# Merchant Transmission Investment with Uncertainty

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**Abstract**

We adapt Bushnell and Stoft (1996) allocation rule for property rights on a merchant transmission network to the case where demand and supply are stochastic. We show that the efficiency properties of this rule remain when there is uncertainty. This suggests that implementation of merchant transmission networks is more robust to uncertainty than it was previously suggested in the literature.

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# 1 Introduction

Electricity transmission networks are said to be merchant if property rights are issued for each transmission line and if the allocation of investment in lines results from decentralized decisions on a free market. Merchant investment is possible in electricity markets when ownership and operation of the transmission network are separated. The transmission congestion rents, which are collected by an "Independent System Operator " (ISO) through network operations, are distributed among individual owners of transmission property rights. Investors in the network receive new property rights according to an allocation rule set by a regulatory agency or by the ISO. This allocation rule takes into account that investment in new transmission lines may modify the transmission capacity of existing lines. Decentralized investment decisions can thus have external effects on the returns of the owners of existing lines.

Bushnell and Stoft (1996) have proposed an allocation rule that eliminates incentives to make investments which would reduce overall welfare. This rule is implementable in a context of demand and supply certainty. Unfortunately, uncertainty in demand and supply is ubiquitous in electricity markets. This has led Joskow and Tirole ([3], p. 49)[1] to conclude that

> "*non-contingent* transmission rights cannot be defined properly to capture the varying valuations of a transmission investment under the many contingencies that characterize real electric power networks and provide the right incentives to support efficient investments. Only *contingent rights* provide the proper incentives".

In this note, we adapt the Bushnell and Stoft [1] allocation rule of non-contingent property rights to the case of stochastic demand and supply. We show that efficiency properties of the initial rule are maintained under uncertainty even though rights are not contingent. This shows that implementing a merchant transmission network is probably less a challenging problem than it is alluded in the literature.

# 2 The Model

## 2.1 Network Operations and Spot Prices

A transmission network links a set of $N$ nodes and serves a set of $K$ agents (respectively with $N$ and $K$ elements). These agents can be generators, distributors,

[1] The section in which appears this passage is not present in the published version ([2]) of this working paper.

industrial consumers, etc, although they can play many roles on the network. For instance, a generator at node $i$ can be a distributor at node $j$. Demand and supply conditions on the network are random. We denote $q_{is}^k$ the net supply of agent $k$ at node $i$ when the state of nature is $s$, $q_s^k = (q_{1s}^k, ..., q_{ns}^k)$, the vector of agent's $k$ net supply. Agent's $k$ generation costs (and/or consumption benefits) are represented by a twice differentiable and strictly convex function, $C^k : \mathbb{R}^N \rightarrow \mathbb{R}$.

We assume that there is no transmission loss on the network[2] so that the capacity of the network to handle a total net supply of $q_s$ is resumed by a polyhedron $F \subset \mathbb{R}^N$. Dispatch on the network is performed ex post in real time by the ISO who manages to minimise the utilitarian loss:

$$\begin{aligned} \min_{q_s,(q_s^1,...,q_s^K)} \quad & \sum_K C^k(q_s^k) \\ \text{s.t.} \quad & \sum_K q_s^k = q_s \\ & q_s \in F \end{aligned} \tag{1}$$

Let $p_s$ and $\mu_s$ be the vectors of shadow prices of the aggregation constraints and the network capacity constraints, respectively. Necessary and sufficient conditions for an optimal solution $(q_s^{1*}, ..., q_s^{K*})$ include:

$$\nabla C^k(q_s^{k*}) = p_s^* \tag{2}$$

If the market is perfectly competitive, the optimal allocation can be obtained in a decentralized manner by letting agents purchase and sale electricity at nodal prices $p_s^*$. Agent $k$ then chooses $q_s^k$ that minimizes his private loss

$$C^k(q_s^k) - p_s^* \cdot q_s^k$$

where the dot denotes the interior product. Let $Q_s^k(p_s^*)$ be the solution to the agent's problem. Since he minimizes a strictly convex function, $Q_s^k(p_s^*)$ uniquely solves the same necessary and sufficient conditions (2) for a minimum. Aggregate net supply is defined as $Q_s(p_s^*) = \sum_K Q_s^k(p_s^*)$.

## 2.2 Transmission Rights and Hedging against Price Risks

A transmission property right of $\tau$ kW on line $ij$ is a financial title that pays at each period $\tau(p_{js} - p_{is})$ to its owner, where $p_{is}$ is the price at node $i$ under state of nature $s$. We use the $N$-vector $t(\tau, i, j) = (0, -\tau, 0, ..., 0, \tau, 0)$, where $t_i = -\tau$ and $t_j = \tau$, to represent such a right and, following Bushnell and Stoft (1996), we

[2]The Appendix of Bushnell and Stoft show how to adapt the model to lossy networks.

call it a "Transmission Congestion Contract" (TCC). Revenue from $t$ under state of nature $s$ is then $p_s \cdot t(\tau, i, j)$. Letting $T^k$ represent the set of TCCs owned by agent $k$, the combination of titles of the agent is given by $t^k = \sum_{t \in T^k} t(\tau, i, j)$ and the agent's expected transmission revenue is

$$E_s(p_s) \cdot t^k = \sum_s \pi_s \sum_{i=1}^{N} p_{is} t_i^k$$

where $\pi_s$ is the probability of state $s$.

We define agent's $k$ net benefit function under spot prices $p_s$ as the sum of transmission revenue, electricity supply revenue and cost:[3]

$$\Pi^k(p_s, t^k) = p_s \cdot Q_s^k(p_s) + p_s \cdot t^k - C^k(Q_s^k(p_s))$$

An agent can then eliminate transmission price risk by owning, at each node $i$,TCCs in quantity

$$t_i^k = \frac{-E_s(p_{is} Q_{is}^k(p_s))}{E_s(p_{is})} \tag{3}$$

since TCCs revenue $E_s(p_s) \cdot t^k = \sum_i E_s(p_{is}) t_i^k$ will exactly compensate for transmission costs $E_s(p_s \cdot Q_s^k) = E_s(\sum_i p_{is} Q_{is}^k)$.

Lemma 1 shows that if an agent is fully protected against transmission price risk, i.e. if titles are bought according to (3), he always gain from a change in expected prices that increase production value at least as much than TCCs values. This comes from the fact that he can always rearrange its output vector in order to exploit the price change.[4]

**Lemma 1.** *Let $\Delta E_s(\Pi^k(t)) = E_s(\Pi^k(p'_s, t)) - E_s(\Pi^k(p_s, t))$ be the change in net benefit after a change in expected prices from $E_s(p_s)$ to $E_s(p'_s) \neq E_s(p_s)$ and let $k$ be an agent owning a vector $t^k$ that satisfies (3). If $E_s(p'_s \cdot Q_s^k(p_s)) + E_s(p'_s) \cdot t^k \geq 0$, then $\Delta E_s(\Pi^k(t^k)) > 0$.*

[3] If an agent is a consumer, net supply revenue represents electricity expenditure and cost represents consumer's surplus or utility.

[4] The lemma and theorem of this paper, as well as their proofs, are equivalent to those presented in Bushnell and Stoft (1996). The differences come from their adaptations to uncertainty.

**Proof.**

$$\begin{aligned}
\Delta E_s(\Pi(t^k)) &= \sum_s \pi_s \left\{ \left[p'_s \cdot Q^k_s(p'_s) - p_s \cdot Q^k_s(p_s)\right] + \left[p'_s \cdot t^k - p_s \cdot t^k\right] \right. \\
&\qquad \left. -[C^k((Q^k_s(p'_s)) - C^k(Q^k_s(p_s))] \right\} \\
&= \sum_s \pi_s \left\{ \left[p'_s \cdot Q^k_s(p'_s) + p'_s \cdot t^k\right] \right. \\
&\qquad \left. - \left[C^k(Q^k_s(p'_s)) - C^k(Q^k_s(p_s))\right] \right\} \\
&\geq \sum_s \pi_s \left\{ \left[p'_s \cdot Q^k_s(p'_s) - p'_s \cdot Q^k_s(p_s)\right] \right. \\
&\qquad \left. - \left[C^k(Q^k_s(p'_s)) - C^k(Q^k_s(p_s))\right] \right\} \qquad (4)
\end{aligned}$$

where the second equality is obtained from (3) and where the inequality comes from the assumption that $E_s(p'_s \cdot (Q^k_s(p_s))) + E_s(p'_s) \cdot t^k \geq 0$. At a nodal equilibrium, $Q^k_s(p'_s)$ will be set such that $p'_s = \nabla C^k\left(Q^k_s(p'_s)\right)$. Substituting for $p'_s$ in (4) gives

$$\begin{aligned}
\Delta E_s(\Pi^k(t^k)) \geq \sum_s \pi_s \left\{ \nabla C^k\left(Q^k_s(p'_s)\right)\left[Q^k_s(p'_s) - Q^k_s(p_s)\right] \right. \\
\left. - \left[C^k(Q^k_s(p'_s) - C^k(Q^k_s(p_s)\right] \right\} \quad (5)
\end{aligned}$$

Convexity of $C^k$ implies that the term in curly brackets in (5) is non-negative whatever is $s$. As a result $\Delta E_s(\Pi^k(t^k)) \geq 0$. If $C$ is strictly convex, then the term in curly brackets is positive. ■

Under (3) the value of initial production (before the change of price) is equal to the value of TCCs at initial prices. Assumption $E_s(p'_s \cdot (Q^k_s(p_s))) + E_s(p'_s) \cdot t^k \geq 0$ thus implies that the change in expected prices are such that production value increases more than the value of TCCs.

Lemma 2 shows that the results of Lemma 1 applies to groups of agents as well as to individual agents. Let $G$ be a group of agents. The group's net supply is $Q^G_s(p_s) = \sum_{k \in G} Q^k_s(p_s)$ and collective ownership of TCCs amounts to $t^G = \sum_{k \in G} t^k$. The group's net benefit is then given by:

$$\Pi^G(p_s, t^G) = \Pi^G(p_s, \sum_{k \in G} t^k) = \sum_{k \in G} \Pi^k(p_s, t^k)$$

and the change of the group's marginal benefit following a change in expected prices from $E_s(p_s)$ to $E_s(p'_s)$ is:

$$\begin{aligned}\Delta E_s(\Pi^G(t^G)) &= E_s(\Pi^G(p'_s, t^G)) - E_s(\Pi^G(p_s, t^G)) \\ &= \sum_{k \in G}(E_s(\Pi^k(p'_s, t^k)) - E_s(\Pi^k(p_s, t^k))) \\ &= \sum_{k \in G} \Delta E_s(\Pi^k(t^k))\end{aligned}$$

**Lemma 2.** *Let $G$ be a group of agents whose TCCs ownership $t^G$ is such*

$$t_i^G = \frac{-E_s(p_{is} Q_{is}^G(p_s))}{E_s(p_{is})}, \forall i \tag{6}$$

*If* $E_s(p'_s \cdot (Q_s^G(p_s))) + E_s(p'_s) \cdot t^G \geq 0$*, then* $\Delta E_s(\Pi^G(t^G)) \geq 0$.

**Proof.** Assume that group $G$ holding of TCCs satisfies (6) and consider the initial distribution of TCCs among group members. A change in TCC distribution does not impact on electrical flows on the network since dispatch is performed independently of the allocation of TCCs. Assume an initial TCC distribution among group members and redistribute the TCCs so that it satisfies (3). Such a redistribution is always feasible since:

$$\sum_k t_i^k = -\sum_k \left[\frac{-E_s(p_i q_s^k(p_s))}{E_s(p_{is})}\right] = \left[\frac{-E_s(p_i \sum_k q_s^k(p_s))}{E_s(p_{is})}\right] = t_i^G$$

So it is possible to redistribute rights so that they satisfy (3) individually. Now consider the price change. By lemma 1, each member of the group will have $\Delta E_s(\Pi^k(t^k)) \geq 0$, so that this is also true that $\Delta E_s(\Pi^G(t^G)) = \sum_{k \in g} \Delta E_s(\Pi^k(t^k)) \geq 0$. Now redistribute the rights in their original pattern. Again, this leaves $\Delta E_s(\Pi(t^G))$ unaffected. ■

## 2.3 Allocation Rule and Investment Incentives

As financial rights, TCCs can in principle be emitted without any relation to the network operations. However, TCCs are generally issued by the ISO and ISO revenues are composed of transmission congestion rents. To insure that the ISO breaks even, one can constrain that total TCCs issued, $T = \sum_k \sum_{t(\tau,i,j) \in T^k} t(\tau, i, j)$, be such that

$$E_s(p_s^*) \cdot T = E_s(-p_s^* \cdot Q_s(p_s^*)) \tag{7}$$

where $p_s^*$ is the vector of nodal equilibrium prices.

**Lemma 3.** *If $T$ satisfies constraint (7), aggregate expected net benefit corresponds to expected welfare, i.e.*

$$E_s(\Pi(p_s^*, T)) = E_s(W(Q_s(p_s^*))$$

*where* $\Pi(p_s, T) = \sum_K \Pi^k(p_s, t^k)$ *and* $W(Q_s(p_s)) = -\sum_K C^k(Q_s^k(p_s))$

**Proof.** By definition,

$$E_s(\Pi(p_s^*, T)) = E_s\left[p_s^* \cdot Q_s(p_s^*) - \sum_k C^k(Q_s^k(p_s^*))\right] + E_s(p_s^* \cdot T) \qquad (8)$$

But $E_s(p_s^* \cdot T) = E_s(-p_s^* \cdot Q(p_s^*))$, which implies that

$$E_s(\Pi(p_s^*, T)) = E_s(-\sum_k C^k(Q_s^k(p_s^*))).$$

As a result, $E_s(\Pi(p_s^*, T)) = E_s(W(Q_s(p_s^*)))$. ■

When transmission property rights is created over an existing network, initial allocation rule used is the simplest that meets constraint (7).

**Definition 1.** *The* ***initial allocation rule*** *is such that:*

$$T_i = \frac{-E_s(p_{is}^* Q_{si}(p_s))}{E_s(p_{is}^*)}, \forall i \qquad (9)$$

Investment in transmission lines modify network configuration. The allocation rule for investment warrants that the ISO will break even after the investment as it did before the investment.

**Definition 2.** *The* ***investment allocation rule*** *grants a modifier of the grid the right to take any set $\hat{T}$ of TCCs that is such that the dispatch corresponding to $-(T + \hat{T})$ satisfies (9) under the new grid configuration, where $T$ is the previously allocated set of contracts. We thus have*

$$\hat{T}_i = \frac{-E_s(p'_{is} Q_{si}(p'_s))}{E_s(p'_{is})} - T_i, \forall i \qquad (10)$$

*where $p'_s$ represents the vector of equilibrium nodal prices after investment under state of nature $s$.*

**Proposition 1.** *Consider a modification to the grid such that $E_s(p'_s \cdot Q_s(p_s^*)) + E_s(p'_s \cdot T) \geq 0$. If the initial allocation satisfies (9) and the investment is granted TCCs according to (10), then the investor's rights have a negative value: $E_s(p'_s \cdot \hat{T}) < E_s \Delta W$.*

**Proof.** From Lemma 3, we have:

$$E_s(\Pi(p_s^*, T)) = E_s(W(Q_s(p_s^*))) \quad (11)$$

$$E_s(\Pi(p_s', T + \hat{T})) = E_s(\Pi(p_s', T)) + E_s(p_s' \cdot \hat{T}) = E_s(W(Q_s(p_s'))) \quad (12)$$

Substracting the first equality from the second one, we obtain:

$$E_s(p_s' \cdot \hat{T}) = E_s\Delta W - E_s\Delta\Pi(T) \quad (13)$$

Since $E_s\Delta\Pi(T) > 0$, we proved the result. ■

**Corollary 1.** *Consider a modification to the network that is such that*
*(i)* $E_s\Delta W < 0$
*(ii)* $E_s(p_s' \cdot Q_s(p_s^*)) + E_s(p_s' \cdot T) \geq 0$
*then* $E_s(p_s' \cdot \hat{T}) < 0$.

**Proof.** This follows directly from Proposition 1. ■

The proposition and its corollary describe the relation between transmission right allocations and externalities of investment on the network. From the corollary, if an investment reduces welfare, the value of TCCs that would obtain an investor is negative. This eliminates incentives to invest. However, from the proposition, we see that it is possible that $E_s(p_s' \cdot \hat{T}) < 0 < E_s\Delta W$, so that an investment that is socially desirable is not undertaken.

The important point to notice is that these results are exactly the same than those obtained by Bushnell and Stoft (1996) under certainty. They are obtained even though property rights are not contingent.

# 3 Conclusion

Efficiency properties of merchant transmission networks that have been proven in the economics literature depend on strong assumptions that are generally not met in actual electricity markets.[5] But this does not mean that these efficiency properties are not robust to some departures of assumptions. In this note, we have defined non-contingent property rights that preserve the efficiency properties proven by Bushnell and Stoft. As a result, the merchant transmission investment is probably easier to implement in face of uncertainty than it has been believed. This is not to deny the important gap that still exists between models of merchant

[5] See Joskow and Tirole Joskow and Tirole (2005) for a precise account of these assumptions and problems encountered in implementing merchant transmission investment when these conditions are not met.

transmission investment and reality,[6] but in face of the potential gains of merchant investment, we believe it is important to assess the exact consequences of this gap.

# References


[1] BUSHNELL, J.B. and S.E. STOFT, "Electric Grid Investment Under a Contract Network Regime", *Journal of Regulatory Economics,* 10 (1996), 61-79.

[2] JOSKOW, P. and J. TIROLE, "Merchant Transmission Investment", *Journal of Industrial Economics*, 53 (2005), 233-264.

[3] JOSKOW, P. and J. TIROLE, "Merchant Transmission Investment", mimeo.


[6]For instance, this note relies on the assumption that nodal prices are set on perfectly competitive wholesale markets while these markets are in reality characterized by ologopolistic strategic behavior.