.121)	–	0.034 (0.684)	0.100 (0.536)
R^2	0.272	0.126	0.257	0.325	0.246

Notes: Estimates for the intercept are omitted. p-values are in parentheses.

Table 8: **Selected Empirical Results – Disaggregated Model: D5-MA-O**

Param./Region	B.C.	PR	ONT	QC	ATLA
y_{t-1}	-0.787 (0.000)	0.042 (0.882)	0.524 (0.009)	0.092 (0.748)	-0.565 (0.005)
y_{t-2}	-0.155 (0.034)	0.142 (0.131)	-0.069 (0.191)	0.086 (0.361)	-0.080 (0.153)
$y^{x.i}_{t-1}$	0.947 (0.001)	0.497 (0.013)	0.359 (0.000)	0.531 (0.000)	0.406 (0.034)
$y^{x.i}_{t-2}$	0.492 (0.035)	-0.170 (0.236)	-0.089 (0.521)	-0.104 (0.525)	0.383 (0.014)
s_{t-1}	0.497 (0.711)	-0.244 (0.800)	0.198 (0.739)	-0.669 (0.298)	0.010 (0.788)
s_{t-2}	-1.362 (0.318)	0.150 (0.881)	-0.256 (0.670)	0.488 (0.459)	0.200 (0.791)
e_{t-1}	-0.088 (0.263)	0.003 (0.951)	-0.005 (0.897)	0.012 (0.765)	0.011 (0.815)
e_{t-2}	0.108 (0.179)	0.014 (0.758)	0.102 (0.008)	0.069 (0.065)	0.087 (0.060)
θ	0.463 (0.015)	-0.097 (0.737)	-0.379 (0.048)	-0.323 (0.288)	0.434 (0.020)
θ^{ONT}	-0.388 (0.069)	-0.183 (0.256)	–	0.063 (0.554)	0.217 (0.110)
R^2	0.204	0.106	0.234	0.301	0.216

Notes: Estimates for the intercept are omitted. p-alues are in parentheses.

Table 9: Selected Empirical Results – Disaggregated Model: D3-MA-O

Param./Region	B.C.	PR	ONT	QC	ATLA
y_{t-1}	-0.110 (0.559)	-0.374 (0.038)	0.182 (0.332)	-0.126 (0.567)	0.045 (0.797)
$y^{x.i}_{t-1}$	0.629 (0.016)	0.497 (0.002)	0.310 (0.004)	0.468 (0.001)	0.379 (0.012)
y^{US}_{t-1}	0.441 (0.012)	0.141 (0.178)	0.243 (0.011)	0.197 (0.032)	0.112 (0.352)
s_{t-1}	-0.445 (0.511)	0.016 (0.919)	0.029 (0.880)	-0.392 (0.320)	0.086 (0.789)
θ	-0.256 (0.002)	0.214 (0.017)	-0.094 (0.582)	-0.080 (0.674)	-0.194 (0.081)
θ^{ONT}	-0.181 (0.139)	-0.172 (0.016)	–	0.062 (0.545)	0.166 (0.021)
R^2	0.203	0.097	0.215	0.289	0.211

Notes: Estimates for the intercept are omitted. p-values are in parentheses.

Table 10: Selected Empirical Results – Disaggregated Model: D10-MA

Param./Region	B.C.	PR	ONT	QC	ATLA
y_{t-1}	-0.810 (0.002)	0.204 (0.882)	0.589 (0.001)	0.328 (0.005)	-0.274 (0.181)
y_{t-2}	-0.156 (0.000)	-0.010 (0.050)	-0.072 (0.004)	0.226 (0.002)	-0.116 (0.105)
y^{US}_{t-1}	0.540 (0.002)	0.278 (0.008)	0.298 (0.002)	0.346 (0.000)	0.413 (0.000)
y^{US}_{t-2}	0.631 (0.005)	0.065 (0.785)	-0.023 (0.789)	-0.094 (0.469)	0.137 (0.000)
s_{t-1}	0.491 (0.738)	-0.119 (0.857)	0.299 (0.556)	-0.101 (0.041)	0.019 (0.923)
s_{t-2}	-1.545 (0.290)	0.025 (0.917)	-0.381 (0.483)	0.099 (0.239)	0.034 (0.910)
e_{t-1}	-0.125 (0.107)	-0.025 (0.537)	-0.005 (0.898)	0.021 (0.602)	0.063 (0.179)
e_{t-2}	0.129 (0.116)	0.015 (0.716)	0.090 (0.024)	0.047 (0.227)	0.043 (0.315)
p^n_{t-1}	0.015 (0.371)	0.024 (0.013)	-0.011 (0.211)	-0.007 (0.003)	-0.010 (0.000)
p^n_{t-2}	0.001 (0.956)	0.007 (0.510)	-0.001 (0.868)	-0.003 (0.744)	0.006 (0.565)
θ	0.426 (0.029)	-0.333 (0.421)	-0.608 (0.000)	-0.571 (0.000)	0.183 (0.295)
R^2	0.195	0.113	0.207	0.242	0.135

Notes: Estimates for the intercept are omitted. p-values are in parentheses.

Table 11: Selected Empirical Results – Disaggregated Model: D4

Param./Region	B.C.	PR	ONT	QC	ATLA
y_{t-1}	−0.434 (0.000)	−0.054 (0.498)	0.062 (0.463)	−0.175 (0.025)	−0.173 (0.022)
y_{t-2}	−0.154 (0.062)	0.157 (0.044)	−0.149 (0.065)	0.032 (0.681)	−0.196 (0.007)
$y_{t-1}^{x,i}$	0.594 (0.009)	0.257 (0.038)	0.392 (0.000)	0.527 (0.000)	0.612 (0.000)
$y_{t-2}^{x,i}$	0.072 (0.670)	−0.182 (0.137)	0.103 (0.313)	−0.091 (0.380)	0.151 (0.275)
y_{t-1}^{US}	0.365 (0.048)	0.155 (0.160)	0.200 (0.044)	0.174 (0.056)	0.155 (0.157)
y_{t-2}^{US}	0.244 (0.193)	0.125 (0.261)	0.102 (0.305)	0.066 (0.464)	−0.112 (0.322)
s_{t-1}	0.544 (0.683)	−0.261 (0.343)	0.191 (0.701)	−0.636 (0.000)	−0.157 (0.853)
s_{t-2}	−1.372 (0.307)	0.069 (0.843)	−0.128 (0.860)	0.441 (0.292)	0.322 (0.719)
e_{t-1}	−0.151 (0.045)	−0.005 (0.885)	0.017 (0.648)	0.018 (0.576)	0.052 (0.187)
e_{t-2}	0.137 (0.068)	0.011 (0.730)	0.083 (0.035)	0.052 (0.154)	0.014 (0.697)
R^2	0.227	0.111	0.249	0.312	0.240

Notes: Estimates for the intercept are omitted. p-values are in parentheses.

Figure 1: Quarterly Output Growth Series

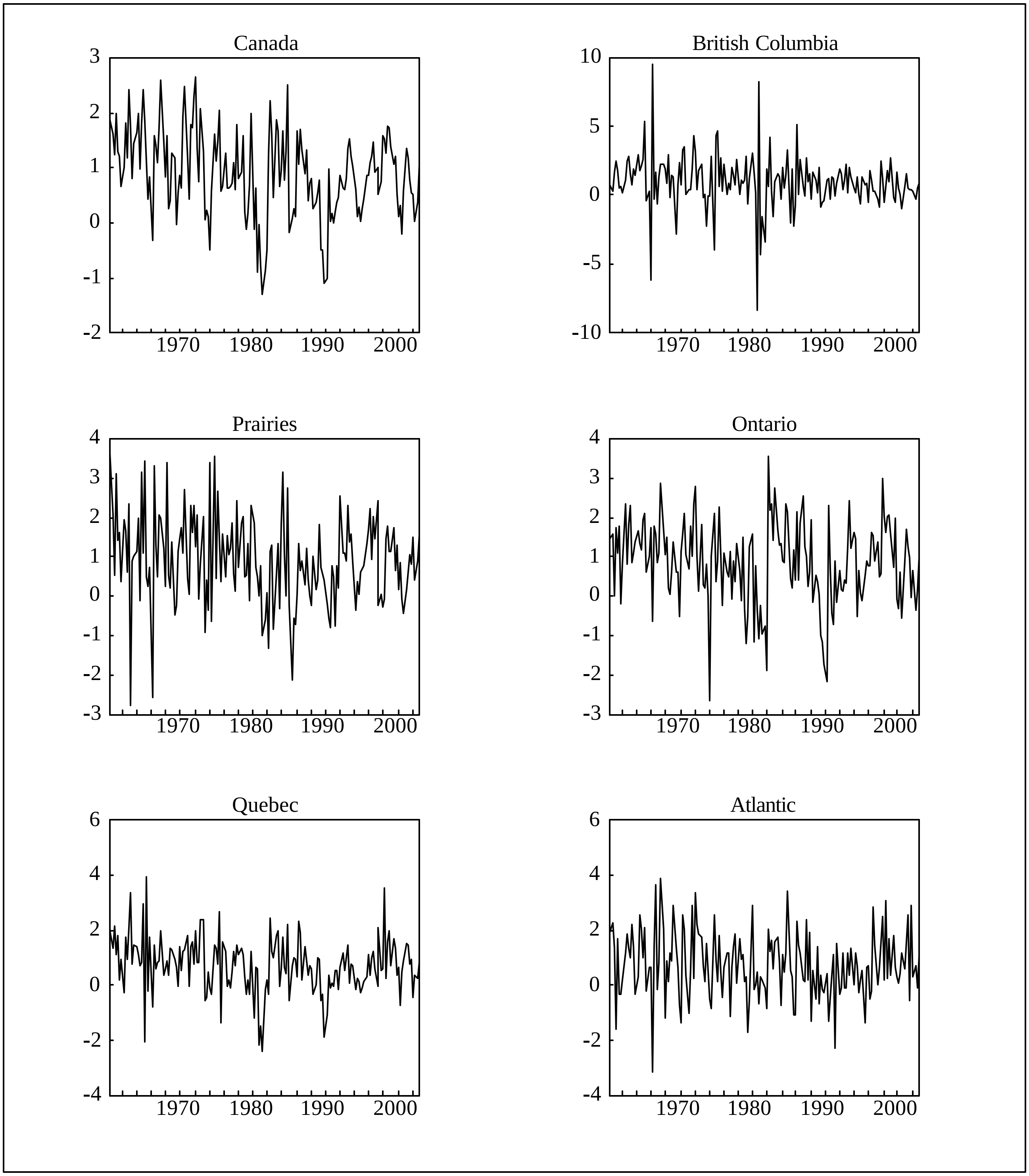


Figure 2: Exogenous Variables

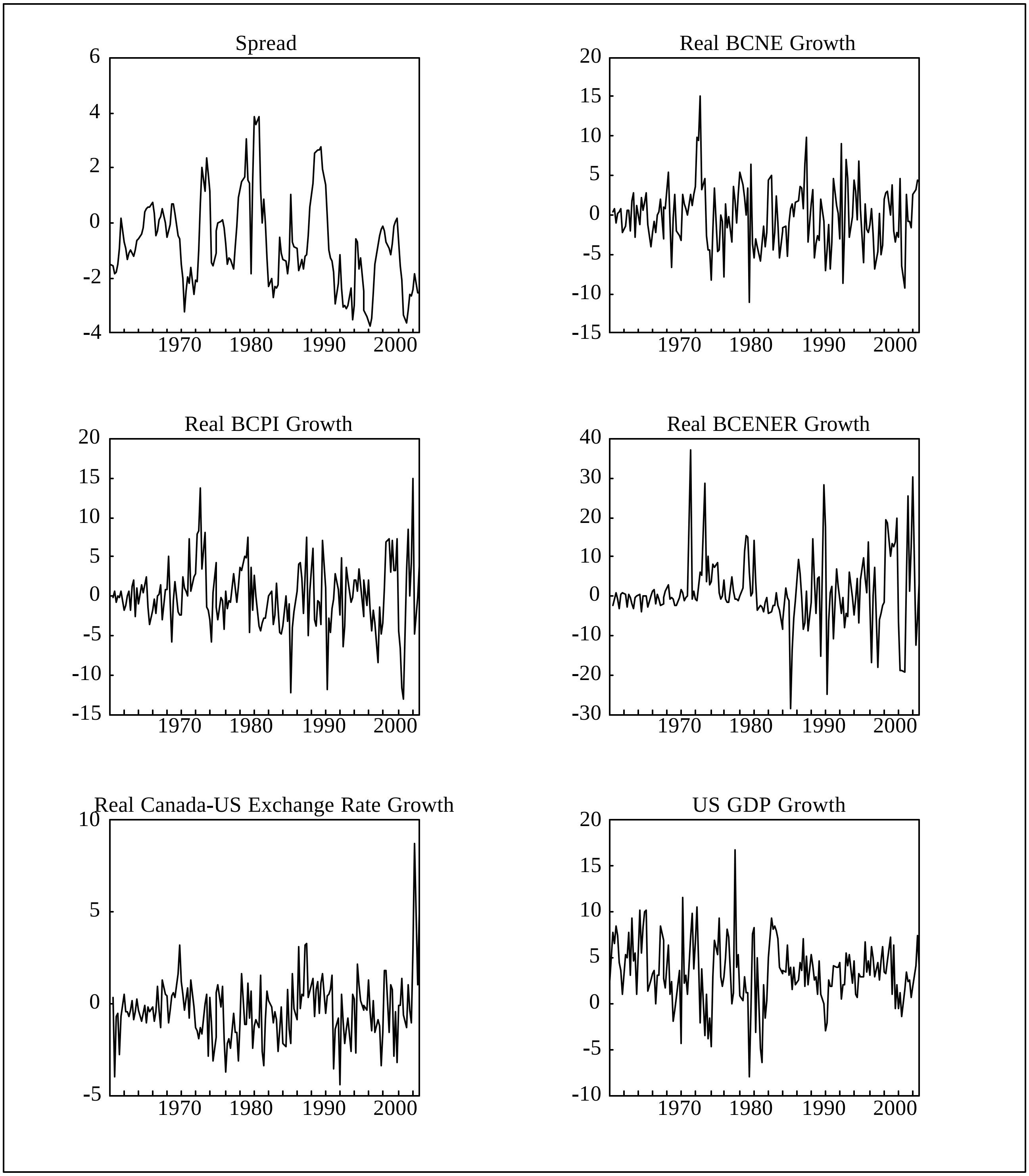


Figure 3: Actual vs One-Step-Ahead Forecasts of Aggregate and Disaggregate Models

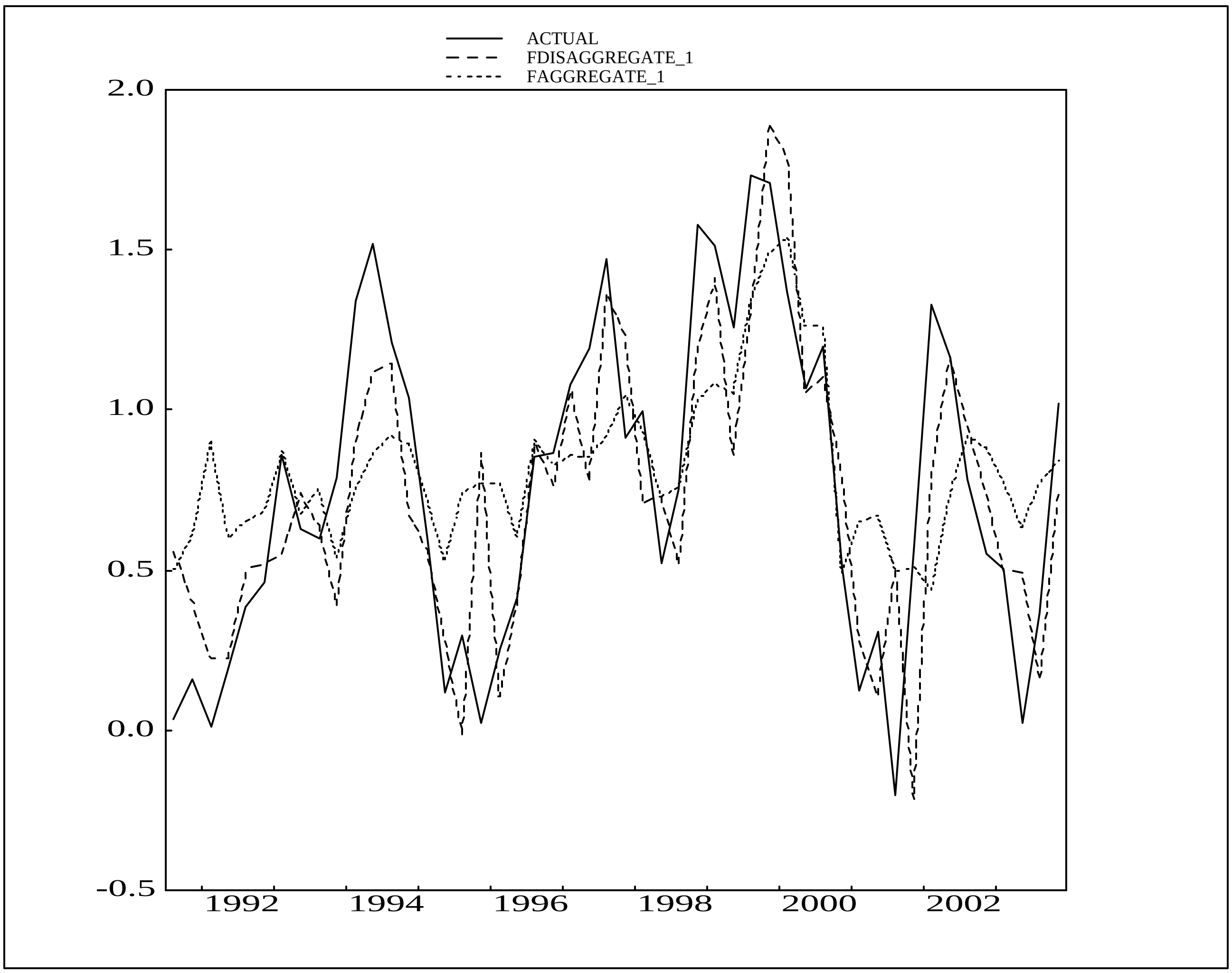


Figure 4: Actual vs 8-Step-Ahead Forecasts of Aggregate and Disaggregate Models

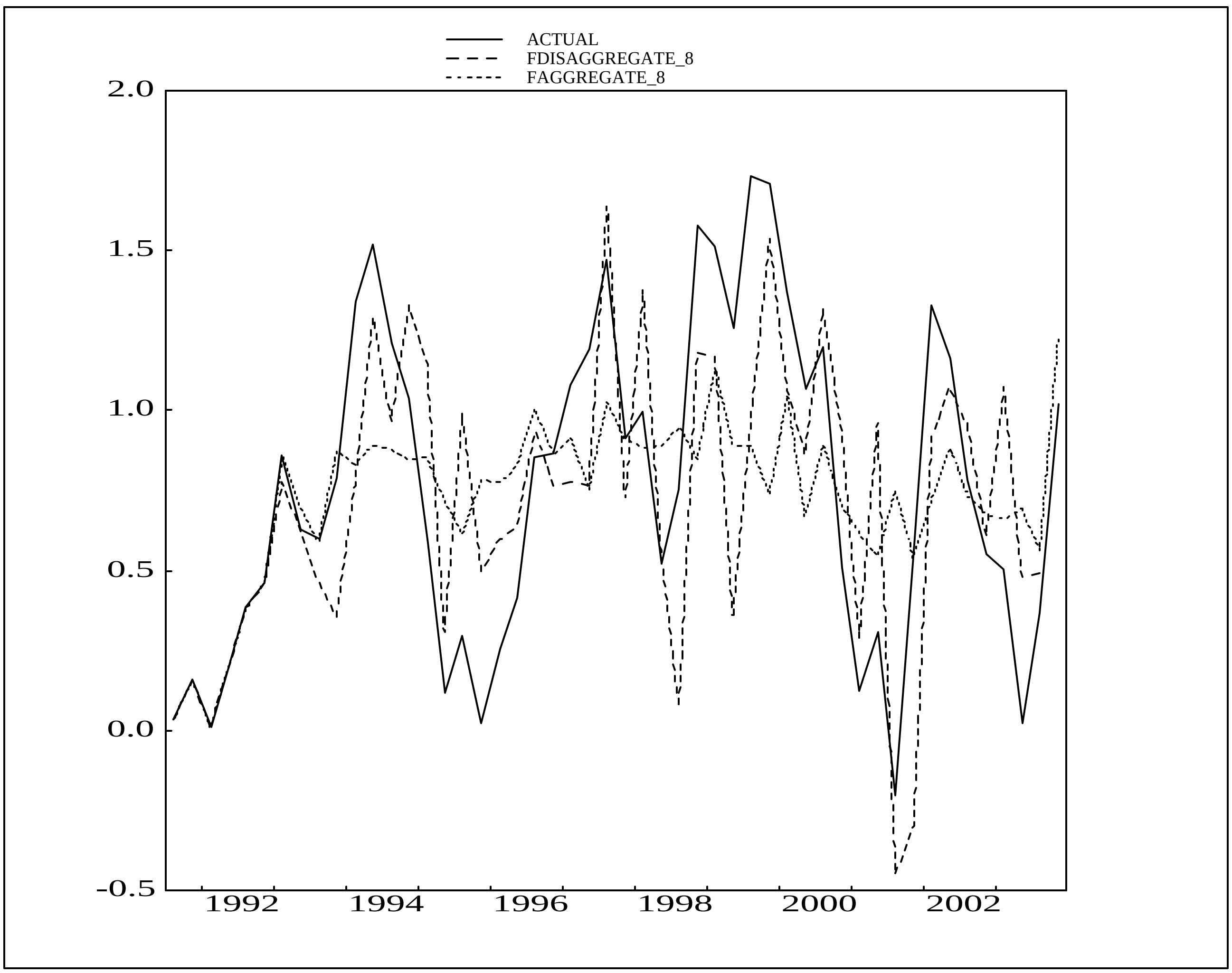
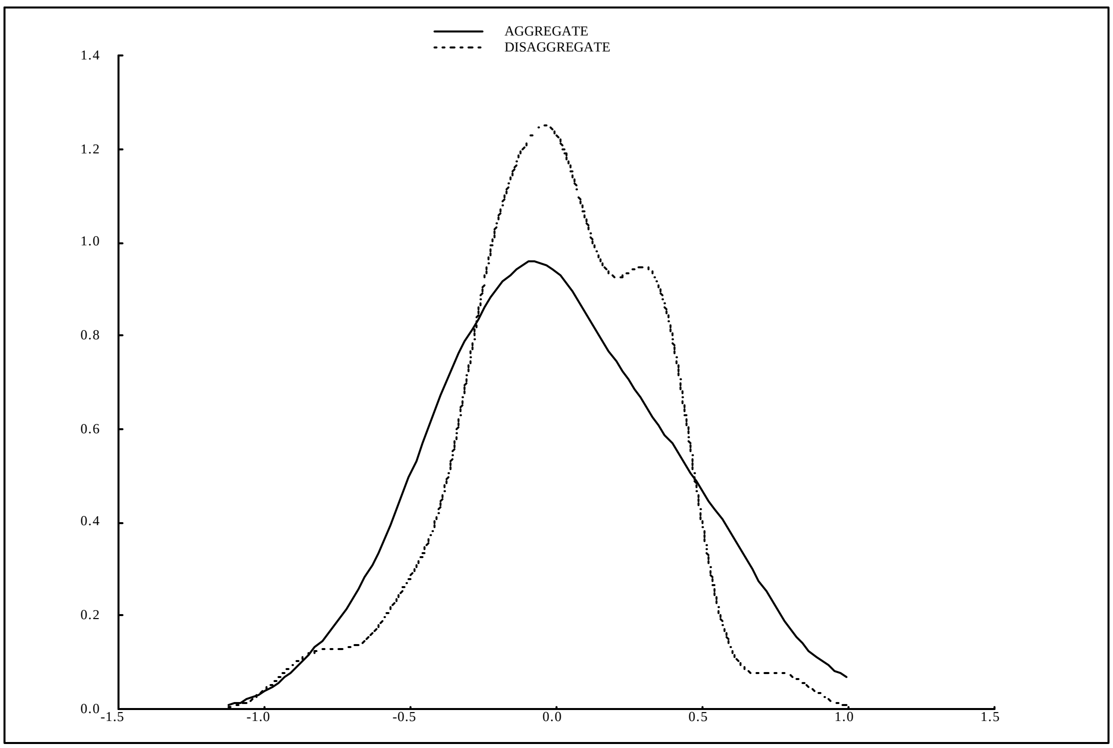


Figure 5: One-Step-Ahead Forecast Error Densities



Appendix A: Data Source*

Y_t	Canadian GDP: v1992067
y_t^{US}	U.S. GDP growth: Bureau of Economic Analysis
y_t^i	Regional GDP estimates: Conference Board
s_t	Interest rate spread: v122491–v122487
e_t	Real exchange rate: CANDA/U.S. noon spot rate multiplied by the ratio of the Canadian GDP deflator over the U.S. GDP deflator, v121716
p_t^c, p_t^e, p_t^n	Real commodity prices: Bank of Canada Commodity Price Index**

*v numbers are Statistics Canada CANSIM reference numbers.

**For details, see Hirsch (2003).

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