Bundling under the Threat of Parallel Trade

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Abstract

This paper examines the use of bundling by a firm that sells in two national markets and faces entry by parallel traders. The firm can bundle its main product, -a tradable good- with a non-traded service. It chooses between the strategies of pure bundling, mixed bundling and no bundling. The paper shows that in the low-price country the threat of grey trade elicits a move from mixed bundling, or no bundling, towards pure bundling. It encourages a move from pure bundling towards mixed bundling or no bundling in the high-price country. The set of parameter values for which the profit maximizing strategy is not to supply the low price country is smaller than in the absence of bundling. The welfare effects of deterrence of grey trade are not those found in conventional models of price arbitrage. Some consumers in the low-price country may gain from the threat of entry by parallel traders although they pay a higher price. This is due to the fact that the firm responds to the threat of arbitrageurs by increasing the amount of services it puts in the bundle targeted at consumers in that country. Similarly, the threat of parallel trade may affect some consumers in the high-price country adversely.

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1 Introduction

Parallel trade takes place when a product introduced in a national market for sale to local buyers is diverted to another territory via distribution channels that are neither set up, nor authorized by the party that made the first sale. Parallel trade - also called grey trade - is not the same as trade in counterfeited goods. Products that circulate in parallel trade are genuine. They are marketed first by a person who often holds intellectual property rights in these products, or by a licensee of such person. What sets parallel trade apart from ordinary commerce is the diversion of products from the markets ostensibly targeted by the right holder.

The import of grey trade as a share of total trade is difficult to assess in quantitative terms. Within the European Union the share appears highest for musical recordings, soft drinks, cosmetics and perfumes.[1] A recent decision of the French Competition Commission cites an IMS Health estimate of 4%-5% for pharmaceuticals in the European market.[2]

Parallel trade takes place primarily in response to cross-country price disparities. As such, it curbs manufacturers' capacity to segment national markets. For that reason, firms threatened by grey imports take price and non-price measures to curtail grey trade.[3] They may reduce the quantities delivered in territories where prices are low, or determine that warranties are valid only within the territory of first sale. They may also call on technical devices to preclude the utilization in one country of an article originally sold in another territory[4], or take action to create in the mind of consumers a belief that grey goods are counterfeit, pirated, or of lesser quality. They may rely on patent, copyright, or trademark law to prevent the import of grey product into territories in which their intellectual property is protected.[5]

[1] National Economic Research Associates (1999).

[2] The penetration reaches 15% of the local market in The Netherlands. See Commission de la concurrence (2005).

[3] In a recent empirical paper Kyle (2005) finds evidence of price and non-price responses of pharmaceutical companies to the threat of parallel trade. The Bayer case provides a good illustration. To curtail parallel exports from France and Spain to the UK, the pharmaceutical firm puts a limit on orders originating from distributors in the exporting countries. Bayer was fined by the European Commission for violating Article 81(1) of the Treaty of Rome. This decision was overturned on appeal, and the annulment was confirmed by the European Court of Justice. See Joined Cases C-2/01 P and C-3/01 P: Bundesverband der Arzneimittel-Importeure eV against Commission of the European Communities at http://curia.eu.int/.

[4] This is the outcome of the regional coding systems of DVD's which is said to be designed to protect against piracy; see Dunt et al. (2001).

[5] The key variable in this regard is the exhaustion regime.Under a regime of na-

The welfare effects of parallel trade are ambiguous. On one hand, grey imports increase global welfare by mitigating price differences across markets; on the other hand, they may bring about a reduction of output below the level attained under discriminatory pricing. Malueg and Schwartz (1994) show that when there is a large disparity in the willingness to pay across national markets, a mixed regime of discrimination across groups of countries but not within groups, produces greater world welfare than uniform pricing. In a subsequent paper Szymanski and Valetti (2005) establish conditions under which the benefits from parallel trade to consumers in high valuation countries come at the expense of a reduction in quality. The interaction of the parallel import regime with the tariff as a determinant of welfare is examined by Knox and Richardson (2002).

Interestingly, there exist circumstances under which manufacturers benefit from the activity of parallel traders. Anderson and Ginsburgh (1999), and Haller and Jeanneret (1999) show that when the grey product is believed by consumers to be of lesser quality than 'authorized' product, or when consumers differ from each other in respect to the cost of acquiring grey products, manufacturers can bring parallel trade into play as a means of dividing a national market into a segment that disburses a premium price for legitimate' product, and another segment that acquires grey imports at a lower price.[6] In a recent paper Raff and Schmitt (2005) establish that when demand is random and distributors must order before demand is revealed to them, a manufacturer may benefit by letting the distributors engage in parallel trade. Such trade allows the distributors to ship product abroad if the local demand is lower than expected. This reduces distributors' risk, and encourages them to order

tional exhaustion, the person holding an intellectual property right waives the right to prohibit resale of the product in which the right is held within the national territory of the country where it was first sold legally. Under an international exhaustion regime, the original right holder forfeits the power to control resale within the national boundaries as soon as the product is legally put into circulation anywhere in the world. The particular regime a country chooses may vary from one type of IPR to another. Regimes may also be product-specific. The EU has adopted a regime of regional exhaustion with respect to trademarks. This means that it applies national exhaustion but treats the whole territory of the Union as a single country. The WTO does not limit countries' freedom to choose their exhaustion regime as long as it is not discriminatory. Crampes et al. (2005) provide an analysis of the position taken by European competition authorities on parallel trade among member countries and between member countries and outsiders.

[6] Anderson and Ginsburgh (1999) show that when two countries are served by a monopolistc manufacturer, changes in the cost of price arbitrage have an ambiguous effect on profits and welfare The reason is that while increases in the cost of arbitrage make it easier to engage in third degree discrimination, they also reduce the advantages from consumer segmentation in the home market.

larger quantities from the manufacturer.

Ganslandt and Markus (2005) have examined parallel trade in a framework that assumes a monopolistic producer who serves two markets via local distributors that enjoy a monopoly in each country. The manufacturer cannot prevent one of the distributors to sell in both markets. When setting the wholesale price, the manufacturer seeks to reconcile the objective of curtailing parallel trade with the objective of limiting the harm from double marginalisation. They find that for some parameter values a reduction in the trading cost brings about a cross country divergence rather than a convergence of prices.

Horn and Shy (1996) look at grey trade in the framework of a model that stands out from the aforementionned papers in two respects: a) They allow duopolistic interaction by making each market accessible to two firms; b) the firms take non-price action to lower the return from price arbitrage. Each firm limits the profitability of price arbitrage by bundling its tradeable product with an exogenously determined quantity of a non-tradeable commodity that has no resale value in the local market. Arbitrage is precluded because the arbitrageur who purchases a bundle in one country can only resell the traded good. They find that one firm bundles in equilibrium, and that such bundling allows both firms to escape Bertrand competition.

This paper is related to Horn and Shy in that it also looks at bundling between a traded and a non-traded good. However, there are several differences. This paper assumes a single producer. Therefore, the avoidance of Bertrand competition cannot be a motive for bundling. Also the firm adopts a bundling strategy even when the markets are segmented. What elicits bundling is the fact that consumers' utility is a function of the amount of services consumed in conjunction with the product. Bundling the product with services- the quantity of which is endogeneous in the paper - allows better extraction of consumer surplus.

The central question addressed in the paper is how a threat of entry by parallel traders affects prices, and how it determines the form of bundling chosen by the firm. The firm decides between pure bundling, mixed bundling and the sale of the tradable alone. Under pure bundling, it offers nothing but a package that includes one unit of product and a specific amount of services. Under mixed bundling it gives consumers the additional option of purchasing the product alone, that is without any services added. Examples of such services are warranties with validity limited to the territory of the country of first sale, and customer support given to clients who purchase from local distributors.

The second section of the paper introduces the notation and sets out the basic assumptions. The third section derives a solution for the

benchmark case of segmented markets. It finds that mixed bundling dominates pure bundling in terms of profits and consumer surplus, and that the extent of dominance is an increasing function of the amount of services contained in the bundle.

The fourth section explores the response of the firm to the threat of entry of parallel traders who seek to profit from a cross-country disparity in prices. It shows that the threat of entry encourages a move from mixed bundling, or from the sale of traded good alone towards pure bundling in the country in which price would be lower if markets were segmented. The same threat brings forth a move from pure bundling in the country where price would be highest under segmentation toward mixed bundling or the sale of the product alone. The set of parameter values for which the profit maximizing strategy is not to supply the low price country, is smaller than in the absence of bundling. Also the welfare effects of deterrence of price arbitrage are not necessarily those found in the conventional model where the option of bundling does not exist. As in the conventional model, some consumers in the low-price country lose because they pay a higher price. However, there are also consumers in the low-price country who gain. The reason is that the firm responds to the threat of price arbitrage by increasing the amount of services that it puts in the bundle. Consumers in the low price country who gain benefit more from the extra services than they lose from the higher price.

The final section of the paper revisits some the assumptions of the model. It discusses the relevance of the model in light of the fact that no parallel trade exists in equilibrium and offers perturbations that generate a positive amount of grey trade.

2 Assumptions and notation

A firm serves two national markets; a home market indexed "h", and a foreign market indexed "f". The firm is the sole producer of a homogeneous good and a service that is consumed in conjunction with that good.

Each national market has a continuum of consumers with Mussa-Rosen preferences. The consumers are identified by their taste parameter θ, which is distributed uniformly over the support $[0, b^k]$ in country k, where $k = \{h, f\}$, and $b^h > b^f$. Consumers purchase either one unit of the product or none at all. The consumer θ who spends $p(z)$ for a package that contains one unit of product and z units of service gets a surplus $CS(z) = \theta(1+z) - p(z)$. The services do not generate any utility when consumed without the product.

The cost of producing the product alone is w per unit in the two countries, and $w < b^k$. The firm incurs a cost $qc(z^k) + F$ to produces q

bundles each containing z^k units of service. The cost of service function which is the same in both countries has $c(0) = 0, c'(.) > 0$ and $c"(.) > 0$. The firm incurs a fixed cost F in every country in which it sells services contained in a bundle. There is no outside supplier of services.

The firm can engage in pure bundling, or in mixed bundling. Under pure bundling it offers consumers nothing but a package that contains one unit of product and z units of service. Under mixed bundling it offers the aforementioned package as well as the product alone. The firm may opt for pure bundling in one country and mixed bundling in the other country. Also, the bundle the firm offers in country h need not contain the same amount of services as the bundle it offers in country f.

The firm must determine the following: a) the type of bundling it will engage in; b) the amount of services it will include in the home and foreign packages; c) prices.

Parallel traders are perfect competitors. They engage in international price arbitrage at a cost y per unit. They can only trade in the product. This means that when traders purchase a bundle in one country they can only resell in the other country the product component. The firm does not incur a cost when it trades internationally.[7]

The paper makes a distinction between segmented markets, and markets linked by arbitrage. Markets are segmented when trade between them is impossible, or when the trading cost y is larger than the gap in prices that the firm would set in the two countries if trade were in fact ruled out.

Segmentation requires that there be no consumer who derives from a unit of product stripped of services and imported via a parallel channel, a surplus larger than the surplus that the consumer derives from purchase of a package, or a product alone obtained via an "official" channel.

Upper case letters denote prices and quantities in segmented markets; lower case letters denote the same in markets linked by arbitrage. For the case of segmentation P_i^k denotes the price at which the firm offers the bundle in country $k = \{h, f\}$ when it chooses bundling option i, where $i = \{B \equiv pure\ bundling, M \equiv mixed\ bundling\}$. Z_i^k denotes the number of units of service included in the package in country k under the bundling option i. R^k denotes the price of the product alone in country k. Similarly, p_i^k, r^k and z_i^k denote prices and service quantities when markets are linked by arbitrage.

[7] This simplifies the analysis because the results are unaffected by the location of the firm's production facilities.

3 The benchmark case: national markets are segmented.

We proceed in three steps. First, we show that under mixed bundling the firm includes a larger amount of services in the bundle than under pure bundling. Next, we establish that when the firm offers the same amount of services per product under the two bundling regimes, mixed bundling generates a higher profit. Finally, we prove that the profits from pure bundling are higher than those from mixed bundling when the amount of services under the two options is chosen optimally. Because the qualitative results are the same for each country when markets are segmeted, we do not use the superscripts h and f in this section.

3.1 Pure bundling

Under pure bundling the firm sells to consumers who have a preference index $\theta \in [\underline{\Theta}_B, b]$ where $\underline{\Theta}_B = \frac{P_B}{1+Z_B}$.

The profit from pure bundling in a single country, denoted $\Pi_B = \frac{1}{b}\left[P_B - w - c(Z_B)\right]\left[b - \underline{\Theta}_B\right] - F$ is maximized when the price of the bundle and the amount of services it contains satisfy the conditions (1) and (2) below[8]

(1) $P_B = \frac{1}{2}\left[b\left(1 + Z_B\right) + w + c(Z_B)\right] = \frac{1}{2}\left[(b+w) + Z_B(b + \frac{c(Z_B)}{Z_B})\right]$

(2) $c'(Z_B) = \underline{\Theta}_B$ or $\frac{1}{2}\left[(b - \frac{c(Z_B)}{Z_B}) + \frac{1}{1+Z_B}(w - \frac{c(Z_B)}{Z_B})\right] - [c'(Z_B) - \frac{c(Z_B)}{Z_B}] = 0$

To constitute an equilibrium the values of Z_B and P_B determined by (1) and (2) must generate a non-negative profit margin and market area. This requires $(b - w) + Z_B(b - \frac{c(Z_B)}{Z_B}) > 0$ for $Z_B > 0$.[9]

Conditions (1) and (2) entail

$$\frac{\partial Z_B}{\partial b} > 0 \text{ and } \frac{\partial P_B}{\partial b} = (1 + Z_B)\, c''(Z_B)\left[b - \frac{P_B}{1+Z_B}\right] + c'(Z_B)\frac{\partial Z_B}{\partial b} > 0$$

For any given level of services, price increases when b increases. The marginal consumer's preference index also increase with b. Because the firm chooses the service level according to the marginal consumer's reservation price, it responds to the increase in b by raising the amount of services contained in the bundle. This brings forth a further increase in P_B.

[8] For computational details see the appendix.

[9] Note from (2) that $Z_B > 0$ requires $c'(0) < (b + w + c(0))/2$.

For later reference it is useful to substitute (1) into the profit function and write

$$(3) \qquad \Pi_B = \frac{\left[(b-w)+Z_B\left(b-\frac{c(Z_B)}{Z_B}\right)\right]^2}{4b(1+Z_B)} - F$$

3.2 Mixed bundling

Consumers who buy the bundle have a preference index θ such that $\theta(1+Z_M) - P_M \geq \max[0, \theta - R]$; consumers who purchase the product alone have preference index θ where $\theta - R > \max[0, \theta(1+Z_M)) - P_M]$. Profits are $\Pi_M = \frac{1}{b}\left\{[P_M - w - c(Z_M)]\left[b - \widetilde{\Theta}_M\right] + [R-w]\left[\widetilde{\Theta}_M - R\right]\right\} - F$ where $\widetilde{\Theta}_M = \frac{P_M - R}{Z_M}$ indexes the consumer who is indifferent between the bundle and the product alone.

First order conditions consistent with the existence of two classes of buyers are

$$(4) \qquad R = \tfrac{1}{2}[w+b]$$

$$(5) \qquad P_M = R + \tfrac{Z_M}{2}\left(b + \tfrac{c(Z_M)}{Z_M}\right) = \tfrac{1}{2}[b(1+Z_M) + w + c(Z_M)]$$ [10]

Condition (4) shows that the price of the product alone does not depend on the amount of services contained in the bundle. In fact, it is the very same price the firm would set if it did not offer any services. The firm sets prices as if selling two unrelated goods; a product alone at the monopoly price R, and a package of services at the monopoly price $\frac{1}{2}[bZ_M + c(Z_M)]$. The convexity of $c(z)$ and condition (5) entail that the price premium per unit of service - that is $\frac{P_M - R}{Z_M}$ - increases in the number of units of service contained in the bundle. Conditions (1) and (5) imply that when $Z_M = Z_B$ the price of the bundle is the same under the pure and mixed bundling regimes.

Jointly, (4) and (5) yield

$$(6) \qquad \Pi_M = \tfrac{1}{4b}\left\{[b-w]^2 + Z_M\left[b - \tfrac{c(Z_M)}{Z_M}\right]^2\right\} - F$$

The amount of services that maximizes profits satisfies

$$(7) \qquad c'(Z_M) = \widetilde{\Theta}_M = \tfrac{1}{2}\left[b + \tfrac{c(Z_M)}{Z_M}\right]$$

or equivalenty $\frac{1}{2}\left[b - \frac{c(Z_M)}{Z_M}\right] - [c'(Z_M) - \frac{c(Z_M)}{Z_M}] = 0$.[11]

[10] See appendix for derivation of (3) and (4).

[11] For derivation see the appendix.

It is straightforward to show that the existence of an equilibrium that has positive profit margin and market area for the bundle as well as for the product alone requires that (8) below holds

(8) $\quad b > \frac{c(Z_M)}{Z_M} > w$[12]

Specifically when $\frac{c(Z_M)}{Z_M} < w$ the firm engages in pure bundling; when $\frac{c(Z_M)}{Z_M} > b$ the firm sells no services, it limits itself to the product alone. Note that condition (8) entails $(b-w)+Z_B(b-\frac{c(Z_B)}{Z_B}) > 0$ implying that a pure bundling equilibrium exists when a mixed equilibrium exists.

3.3 Pure bundling *vs.* mixed bundling

The following lemma compares profits Π, consumers' welfare CS and total welfare $W = \Pi + CS$ under pure and mixed bundling.

Lemma 1: When there exists a mixed bundling equilibrium with exogenous $Z > 0$,

$$i) \quad \Pi_M(Z) - \Pi_B(Z) = \frac{1}{4b}\frac{Z}{1+Z}\left[\frac{c(Z)}{Z} - w\right]^2 > 0,$$
$$ii) \quad CS_M(Z) > CS_B(Z),$$
$$iii) \quad W_M(Z) > W_B(Z).$$

Proof : Part i) follows from subtraction of (3) from (6). Part ii) follows from (1) and (5) which show that the price of the bundle is the same under the two regimes when Z is the same. The implication is that consumers who purchase the bundle enjoy the same surplus under both regimes. But then it must be true by revealed preference that consumers who purchase the product alone under the mixed regime and also purchase under pure regime must be better off under the former. Revealed preferences also entails that consumers who make a purchase under the mixed regime and do not purchase under pure regime are better off under the former. Part iii) follows directly from i) and ii).

A mixed regime produces larger profits because it allows the firm to differentiate it product. Under the mixed regime the firm can sell at a low price to consumers with low θ and still collect a premium from consumers with high θ who purchase the bundle.

Because mixed bundling yields higher profits than pure bundling for all Z, it also yields higher profits when $Z = Z_B$ that is when Z is chosen to maximize the profits under the pure bundling regime. Therefore, it is surely true that $\Pi_M(Z_M) > \Pi_B(Z_B)$ where Z_M is chosen to maximize the profits from mixed bundling.

[12] Condition (8) cannot be met when $c'(0) > b$ and/or F is large.

The next lemma compares the quantity of services Z and the price P under pure and mixed bundling,

Lemma 2: When there exists a mixed bundling equilibrium in which the amount of services is chosen by the firm,

i) $Z_M > Z_B$ and $P_M > P_B$;

ii) $Z_M = \arg\max W(Z)$ when prices are given by (4) and (5)

Proof: Part *i)* follows from concavity of the profit function. From the first order conditions (2) and (7), we have $\frac{\partial \Pi_B}{\partial Z_B} < 0$ when Z_B is determined by (7) instead of (2). Therefore, it must be true that the equilibrium amount of services under pure bundling is smaller than under mixed bundling. And, if the latter is true one has $P_M > P_B$ by (1) and (5).

To show part *ii)* note that

$$W_M(Z) = \frac{1}{b}\left\{ \int_R^b (\theta - w)\, d\theta + \int_{\widetilde{\Theta}_M(Z)}^b [\theta Z - c(Z)] d\theta \right\} - F$$
$$= \frac{1}{b}\int_R^b (\theta - w)\, d\theta + \frac{3}{8} Z \left[b - \frac{c(Z)}{Z} \right]^2 - F$$

where $\widetilde{\Theta}_M(Z)$ denotes the marginal buyers of the bundle when the amount of services is Z and prices satisfy (4) and (5). Therefore, $\frac{dW_M(Z)}{dZ} = \frac{3}{8}\left\{ \left[b - \frac{c(Z)}{Z} \right] \left[b - 2c'(Z) + \frac{c(Z)}{Z} \right] \right\}$ [13]. Because $\left[b - \frac{c(Z)}{Z} \right] > 0$, it follows that $\frac{dW_M(Z)}{dZ} = 0$ when $\left[b - 2c'(Z) + \frac{c(Z)}{Z} \right] = 0$ which is the same as condition (7).

The intuition behind part *i)* is a simple one. Assume that starting from the equilibrium under the mixed regime one prohibits the sale of the product alone. With bundle price and services unchanged, consumers with θ slightly below $\widetilde{\Theta}_M$ will now purchase the bundle; consumers with θ significantly below $\widetilde{\Theta}_M$ who purchase the product alone under the mixed regime, will now refrain purchasing. Because the marginal buyer now has $\theta < \widetilde{\Theta}_M$, the optimal policy is to lower the amount of services contained in the bundle as well as the price.

Part *ii)* is more surprising. To see why the profit maximizer and the welfare maximizer choose the same amount of services note from

[13] For derivation see the appendix.

$\frac{dW_M(Z)}{dZ} = -\left[\widetilde{\Theta}_M Z - c(Z)\right]\frac{\partial\widetilde{\Theta}_M}{\partial Z} + \int\limits_{\widetilde{\Theta}_M(Z)}^{b}[\theta - c'(Z)]d\theta = 0$ that when the number of buyers of the bundle is given welfare is maximized when the marginal cost of services equals the marginal willingness to pay for services by the buyer of the bundle whose marginal willingness to pay for services is the average. Also, note that $\frac{\partial\Pi_M}{\partial Z} = \frac{\partial\left[\widetilde{\Theta}_M Z - c(Z)\right]\left[b-\widetilde{\Theta}_M\right]}{\partial Z} = -\left[\widetilde{\Theta}_M Z - c(Z)\right]\frac{\partial\widetilde{\Theta}_M}{\partial Z} + \left[\widetilde{\Theta}_M + Z\frac{\partial\widetilde{\Theta}_M}{\partial Z} - c'(Z)\right]\left[b - \widetilde{\Theta}_M\right]$.

The latter shows that when the number of buyers of the bundle is *given*, the firm maximizes profits by setting the amount of services so that the marginal cost of services equals the marginal revenue for services derived from the *marginal* buyer of the bundle. That marginal revenue increases when Z increases because the bundle contains more services *and* because the premium that the marginal buyer is willing to pay per unit of service increases when the number of services contained in the bundle inceases. Result *ii*) holds because in equilibrium, the *marginal* revenue from services that the profit maximizing firm extracts from the *marginal* buyer of the bundle equals the *average* price premium that the average buyer is willing to pay per unit of service.

A corrolary of Lemma 2 is that for endogeneous Z total welfare is higher under mixed bundling than under pure bundling. The reason is as follows: By Lemma 1, $W_M(Z) > W_B(Z)$ for all Z, and by Lemma 2, $Z_M = \arg\max W(Z)$ when prices are given by (5) and (6). Therefore is is true that $W_M(Z_M) > W_B(Z_B)$.

To compare consumer welfare under the two bundling regimes we define the following preference indexes:

$\underline{\Theta}_M \equiv \frac{1}{2}[b + w]$ index of the marginal buyer of the product alone under mixed bundling;

$\underline{\Theta}_B \equiv \frac{1}{2}[b + \frac{w+c(Z_B)}{1+Z_B}]$ index of the marginal buyer under pure bundling;

$\widetilde{\Theta}_M \equiv \frac{1}{2}[b + \frac{c(Z_M)}{Z_M}]$ index of the buyer who is indifferent between the product alone and the bundle under mixed bundling;

$\underset{\longleftrightarrow}{\Theta} \equiv \frac{1}{2}[b + \frac{c(Z_B)}{Z_B}]$ index of the buyer of the product alone under mixed bundling who is as well off as under pure bundling;

$\overleftrightarrow{\Theta} \equiv \frac{p_M - p_B}{Z_M - Z_B} = \frac{1}{2}\left[b + \frac{c(Z_M)-c(Z_B)}{Z_M - Z_B}\right]$ index of the buyer of bundle under mixed bundling who is as well off as under pure bundling.[14]

[14] See appendix for the derivation of $\underset{\longleftrightarrow}{\theta}$ and $\overleftrightarrow{\theta}$.

The following lemma is sufficient to establish that some consumers are better off under mixed bundling than under pure bundling and others are worse off.

Lemma 3 : The ranking of the preference indexes of marginal consumers is $\underline{\Theta}_M < \underline{\Theta}_B < \underset{\longleftrightarrow}{\Theta} < \widetilde{\Theta}_M < \overset{\longleftrightarrow}{\Theta} < b$.

Proof: The conditions $\underline{\Theta}_M < \underline{\Theta}_B < \underset{\longleftrightarrow}{\Theta}$ follow directly from (8), while $\underset{\longleftrightarrow}{\Theta} < \widetilde{\Theta}_M < \overset{\longleftrightarrow}{\Theta}$ follows from Lemma 2, $c"(.) > 0$ and (8). Also, $c'(.) > 0$, $c"(.) > 0$ and $Z_M > Z_B$ entail $c'(Z_M) > \frac{c(Z_M)-c(Z_B)}{Z_M-Z_B}$. Because $c'(Z_M) = \widetilde{\Theta}_M < b$ it must be true that $\overset{\longleftrightarrow}{\Theta} < b$.

The consumers with $\theta \in [\underline{\Theta}_M, \underline{\Theta}_B]$ are better off under the mixed regime since it is the only regime under which they make a purchase. Consumers with $\theta \in \left[\underline{\Theta}_B, \underset{\longleftrightarrow}{\Theta}\right]$ purchase the bundle in the pure regime and the product alone under the mixed regime. They are better off under the latter because they value the opportunity to purchase at a lower price more than they value the services they acquire under the pure bundling regime. Consumers with $\theta \in \left[\underset{\longleftrightarrow}{\Theta}, \widetilde{\Theta}_M\right]$ purchase the product alone under the mixed regime. They are worse off than under the pure regime because the extra services they get in the mixed regime do not compensate for the higher price. For that reason they opt for the product alone in the mixed regime. Consumers with $\theta \in \left[\widetilde{\Theta}_M, \overset{\longleftrightarrow}{\Theta}\right]$ purchase the bundle under the mixed regime, but they are worse off than under the pure regime. The reason is that the extra services found in the bundle under the mixed regime do not compensate for the higher price. However, unlike the consumers with lower θ, the higher price is insufficient to induce a purchase of the product alone. Finally, consumers with $\theta \in \left[\overset{\longleftrightarrow}{\Theta}, b\right]$ are better off under mixed bundling because the extra utility from additional services outweighs the disutility from a higher price.

4 National markets linked by arbitrage.

This section assumes that the cost of cross-country price arbitrage is sufficiently small to allow profitable entry by parallel traders when the firm sets segmentation prices and services levels. Nevertheless, parallel trade does not take place in equilibrium. This is a direct consequence of

the assumption that the firm can ship product to both markets at zero cost, whereas parallel traders incur a positive trading cost. [15]

In conventional models that do not consider the option of bundling, a threat of entry by parallel traders brings about a convergence of prices as long the firm serves both markets. When there is a large asymmetry in market sizes and/or willingness to pay, the firm may withdraw from the smaller/poorer market. The bundling option opens up additional margins of adjustment. The firm can now also relax the arbitrage constraint by switching bundling regimes. Specifically, the profitability of parallel trade is reduced by a switch in the foreign country from the product alone to pure bundling, or from mixed bundling towards pure bundling, Similarly, the profitability of parallel trade falls when the firm switches from pure bundling towards mixed bundling in the home county. The existence of additional margins of adjustment implies that the range of parameter values for which the firm serves both countries is larger.

To explore the firm's responses to a threat of parallel trade it is useful to start by listing the range of possible equilibria under segmentation. Table 1 gives an overview of equilibria, and states the conditions under which they occur.

Table 1
Equilibrium configurations under market segmentation[16]

home / *foreign*	mixed bundling M^h	pure bundling B^h	product alone A^h
mixed bundling M^f	$b^h > \frac{c(Z_M^h)}{Z_M^h} > w$ $b^f > \frac{c(Z_M^f)}{Z_M^h} > w$	impossible	impossible
pure bundling B^f	$b^h > \frac{c(Z_M^h)}{Z_M^h} > w$ $b^f > w > \frac{c(Z_M^f)}{Z_M^f}$	$b^h > w > \frac{c(Z_M^h)}{Z_M^h}$ $b^f > w > \frac{c(Z_M^f)}{Z_M^f}$	impossible
product alone A^f	$b^h > \frac{c(Z_M^h)}{Z_M^h} > w$ $\frac{c(Z_M^f)}{Z_M^f} > b^f > w$	$b^h > w > \frac{c(Z_M^h)}{Z_M^h}$ $\frac{c(Z_M^f)}{Z_M^f} > b^f > w$	$\frac{c(Z_M^h)}{Z_M^h} > b^h > w$ $\frac{c(Z_M^f)}{Z_M^f} > b^f > w$

[15] The implication of that assumption is that any cross-country reallocation of output by traders, can be replicated by the manufacturer, and that under such replication the firm earns a higher profit than in the absence of replication.

[16] The conditions that appear in the cells of table 1 refer to mixed bundling . The reason is that whenever $\Pi_M(Z_M) > 0$ for $Z_M > 0$, one has $\Pi_M(Z_M) > \Pi_B(Z_B)$ (see section 3.3).

Table 1 indicates that there cannot exist a segmentation equilibrium under which the firm engages in pure bundling in the home country and mixed bundling in the foreign country. Indeed, because $b^h > b^f$, condition (7) entails $c(Z_M^h)/Z_M^h > c(Z_M^f)/Z_M^f$. But then, it cannot be true that $c(Z_M^h)/Z_M^h < w < c(Z_M^f)/Z_M^f$ which, by (8), is required if the equilibrium is (B^h, M^f). The combination A_h and M_f cannot arise either. Indeed, by (7) mixed bundling in the foreign country requires $b^f > c'(Z_M^f) > c(Z_M^f)/Z_M^f$. By (8), selling the product alone requires that $c\prime(Z) \geq \frac{c(Z)}{Z} > b^h$ for all $Z \geq 0$. But, both conditions cannot hold simultaneously as $Z_M^f > 0$, and $b^h > b^f$. A similar reasoning rules out the combination (A^h, B^f).

The remainder of this section explores possible adjustments for each of the feasible segmentation equilibria displayed in Table 1. It explores the response of the firm to a changes in the cost of trading when the service cost function has the form $c(z) = mz^a + tz$ where $a > 0$. MATHLAB software generates numerical illustrations of adjustments. The richest set of responses emerges for parameter values which under segmentation yield pure bundling in the home country and sales of the product alone in the foreign country. We discuss this case in more detail than the other cases.

4.1 Case 1: The segmented equilibrium has mixed bundling in the home and foreign markets

Case 1 breaks down as follows: Subcase 1a is where the trading cost y is smaller than the cross-country gap between the prices of the product alone, but is larger than the gap between the home price of the product alone, and the price of the foreign bundle. Subcase 1b is where the trading cost is smaller than the gap between the segmentation price of the product alone in the home country, and the segmentation price of the foreign bundle.

Subcase 1a: $R^h - P_B^f < y < R^h - R^f$. Continuity entails that

when y is only slightly smaller than $R^h - R^f$, the firm maintains a mixed bundling regime in the two countries. If so, it chooses $r^h, r^f, p_M^h, p_M^f, z_M^h$ and z_M^f to maximize the following profit function

$$(9)\ \pi_{MM}(y) = \tfrac{1}{b^h}\left\{\left[p_M^h - w - c(z_M^h)\right]\left[b^h - \widetilde{\theta}_M^h\right] + \left[r^h - w\right]\left[\widetilde{\theta}_M^h - r^h\right]\right\} + \tfrac{1}{b^f}\left\{\left[p_M^f - w - c(z_M^f)\right]\left[b^f - \widetilde{\theta}_M^f\right] + \left[r^f - w\right]\left[\widetilde{\theta}_M^f - r^f\right]\right\} - 2F$$

where $\widetilde{\theta}_M^k = \frac{p_M^k - r^k}{z_M^k}$ for $k = \{h, f\}$ and $r^h = r^f + y$.

Upon defining $\alpha \equiv \frac{b^f}{b^f+b^h}$,the first order conditions in regard to prices and services can be written as (10)- (13) below

(10) $r^f = \frac{1}{2}[\alpha b^h + (1-\alpha)b^f + w] - \alpha y$

(11) $r^h = \frac{1}{2}[\alpha b^h + (1-\alpha)b^f + w] + (1-\alpha)y$

(12) $p_M^k = r^k + \frac{1}{2}[b^k z_M^k + c(z_M^k)]$ for $k = \{h, f\}$

(13) $c'(z_M^k) = \frac{1}{2}[b^k z_M^k + c(z_M^k)]$ for $k = \{h, f\}$

Conditions (10) and (11) show that r^h falls and r^f increases when y decreases. Also, from (12)-(13) and (5)-(7) it follows that $\widetilde{\theta}_M^k = \widetilde{\Theta}_M^k$ and $z_M^k = Z_M^k$ for $k = \{h, f\}$. Because neither the price premium on the bundle -that is $(p_M^k - r^k)$- nor the quantity of services contained in the bundle adjust in reponse to a fall in y, it follows that the lower trading cost benefits all home buyers of the bundle and the product alone. All foreign consumers are affected adversely by a fall in y.

Because the firm's profits fall when y decreases, $\pi_{MM}(y)$ may eventually drop below the profit generated by a segmentation equilibrium that has mixed bundling in the home country and pure bundling in the foreign country. Defining $\widehat{y}$ by $\pi_{MM}(\widehat{y}) = \Pi_M^h + \Pi_B^f$ we can state that (10)-(13) characterize the equilibrium for $y \in (\widehat{y}, R^h - R^f]$, whereas for $y \in [0, \widehat{y}]$ conditions (1),(2) and (4), (5),(7) characterize the equilibrium respectively for the foreign and home country. A switch to pure bundling in the foreign country leaves home consumers as well off as under segmentation. The effect of foreign consumers is given by Lemma 3.

One easily checks that for parameter values $b^h = 2.6, b^f = 1.7, w = 0$, $c(z) = \frac{1}{2}z^2$ and $F = 0$, and $y > 0.25$, the firm earns higher profits from mixed bundling in both countries. For lower y it earns higher profits by engaging in mixed bundling in the home country and pure bundling in the foreign country.

Subcase 1b: $0 < y < R^h - P_B^f$. Clearly, the mere switch from a mixed regime to a pure regime in the foreign country does no longer deter parallel trade if prices are set at the level of the segmented equilibrium.

Continuity of the profit function entails that when y is only slightly smaller than $R^h - P_B^f$ the equilibrium prices and service levels are obtained by maximization of

(14) $\pi_{MB}(y) = \frac{1}{b^h}\left\{\left[p_M^h - w - c(z_M^h)\right]\left[b^h - \widetilde{\theta}_M^h\right] + \left[r^h - w\right]\left[\widetilde{\theta}_M^h - r^h\right]\right\} +$
$\frac{1}{b^f}\left\{\left[p_B^f - w - c(z_B^f)\right]\left[b^f - \frac{p_B^f}{1+z_B^f}\right]\right\} - 2F$

$$\text{subject to } \widetilde{\theta}_M^h = \frac{p_M^h - r^h}{z_M^h} \text{ and } r^h = p_B^f + y.$$

Upon defining $\beta \equiv \frac{b^f(1+z_B^f)}{b^h+b^f(1+z_B^f)}$ the first order conditions with respect to prices and services can be written as (15) - (19) below[17]

$$(15) \qquad r^h = \frac{1}{2}\left\{\beta(b^h+w)+(1-\beta)[b^f(1+z_B^f)+w+c(z_B^f)+2y]\right\}$$

$$(16) \qquad p_M^h = r^h + \frac{z_M^h}{2}\left[b^h + c(z_M^h)/z_M^h\right]$$

$$(17) \qquad p_B^f = r^h - y = \frac{1}{2}\left\{w+\beta(b_h-2y)+(1-\beta)[b^f(1+z_B^f)+c(z_B^f)]\right\}$$

$$(18) \qquad c'(z_M^h) = \frac{p_M^h - r^h}{z_M^h} = \frac{1}{2}\left[b^h + \frac{c(z_M^h)}{z_M^h}\right]$$

$$(19) \qquad c'(z_B^f)\left[b^f\left(1+z_B^f\right) - p_B^f\right] - \frac{p_B^f}{1+z_B^f}\left[p_B^f - w - c(z_B^f)\right] = 0$$

It follows from (18) and (7) that the amount of services contained in the home country bundle is the same as under the segmentation equilibrium. By (16) then, it also follows that the gap between the price of the bundle and the price of the product alone in the home country are not affected by changes in y. But then, the preference index of the home consumer indifferent between the bundle and the product alone is the same as under the segmentation equilibrium.

Note that (17) and (19) constitute a simultaneous system of equations in z_B^f and p_B^f. We know that $\frac{\partial z_B^f}{\partial y} = \frac{1}{\Delta}\begin{vmatrix} \frac{\partial^2\pi_{MB}}{\partial(p_B^f)^2} & -\frac{\partial^2\pi_{MB}}{\partial y \partial p_B^f} \\ \frac{\partial^2\pi_{MB}}{\partial p_B^f \partial z_B^f} & -\frac{\partial^2\pi_{MB}}{\partial y \partial z_B^f} \end{vmatrix}$ where $\Delta = \left[\frac{\partial^2\pi_{MB}}{\partial(p_B^f)^2}\cdot\frac{\partial^2\pi_{MB}}{\partial(Z_B^f)^2} - \left(\frac{\partial^2\pi_{MB}}{\partial z_B^f \partial p_B^f}\right)^2\right] > 0$ and $\frac{\partial^2\pi_{MB}}{\partial(p_B^f)^2} < 0$ by the second order conditions. Also, $-\frac{\partial^2\pi_{MB}}{\partial y \partial p_B^f} = \frac{1}{b^h}\left[2+\frac{1}{z_B^f}\right] > 0$, $\frac{\partial^2\pi_{MB}}{\partial p_B^f \partial z_B^f} = \frac{1}{b^f}\left\{\frac{p_B^f}{(1+z_B^f)^2} + \frac{p_B^f - w - c(z_B^f)}{1+z_B^f} + \frac{c'(z_B^f)}{1+z_B^f}\right\} > 0$, and $\frac{\partial^2\pi_{MB}}{\partial y \partial z_B^f} = 0$. Therefore $\frac{\partial z_B^f}{\partial y} < 0$. Since (20) entails $\frac{\partial z_B^f}{\partial p_B^f} > 0$, it must be true that $\frac{\partial p_B^f}{\partial y} < 0$.

Because $\pi_{MB}(y)$ falls monotonically as the trading cost falls, there may exist a positive $y = \overleftrightarrow{y}$ such that $\pi_{MB}(\overleftrightarrow{y}) = \Pi_M^h$. The prices and service amounts given by (15)-(19) constitute an equilibrium for $y \in (\overleftrightarrow{y}, R^h - P_B^f)$. For lower y the firm serves only the home market.

As long as the firm serves both countries, all home consumers benefit from a fall in y. This is because both the price of the product alone and the price of the bundle fall, and the amount of services contained in the bundle does not change. Foreign consumers with the low willingness to pay lose because $p_B^f/\ (1+\ z_B^f)$ rises when y falls. However, some foreign

[17] See appendix for computational details

consumers with high θ may gain as their benefit from extra services outweighs the adverse effect of the price increase.

For parameter values $b^h = 4.2$, $b^f = 0.6, w = 0$, $c(z) = \frac{1}{2}z^2$, and $F = 0$, one finds that for $y \in [0.6, 1.8]$ profits are highest when the firm engages in mixed bundling in the home country and pure bundling in the foreign country. As y falls from 1.8 to 0.6, welfare increases monotonically from 7.99 to 9.48; z_B^f increases from 0.31 to 72, and p_B^f falls from 2.30 to 1.36. All consumers in the home country gain, and all consumers in the foreign country lose as y falls. For $y < 0.6$ profits are highest when the firm does not serve the foreign market, and adopts a mixed bundling regime in the home country. However, for $c(z) = 0.15z^2$ and the remaining parameters as above, foreign consumers with preference parameter in the neighborhood of b^h gain when the trading cost falls.

4.1.1 Case 2: The segmentation equilibrium has pure bundling in the foreign country $\left[b^f > w > c(Z_M^f)/Z_M^f\right]$

This case breaks down into subcase 2a where the firm engages in mixed bundling in the home country, and subcase 2b where it engages in pure bundling in the home country.

Subcase 2a: Mixed bundling in the home country $[b^h > c(Z_M^h)/Z_M^h > w]$**.** Pricing is constrained by arbitrage when $y \in [0, R^h - P_B^f]$. For y

slightly below $R^h - P_B^f$, the firm maintains the same bundling regimes as under segmentation. It chooses p_M^h, r^h and p_B^f to maximize $\pi_{MB}(y)$. Prices and service quantities that satisfy (15)-(19) constitute an equilibrium under the same conditions as for subcase 1b.

For the parameter values $b^h = 6$, $b^f = 1.2, w = 0.2$, $c(z) = z^5$ and $F = 0$, the arbitrage constraint is binding for $y < 1.99$. The firm maintains the bundling regimes of segmentation for $y \in (0.40, 1.99]$ For $y < 0.40$ it maximizes profits by serving only the home market.

Subcase 2b: Pure bundling in the home country $[b^h > w > c(Z_M^h)/Z_M^h]$**.** Pricing is constrained when the home consumer who is indifferent between purchasing the home bundle and not purchasing when the home price is set at the segmentation level, derives positive surplus from grey product purchased at the foreign segmentation price plus trading cost. Because the marginal home consumer has preference index $\frac{P_B^h}{1+Z_B^h}$, pricing is constrained by arbitrage when $y \in [0, \frac{P_B^h}{1+Z_B^h} - P_B^f]$. For y close to the upper bound of this interval, the firm maintains pure bundling in both countries. It chooses the prices and services to maximize the profit function

$$\pi_{BB}(y) = \tfrac{1}{b^h}\left\{\left[p_B^h - w - c(z_B^h)\right]\left[b^h - \tfrac{p_B^h}{1+z_B^h}\right]\right\} + \tfrac{1}{b^f}\left\{\left[p_B^f - w - c(z_B^f)\right]\left[b^f - \tfrac{p_B^f}{1+z_B^f}\right]\right\} - 2F$$

where $\frac{p_B^h}{1+z_B^h} = p_B^f + y$. Upon defining $\gamma \equiv \frac{b^f\left(1+z_B^f\right)\left(1+z_B^h\right)}{b^h+b^f\left(1+z_B^f\right)\left(1+z_B^h\right)}$, the first order condition can be written [18]

$$(20)\ p_B^f = \tfrac{1}{2}\left\{\gamma\left[(b^h - 2y) + \tfrac{w+c(z_M^h)}{\left(1+z_B^h\right)}\right] + (1-\gamma)\left(1+z_B^f\right)\left[b^f + \tfrac{w+c(z_B^f)}{\left(1+z_B^f\right)}\right]\right\}$$

$$(21)\ c'(z_B^h) = \tfrac{p_B^h}{1+z_B^h}$$

$$(22)\ \ c'(z_B^f)\left[b^f\left(1+z_B^f\right) - p_B^f\right] = \tfrac{p_B^f}{1+z_B^f}\left(p_B^f - w - c(z_B^f)\right)$$

Because $p_B^h/(1+z_B^h)$ decreases when y falls, it is true by (21) that z_B^h also decreases. And because p_B^f increases, z_B^f must increase by (22). Subcase 2b arises when $b^h = 1.8$, $b^f = 0.6, w = 0, = c(z) = z^6$ and $F = 0$. The arbitrage constraint binds for $y \in [0, 0.389]$ and π_{BB} falls from 0.699 to 0.648 as y declines within this interval. Bundling in both countries remains the dominant strategy for all $y \geq 0$.

4.2 Case 3: Under segmentation product alone is sold in the foreign country $\left[c(Z_M^f)/Z_M^f > b^f > w\right]$.

Segmentation equilibria where the firm sells the product alone in the foreign country requires: a) $c'(z) > b^f$ for all $z > 0$ and/or; b) large F in

[18] For computational details see appendix

relation to gross profits from bundling in the foreign country. Subcase 3a illustrates the response of the firm to a fall of the trading cost when sales of the product alone in the foreign country is attributable to a high marginal cost of services. Subcase 3b illustrates the case where it is due to a large fixed cost of services.The subcase where product alone is sold in both countries is standard and therefore not examined here.

Subcase 3a: Mixed bundling in home country $[b^h > c(Z_M^h)/Z_M^h > w]$**.** Pricing is constrained by arbitrage when $y \in \left[0, R^h - R^f\right]$. For y slightly smaller than the cross country gap in segmentation prices of the product alone, the firm chooses r^h, r^f, p_M^h and z_M^h to maximize

$\pi_{MA}(y) = \frac{1}{b^h}\left\{\left[p_M^h - w - c(z_M^h)\right]\left[b^h - \widetilde{\theta}_h\right] + \left[r^h - w\right]\left[\widetilde{\theta}_h - r^h\right]\right\} + \frac{1}{b^f}\left\{\left[r^f - w\right]\left[b^f - r^f\right]\right\} - F$ subject to $r^h = r^f + y$ and $\widetilde{\theta}_h = \frac{p_M^h - r^h}{z_M^h}$.

The first order conditions have the form (10)-(13).

As y decreases profit maximization may call for discontinuation of supply to the foreign market, (when b^f is sufficiently smaller than b^h), or a switch to pure bundling in the foreign market. While bundling in the foreign country generates lower local profits than selling the product alone, it relieves the arbitrage constraint because it allows a higher foreign price.

As long as the firm maintains the bundling regime of segmentation, a decrease in the trading cost benefits home consumers and affects foreign consumers adversely. A switch to pure bundling in the foreign country will affect the local consumers with the lower θ adversely as they will refrain from purchasing. However, some foreign consumers with high θ may be better off than under segmentation.

When $b^h = 4, b^f = 2, w = 1.1$, $c(z) = z^2 + 2.6z$ and $F = 0$ profit maximization under segmentation calls for the sale of product alone in the foreign country because $c'(z) > b^f$ for $z \geq 0$. This yields $\Pi_M^h = 0.551$ and $\Pi_A^f = 0.101$. As y falls in the interval $[0, 1]$, π_{MA} falls monotonically from 0.652 to 0.496. This removes the arbitrage constraint as $R^h = 2.55$. It is straightforward to calculate that pure bundling in the foreign country yields a larger profit for $y < 0.267$.

Subcase 3b: Pure bundling in home country $[b_h > w > c(Z_M^h)/Z_M^h]$

Pricing is now constrained by arbitrage when $y \in \left[0, \frac{P_B^h}{1+Z_B^h} - R^f\right]$. For y slightly below $\frac{P_B^h}{1+Z_B^h} - R^f$ the firm establishes prices and home service level to maximize $\pi_{BA}(y) = \frac{1}{b^h}\left\{\left[p_B^h - w - c(z_B^h)\right]\left[b^h - \frac{p_B^h}{1+z_B^h}\right]\right\} +$

$\frac{1}{b^f}\left\{\left[r^f - w\right]\left[b^f - r^f\right]\right\} - F$ subject to $\frac{p_B^h}{1+z_B^h} = r^f + y$

Upon defining $\nu \equiv b_f\left(1+z_B^h\right)/b^h + b_f\left(1+z_B^h\right)$, the first order conditions can be written

$$r^f = \frac{1}{2}\left\{\nu\left[b^h - 2y + \frac{w+c(z_B^h)}{\left(1+z_B^h\right)}\right] + (1-\nu)\left[b^f + w\right]\right\} \tag{23}$$

$$c'(z_B^h) = \frac{p_B^h}{1+z_B^h} \tag{24}$$

Because the fall in y brings about a decrease in $\frac{p_B^h}{1+z_B^h}$, it must be true by (24) and $c"(.) > 0$ that z_B^h decreases. If so, all consumers in the foreign country are affected adversely when y falls. Some -and possibly all - consumers in the home country benefit from the fall in y. However, it is also possible that some home consumers (those with θ close to b^h) are affected adversely. This happens when their loss in utility attibutable to fewer services outweighs the benefit from a lower price.

However, as y drops significantly, profit maximization may call for one of the following responses: a) a switch from pure bundling to mixed bundling in the home country; b) a switch in the foreign country from selling the product alone to pure bundling; c) a switch from pure bundling to mixed bundling in the home country *and* a switch in the foreign country from selling the product alone to pure bundling. It is also possible that for y smaller than a critical value the best response is to discontinue sales in the foreign market.

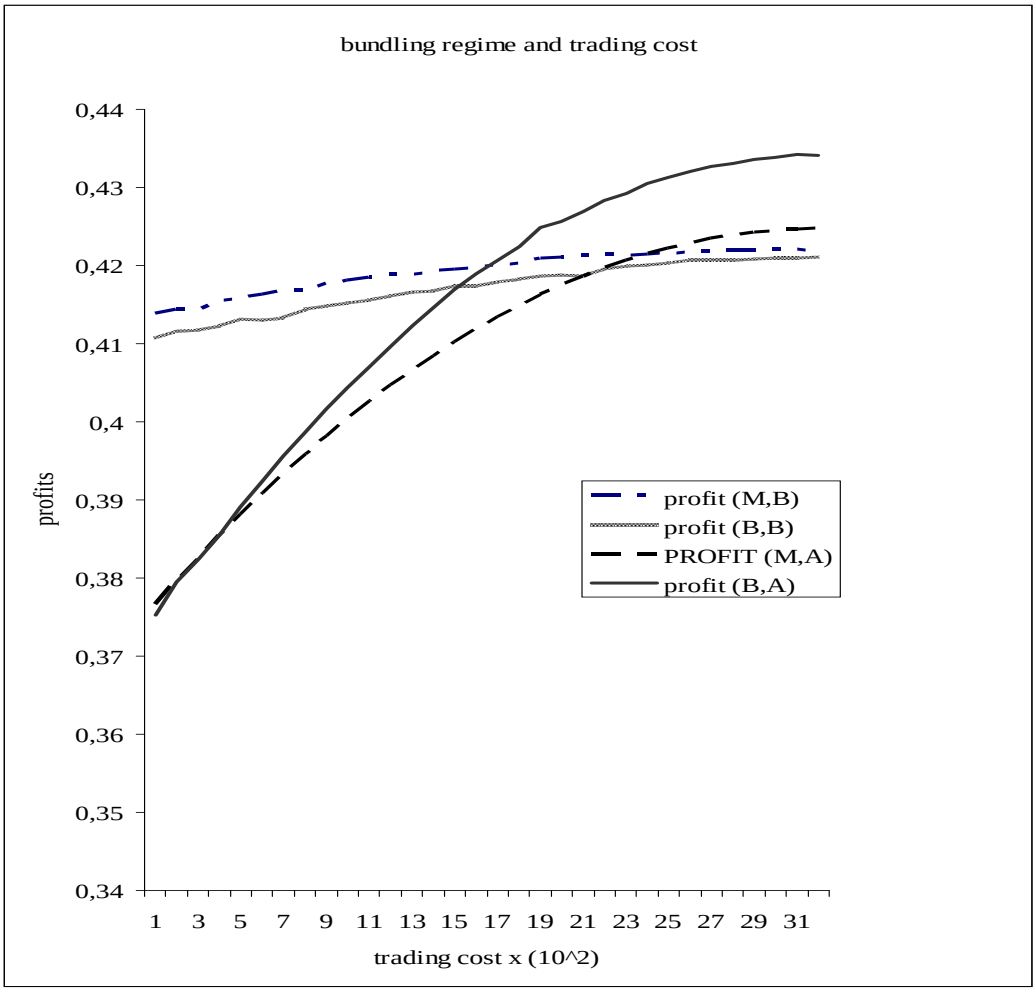
The best response to a fall in y is that which maximizes $\widehat{\pi}(y) = \max\left[\pi_{BA}(y), \pi_{MA}(y), \pi_{BB}(y), \pi_{MB}(y), \Pi_B^h\right]$. The adjustments that are optimal to any fall in y depend on parameter values, in particular the fixed cost of services in the foreign country.

Figure 1 displays π_{BA}, π_{MA}, π_{BB} and π_{MB} as a function of y for $b^h = 1.4, b^f = 0.6, w = 0.2, c(z) = z^6$ and $F = 0.08$.[19] Because $P_B^h/(1+Z_B^h) = 0.787$ and $R^f = 0.40$, we need to consider the profits under each of the aforementionned regimes for $y \in [0, 0.387]$. Figure 1 shows that $\widehat{\pi}(y) = \pi_{BA}(y)$ for $y \in [0.232, 0.387]$ and $\widehat{\pi}(y) = \pi_{MB}$ for $y \in [0, 0.232]$.[20]

Note that for larger F, $\pi_{MB}(y)$ is lower relative to $\pi_{BA}(y)$ and $\pi_{MA}(y)$,

[19] These parameter values yield $\Pi_A^f = 0.067 > \Pi_B^f = 0.0529$ and there exists no mixed bundling equilibrium in the foreign country. Also $\Pi_B^h = 0.368 > \Pi_A^h$ and there exists no mixed bundling equilibrium in the home country.

[20] Note that although $\pi_{BA}(y)$ is larger than $\pi_{MA}(y)$ for $y \in [0, 0.116]$, it is dominated by $\pi_{MB}(y)$. Also note that $\pi_{MB}(0) = 0.408 > \Pi_B^h = 0.368$



implying that the bundling regimes of segmentation remain in force for y smaller than 0.232.

5 Final Remarks

The paper shows how bundling between a traded and a non-traded good can be used to alleviate or eliminate a constraint on cross-country price discrimination due to potential entry by parallel traders. The deterrence of such entry may call for a change in prices, a change in bundling regime, or a combination of both. The optimal response of the firm to the threat of entry depends on the parameters of demand and production cost, and on the cost incurred by parallel traders.

The general direction of the change in bundling regimes that takes place in response to the threat of entry is towards unbundling in the high-

price country, and towards bundling in the low-price country. Specifically, a decline in the cost of arbitrage incurred by traders may elicit in the country where consumers have the lower willingness to pay a move from mixed bundling or no bundling, towards pure bundling. Conversely, it may bring forth in the country with the higher willingness to pay a move from pure bundling towards mixed bundling.

In the particular case where the firm maintains a mixed bundling regime in both countries, a decline in the trading cost lowers the number of consumers who purchase the product alone in the low-price country and increases the number of such consumers in the high-price country. Interestingly, as long as the firm maintains a mixed bundling regime in a country, the number of local consumers who purchase the bundle does not change. This is attributable to the fact that neither the gap between the price of the bundle and the product alone nor the amount of services contained in the bundle are affected by changes in the trading cost. This means that changes in the trading cost affect a country's consumers in the same way under mixed bundling as under no bundling.

Under pure bundling by contrast, the trading cost influences the amount of services. Specifically, under pure bundling in the low-price country a fall in the trading cost brings about an increase in the amount of services contained in the bundle. This happens because the amount of services is determined by the marginal consumer's reservation price, and because the buyer who becomes marginal after the increase in price has a higher reservation price.

It is impossible therefore to conclude - as in the standard literature- that all consumers in the low price country are affected adversely by the threat of parallel trade. Those who derive the highest utility from services may benefit because their gain from additional services outweighs their loss from the higher price. The effect on consumers in the home country is also ambiguous under the pure bundling regime. Indeed, the amount of services contained in the home bundle falls when the trading cost declines. The implication is that some consumers may be worse off even though they now pay less for the bundle.

Adjustments in the number of services in response to a change in the trading cost are more difficult to predict when they are accompanied by a change in the bundling regime. Consider the case where the foreign country switches from mixed bundling to pure bundling. When such switch allows the firm to set segmentation prices without eliciting entry by traders, there are fewer services in the foreign bundle than under segmentation. This benefits some consumers and affects other consumers adversely (see Lemma 3). When the switch does not render the arbitrage constraint ineffective, the firm increases services and price in the foreign

country to deter entry. Therefore it is impossible to infer from the mere observation that the firm engages in pure bundling in the foreign country that services in the foreign bundle would be larger or smaller in the absence of a threat of parallel trade. It is clear that the set of demand and cost parameters for which the firm drops out of the low-price maket is smaller when the firm can avail itself of a bundling option with a non-tradeable.

The absence of parallel trade in equilibrium, is a direct consequence of several assumptions: 1) that only arbitrageurs incur a trading cost ; 2) that production and sales take place simultaneously; 3) that demand is unchanging and 4) that all actors have full information. The first of these assumption is not critical. The assumptions that the manufacturer incurs a lower trading cost than traders is sufficient to insure that he can replicate any output allocation acheived via parallel trade and earn higher profit than under such trade.

The model has also assumed perfect correlation between consumers' valuation of the product and their valuation of the services. This is an a priori a reasonable assumption but it is not critical to the finding that a move towards bundling in the low-price country mitigates the adverse effect othat the treath of price arbitrage has on the manufacturers' profit. It only means that when the firm switches to bundling, some consumers with a strong preference for the product may drop out the market while some who did not purchase before may become buyers of the bundle. However, it remains true that bundling removes threath of arbitrage or mitigates the effect of that treath on profits.

6 References

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7 Appendix

Derivation of (1) and (2)

(A.1) $\frac{\partial \Pi}{\partial Z_B^h} = -c'(Z_B^h)\left(b_h - \frac{P_B^h}{1+Z_B^h}\right) + \frac{P_B^h}{(1+Z_B^h)^2}\left[P_B^h - w - c(Z_B^h)\right] = 0$

(A.2) $\frac{\partial \Pi}{\partial P_B^h} = b_h - \frac{P_B^h}{1+Z_B^h} - \frac{1}{1+Z_B^h}\left[P_B^h - w - c(Z_B^h)\right] = 0$ yields (1).

Substitution of (A.1) into (A.2) yields (2)

Derivation of (4) and (5)

(A.3) $\frac{\partial \Pi_M}{\partial P_M} = b - \widetilde{\Theta} - \frac{\partial \widetilde{\Theta}}{\partial P_M}\left[P_M - w - c(Z_M)\right] + \frac{\partial \widetilde{\Theta}}{\partial P_M}(R-w) = 0$
or $b - \widetilde{\Theta} - \frac{1}{Z_M}\left[P_M - R - c(Z_M)\right] = 0$
or $bZ_M - P_M + R - \left[P_M - R - c(Z_M)\right] = 0$
or $2\left(P_M - R\right) = bZ_M + c(Z_M)$
(A.4) $\frac{\partial \Pi_M}{\partial R} = -\frac{\partial \widetilde{\Theta}}{\partial R}\left[P_M - w - c(Z_M)\right] + \left(\widetilde{\Theta} - R\right) + (R-w)\left[\frac{\partial \widetilde{\Theta}}{\partial R} - 1\right]$

$= 0$ or
$\frac{1}{Z_M}\left\{\left[P_M - w - c(Z_M)\right] + \left[P_M - R - RZ_M\right] - (R-w)\left(1+Z_M\right)\right\} = 0$
or $2(P_M - R) - c(Z_M) + wZ_M - 2RZ_M = 0$
or $bZ_M + c(Z_M) - c(Z_M) + wZ_M - 2RZ_M = 0$ or
(A.5) $R - (b+w)/2 = 0$
From (A.3) and (A.5) $P_M - \frac{1}{2}\left\{b(1+Z_M) = w + c(Z_M)\right\} = 0$

Derivation of (6)

$\Pi_M^h = \frac{1}{b_h}\left[\frac{b_h\left(1+Z_M^h\right) - w - c(Z_M^h)}{2}\right]\left[\frac{b_h - \frac{c(Z_M^h)}{Z_M^h}}{2}\right] + \frac{1}{2}\left[b_h - w\right]\frac{1}{2}\left[\frac{c(Z_M^h)}{Z_M^h} - w\right]$
$= \frac{1}{4b_h}\left\{\left[(b_h - w) + Z_M^h(b_h - \frac{c(Z_M^h)}{Z_M^h})\right]\left[b_h - \frac{c(Z_M^h)}{Z_M^h}\right] + (b_h - w)\left[(b_h - w) - \left(b_h - \frac{c(Z_M^h)}{Z_M^h}\right)\right]\right\}$
$= \frac{1}{4b_h}\left\{(b_h - w)\left[b_h - \frac{c(Z_M^h)}{Z_M^h}\right] + Z_M^h(b_h - \frac{c(Z_M^h)}{Z_M^h})^2 + (b_h - w)^2 - (b_h - w)\left[b_h - \frac{c(Z_M^h)}{Z_M^h}\right]\right\}$

$= \frac{1}{4b_h}\left\{Z_M^h(b_h - \frac{c(Z_M^h)}{Z_M^h})^2 + (b_h - w)^2\right\}$

Derivation of (7)

$\frac{\partial \Pi_M^h}{\partial Z_M^h} = \frac{1}{4b_h}\left[(b_h - \frac{c(Z_M^h)}{Z_M^h})^2 + 2Z_M^h(b_h - \frac{c(Z_M^h)}{Z_M^h})\left(-\frac{c'(Z_M^h)}{Z_M^h} + \frac{c(Z_M^h)}{(Z_M^h)^2}\right)\right]$
$= \frac{1}{4b_h}\left(b_h - \frac{c(Z_M^h)}{Z_M^h}\right)\left[(b_h - \frac{c(Z_M^h)}{Z_M^h} + 2\left(-c'(Z_M^h) + \frac{c(Z_M^h)}{Z_M^h}\right)\right]$
$= \frac{1}{4b_h}\left(b_h - \frac{c(Z_M^h)}{Z_M^h}\right)\left[(b_h + \frac{c(Z_M^h)}{Z_M^h} - 2c'(Z_M^h)\right]$

Proof of Lemma 2(i):From (3) and (6) it follows that for $Z \equiv Z_M^h =$

Z_B^h one has

$\Pi_B^h(Z)-\Pi_M^h(Z) = \frac{1}{4b_h}\left\{\frac{1}{1+Z}\left[(b_h-w)+Z(b_h-\frac{c(Z)}{Z})\right]^2\right\}-\left\{(b_h-w)^2+Z(b_h-\frac{c(Z)}{Z})^2\right\}$

$= \frac{1}{4b_h}\left\{(b_h-w)^2\left(\frac{1}{1+Z}-1\right)+(b_h-\frac{c(Z)}{Z})^2\left(\frac{Z^2}{1+Z}-Z\right)+\frac{2Z(b_h-w)(b_h-\frac{c(Z)}{Z})}{1+Z}\right\} =$

$-\frac{1}{4b_h}\frac{Z}{1+Z}\left[(b_h-w)-(b_h-\frac{c(Z)}{Z})\right]^2 = -\frac{1}{4b_h}\frac{Z}{1+Z}\left[\frac{c(Z)}{Z}-w\right]^2 < 0$

Derivation of $\frac{dW_M(Z)}{dZ}$

$\frac{dW_M(Z)}{dZ} = \int_{\widetilde{\Theta}_M}^{b}[\theta Z-c(Z)]d\theta = \frac{1}{2}\left[b^2-\widetilde{\Theta}^2\right]Z-c(Z)\left[b-\widetilde{\Theta}\right] = Z\left[b-\widetilde{\Theta}\right]\left[\frac{1}{2}(b+\widetilde{\Theta})-\frac{c(Z)}{Z}\right] =$

$Z\left[b-\frac{1}{2}(b+\frac{c(Z)}{Z})\right]\left[\frac{1}{2}(b+\frac{1}{2}(b+\frac{c(Z)}{Z})-\frac{c(Z)}{Z}\right]$

$= \frac{3}{8}Z\left[b-\frac{c(Z)}{Z}\right]^2$.Therefore, $\frac{dW}{dZ} = \frac{3}{8}[\left[b-\frac{c(Z)}{Z}\right]^2+2Z\left[b-\frac{c(Z)}{Z}\right]\left[-\frac{c'(Z)}{Z}+\frac{c(Z)}{Z^2}\right]] =$

$\frac{3}{8}\left\{\left[b-\frac{c(Z)}{Z}\right]\left[b-\frac{c(Z)}{Z}-2c'(Z)+2\frac{c(Z)}{Z}\right]\right\}$

$= \frac{3}{8}\left\{\left[b-\frac{c(Z)}{Z}\right]\left[b-2c'(Z)+\frac{c(Z)}{Z}\right]\right\}$

Derivation of $\underleftrightarrow{\Theta}$ and $\overleftrightarrow{\Theta}$

$\underleftrightarrow{\Theta}-\mathbf{R} = \underleftrightarrow{\Theta}(\mathbf{1}+\mathbf{Z}_B)-\mathbf{P}_B$

$\Leftrightarrow \underleftrightarrow{\Theta}-\frac{1}{2}(\mathbf{w}+\mathbf{b}) = \underleftrightarrow{\Theta}(\mathbf{1}+\mathbf{Z}_B)-\frac{1}{2}[(\mathbf{w}+\mathbf{b})+\mathbf{Z}_B(\mathbf{b}+\frac{c(Z_B)}{Z_B})]$

$\Longleftrightarrow \underleftrightarrow{\Theta}\mathbf{Z}_B = \frac{1}{2}\mathbf{Z}_B(\mathbf{b}+\frac{c(Z_B)}{Z_B})$

$\overleftrightarrow{\Theta}(\mathbf{1}+\mathbf{Z}_M)-\mathbf{P}_M$

$= \overleftrightarrow{\Theta}(\mathbf{1}+\mathbf{Z}_B)-\mathbf{P}_B$

$\Longleftrightarrow \overleftrightarrow{\Theta} = \frac{P_M-P_B}{Z_M-Z_B} = \frac{1}{Z_M-Z_B}\frac{1}{2}\left[w+b+Z_M(b+\frac{c(Z_M)}{Z_M})+w+b-Z_B(b+\frac{c(Z_B)}{Z_B})\right]$

$\frac{1}{2}\frac{1}{Z_M-Z_B}\left[Z_M(b+\frac{c(Z_M)}{Z_M})-Z_B(b+\frac{c(Z_B)}{Z_B})\right] = \frac{1}{2}\frac{1}{Z_M-Z_B}\left[b(Z_M-Z_B)+c(Z_M)-c(Z_B)\right]$

Derivation of (10)-(13)

$\pi_{MM}(y) = \frac{1}{b_h}\left\{\left[p_M^h-w-c(z_M^h)\right]\left[b_h-\widetilde{\theta}_h\right]+\left[r^h-w\right]\left[\widetilde{\theta}_h-r^h\right]\right\}+$

$\frac{1}{b_f}\left\{\left[p_M^f-w-c(z_M^f)\right]\left[b_f-\widetilde{\theta}_f\right]+\left[r^f-w\right]\left[\widetilde{\theta}_f-r^f\right]\right\}$

Let $r = r^f$ and $r^h = r+y$. First order condition are

$\frac{\partial\pi_{MM}(y)}{\partial p_M^h} = b_h-\widetilde{\theta}_h-\frac{\partial\widetilde{\theta}_h}{\partial p_M^h}\left[p_M^h-w-c(z_M^h)-r-y+w\right] = 0$

$b_h z_M^h-p_M^h+r+y-\left[p_M^h-c(z_M^h)-r-y\right] = 0 \Rightarrow \frac{p_M^h-r-y}{z_M^h} =$

$\widetilde{\theta}_h = \frac{1}{2}\left[b_h+\frac{c(z_M^h)}{z_M^h}\right]$

Similarly $\widetilde{\theta}_f = \frac{1}{2}\left[b_f + \frac{c(z_M^f)}{z_M^f}\right]$

$\frac{\partial \pi_{MM}(y)}{\partial r} = \frac{1}{b_h}\left\{\widetilde{\theta}_h - r - y - r - y + w - \frac{\partial \widetilde{\theta}_h}{\partial r}\left[p_M^h - w - c(z_M^h) - r - y + w\right]\right\} +$

$\frac{1}{b_f}\left\{\widetilde{\theta}_f - r - r + w - \frac{\partial \widetilde{\theta}_f}{\partial r}\left[p_M^f - w - c(z_M^f) - r + w\right]\right\} = 0$

$= \frac{1}{b_h}\left\{\widetilde{\theta}_h - 2(r+y) + w + \frac{1}{z_M^h}\left[\widetilde{\theta}_h z_M^h - c(z_M^h)\right]\right\} + \frac{1}{b_f}\left\{\widetilde{\theta}_f - 2r + w + \frac{1}{z_M^f}\left[\widetilde{\theta}_f z_M^f - c(z_M^f)\right]\right\} = 0$

$= \frac{1}{b_h}\left\{2\widetilde{\theta}_h - 2(r+y) + w - \frac{c(z_M^h)}{z_M^h}\right\} + \frac{1}{b_f}\left\{2\widetilde{\theta}_f - 2r + w - \frac{c(z_M^f)}{z_M^f}\right\} = 0$

$= \frac{1}{b_h}\left\{b_h + \frac{c(z_M^h)}{z_M^h} - 2(r+y) + w - \frac{c(z_M^h)}{z_M^h}\right\} + \frac{1}{b_f}\left\{b_f + \frac{c(z_M^f)}{z_M^f} - 2r + w - \frac{c(z_M^f)}{z_M^f}\right\} = 0$

$= \frac{1}{b_h}\left\{b_h - 2(r+y) + w\right\} + \frac{1}{b_f}\left\{b_f - 2r + w\right\} = 0 \Rightarrow 2r\left[\frac{1}{b_h} + \frac{1}{b_f}\right] = \frac{1}{b_h}\left\{b_h - 2y + w\right\} + \frac{1}{b_f}\left\{b_f + w\right\}$

$\Rightarrow r = \frac{1}{2}[\alpha\left\{b_h - 2y + w\right\} + (1-\alpha)\left\{b_f + w\right\}]$

$\frac{\partial \pi_{MM}(y)}{\partial z_M^h} = \frac{1}{b_h}\left\{-c(z_M^h)\left[b_h - \widetilde{\theta}_h\right] - \frac{\partial \widetilde{\theta}_h}{\partial z_M^h}\left[p_M^h - w - c(z_M^h) - r - y + w\right]\right\} = 0 \Rightarrow$

$$-c(z_M^h)\left[b_h - \widetilde{\theta}_h\right] + \frac{\widetilde{\theta}_h}{z_M^h}\left[\widetilde{\theta}_h z_M^h - c(z_M^h)\right] = 0$$

$$-c(z_M^h)\left[b_h - \widetilde{\theta}_h\right] + \widetilde{\theta}_h\left[\widetilde{\theta}_h - c(z_M^h)/z_M^h\right] = 0$$

$$-c(z_M^h)\left[b_h - \widetilde{\theta}_h\right] + \widetilde{\theta}_h\left[\widetilde{\theta}_h - c(z_M^h)/z_M^h\right] = 0$$

$$-c(z_M^h)\tfrac{1}{2}\left[b_h - c(z_M^h)/z_M^h\right] + \widetilde{\theta}_h\tfrac{1}{2}\left[b_h - c(z_M^h)/z_M^h\right] = 0$$

Derivation of (15)-(19)

$\pi_{MB}(y) = \frac{1}{b_h}\left\{\left[p_M^h - w - c(z_M^h)\right]\left[b_h - \widetilde{\theta}_h\right] + \left[r^h - w\right]\left[\widetilde{\theta}_h - r^h\right]\right\} +$

$\frac{1}{b_f}\left\{\left[p_B^f - w - c(z_B^f)\right]\left[b_f - \frac{p_B^f}{1+z_B^f}\right]\right\}$

$\pi_{MB}(y) = \frac{1}{b_h}\left\{\left[p_M^h - w - c(z_M^h)\right]\left[b_h - \widetilde{\theta}_h\right] + \left[r^h - w\right]\left[\widetilde{\theta}_h - r^h\right]\right\} +$

$\frac{1}{b_f}\left\{\left[r^h - y - w - c(z_B^f)\right]\left[b_f - \frac{r^h - y}{1+z_B^f}\right]\right\}$

Substitution of the constraints into the profit function and differentiation with respect to r^h and p_M^h yields

$\frac{\partial \pi}{\partial r^h} = \frac{1}{b_h z_M^h}\left\{2(p_M^h - r^h) - 2z_M^h r^h + w z_M^h - c(z_M^h)\right\}$

$+\frac{1}{b_f(1+z_B^f)}\left\{b_f(1+z_B^f) + w + c(z_B^f) - 2\left(r^h - y\right)\right\} = 0$

$\frac{\partial \pi}{\partial p_M^h} = \frac{1}{b_h z_M^h}\left\{b_h z_M^h + c(z_M^h) - 2(p_M^h - r^h)\right\} = 0$

Jointly they yield:

$\frac{1}{b_h z_M^h}\left\{b_h z_M^h + c(z_M^h) - 2z_M^h r^h + w z_M^h - c(z_M^h)\right\}$
$+\frac{1}{b_f(1+z_B^f)}\left\{b_f(1+z_B^f) + w + c(z_B^f) - 2\left(r^h - y\right)\right\} = 0$
or $\frac{1}{b_h z_M^h}\left\{b_h z_M^h + w z_M^h\right\} + \frac{1}{b_f(1+z_B^f)}\left\{b_f(1+z_B^f) + w + c(z_B^f) + 2y\right\}$

$= 2r^h[\frac{1}{b_h} + \frac{1}{b_f(1+z_B^f)}] = 2r^h\frac{b_h + b_f(1+z_B^f)}{b_h b_f(1+z_B^f)}$

Define $\beta \equiv \frac{b_f(1+z_B^f)}{b_h + b_f(1+z_B^f)}$ to get

$r^h = \frac{1}{2}\left\{\beta\left[b_h + w\right] + (1-\beta)[b_f(1+z_B^f) + w + c(z_B^f) + 2y]\right\}$
$= \frac{1}{2}\left\{w + \beta b_h + (1-\beta)[b_f(1+z_B^f) + c(z_B^f) + 2y]\right\}$
$p_M^h = r^h + \frac{z_M^h}{2}\left[b_h + c(z_M^h)/z_M^h\right]$
$p_B^f = r^h - y = \frac{1}{2}\left\{w + \beta b_h + (1-\beta)[b_f(1+z_B^f) + c(z_B^f) + 2y] - 2y\right\} =$
$\frac{1}{2}\left\{w + \beta(b_h - 2y) + (1-\beta)[b_f(1+z_B^f) + c(z_B^f)]\right\}$
Also, $\frac{\partial\pi}{\partial z_M^h} = \frac{p_M^h - r^h}{\left(z_M^h\right)^2}\left[p_M^h - w - c(z_M^h) - (r^h - w)\right] - c'(z_M^h)\left[b_h - \frac{p_M^h - r^h}{z_M^h}\right]$

or

$= \frac{p_M^h - r^h}{z_M^h}\left[b_h z_M^h - c(z_M^h)\right] - c'(z_M^h)\left[b_h z_M^h - c(z_M^h)\right] = 0$ or
$c'(z_M^h) = \frac{p_M^h - r^h}{z_M^h} = \frac{1}{2}\left[b_h + \frac{c(z_M^h)}{z_M^h}\right]$

$\frac{\partial\pi}{\partial z_B^f} = -c'(z_B^f)\left[b_f - \frac{r^h - y}{1+z_B^f}\right] + \frac{r^h - y}{(1+z_B^f)^2}\left[r^h - y - w - c(z_B^f)\right] = 0$

or

$= -c'(z_B^f)\left[b_f\left(1 + z_B^f\right) - \left(r^h - y\right)\right] + \frac{r^h - y}{1+z_B^f}\left[r^h - y - w - c(z_B^f)\right] =$
0 or using (...)
$c'(z_B^f)\left[b_f\left(1 + z_B^f\right) - p_B^f\right] = \frac{p_B^f}{1+z_B^f}\left[p_B^f - w - c(z_B^f)\right] = 0$

Derivation of (20)-(22)

$\pi_{BB}(y) = \frac{1}{b_h}\left\{\left[(p_B^f + y)(1+z_B^h) - w - c(z_B^h)\right]\left[b_h - (p_B^f + y)\right]\right\}$
$+\frac{1}{b_f}\left\{\left[p_B^f - w - c(z_B^f)\right]\left[b_f - \frac{p_B^f}{1+z_B^f}\right]\right\}$

$\frac{\partial\pi_{BB}(y)}{\partial p_B^f} = \frac{1}{b_h}\left\{(1+z_B^h)\left[b_h - (p_B^f + y)\right] - \left[(p_B^f + y)(1+z_B^h) - w - c(z_M^h)\right]\right\} +$
$\frac{1}{b_f}\left\{\left[b_f - \frac{p_B^f}{1+z_B^f}\right] - \frac{1}{1+z_B^f}\left[p_B^f - w - c(z_B^f)\right]\right\} = 0 \Rightarrow$

$\frac{(1+z_B^h)}{b_h}\left\{(b_h-2y)+\frac{w+c(z_M^h)}{(1+z_B^h)}\right\}+\frac{1}{b_f}\left\{b_f+\frac{w+c(z_B^f)}{\left(1+z_B^f\right)}\right\}=2p_B^f\left[\frac{1+z_B^h}{b_h}+\frac{1}{b_f\left(1+z_B^f\right)}\right]=$
$2p_B^f\left(1+z_B^h\right)\left[\frac{1}{b_h}+\frac{1}{b_f\left(1+z_B^f\right)\left(1+z_B^h\right)}\right]=$
$2p_B^f\left(1+z_B^h\right)\left[\frac{b_h+b_f\left(1+z_B^f\right)\left(1+z_B^h\right)}{b_hb_f\left(1+z_B^f\right)\left(1+z_B^h\right)}\right]\Rightarrow$
$\frac{1}{b_h}\left\{(b_h-2y)+\frac{w+c(z_M^h)}{(1+z_B^h)}\right\}+\frac{1}{b_f}\left\{b_f+\frac{w+c(z_B^f)}{\left(1+z_B^f\right)}\right\}=2p_B^f\left[\frac{1+z_B^h}{b_h}+\frac{1}{b_f\left(1+z_B^f\right)}\right]=$

$2p_B^f\left[\frac{b_h+b_f\left(1+z_B^f\right)\left(1+z_B^h\right)}{b_hb_f\left(1+z_B^f\right)\left(1+z_B^h\right)}\right]\Rightarrow$

$p_B^f=\frac{1}{2}\{\frac{b_f\left(1+z_B^f\right)\left(1+z_B^h\right)}{b_h+b_f\left(1+z_B^f\right)\left(1+z_B^h\right)}\left[(b_h-2y)+\frac{w+c(z_M^h)}{(1+z_B^h)}\right]$

$+\frac{b_h\left(1+z_B^h\right)\left(1+z_B^f\right)}{b_h+b_f\left(1+z_B^f\right)\left(1+z_B^h\right)}\left[b_f+\frac{w+c(z_B^f)}{\left(1+z_B^f\right)}\right]\}$

$p_B^f=\frac{1}{2}\{\frac{b_f\left(1+z_B^f\right)}{b_h+b_f\left(1+z_B^f\right)\left(1+z_B^h\right)}\left[(b_h-2y)\left(1+z_B^h\right)+w+c(z_M^h)\right]$
$+\frac{b_h\left(1+z_B^h\right)}{b_h+b_f\left(1+z_B^f\right)\left(1+z_B^h\right)}\left[b_f\left(1+z_B^f\right)+w+c(z_B^f)\right]\}$

$\frac{\partial\pi_{BB}(y)}{\partial z_B^h}=\frac{1}{b_h}\left\{\left[(p_B^f+y)-c'(z_B^h)\right]\left[b_h-(p_B^f+y)\right]\right\}=0$
or $(p_B^f+y)-c'(z_B^h)=0$

$\frac{\partial\pi_{BB}(y)}{\partial z_B^f}=\frac{1}{b_f}\left\{-c'(z_B^f)\left[b_f-\frac{p_B^f}{1+z_B^f}\right]+\left[p_B^f-w-c(z_B^f)\right]\frac{p_B^f}{(1+z_B^f)^2}\right\}=0$

Derivation of (23)-(24)

$\pi_{BA}(y)=\frac{1}{b_h}\left\{\left[(r^f+y)\left(1+z_B^h\right)-w-c(z_B^h)\right]\left[b_h-r^f\right]\right\}+\frac{1}{b_f}\left\{\left[r^f-w\right]\left[b_f-r^f\right]\right\}$
$\frac{\partial\pi_{BA}(y)}{\partial r^f}=\frac{1}{b_h}\left\{(b_h-r^f-y)\left(1+z_B^h\right)-\left(r^f+y\right)\left(1+z_B^h\right)+w+c(z_B^h)\right\}+$
$\frac{1}{b_f}\left\{b_f-r^f-r^f+w\right\}=0$
or
$\frac{1}{b_h}\left\{(b_h-2y)\left(1+z_B^h\right)+w+c(z_B^h)\right\}+\frac{1}{b_f}\left\{b_f+w\right\}=2r^f\left[\frac{1+z_B^h}{b_h}+\frac{1}{b_f}\right]=$
$2r^f\left[\frac{b_h+b_f\left(1+z_B^h\right)}{b_hb_f\left(1+z_B^h\right)}\right]$
or
$r^f=\frac{1}{2}\left\{\frac{b_f\left(1+z_B^h\right)}{b_h+b_f\left(1+z_B^h\right)}\left[(b_h-2y)+\frac{w+c(z_B^h)}{\left(1+z_B^h\right)}\right]+\frac{b_h}{b_h+b_f\left(1+z_B^h\right)}\left[b_f+w\right]\right\}.$
Therefore,
$p_B^h=\frac{1+z_B^h}{2}\left\{\frac{b_f\left(1+z_B^h\right)}{b_h+b_f\left(1+z_B^h\right)}\left[b_h+\frac{w+c(z_B^h)}{\left(1+z_B^h\right)}\right]+\frac{b_h}{b_h+b_f\left(1+z_B^h\right)}\left[b_f+w-2y\right]\right\}$

$$\frac{\partial \pi_{BA}(y)}{\partial z_B^h} = 0 \Rightarrow r^f + y - c'(z_B^h) = 0$$