(Draft for comments)

An Optimized Monetary Policy Rule for ToTEM

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Abstract

This paper examines and proposes a monetary policy rule for TOTEM, a DSGE model of the Canadian economy presently under development in the Research Department of the Bank of Canada. We consider simple instrument rules such as Taylor-type and Inflation Forecast-Based rules. The recommended rule minimizes a loss function that reflects the assumed preferences of the monetary authority over inflation and output fluctuations as well as over the variability of its instrument.

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1 Introduction

The goal of this paper is to propose an optimal monetary policy rule to be used in ToTEM, which is an open economy DSGE model of the Canadian economy. The intended use of this model is to produce economic projections and to perform policy analysis experiments. For the purpose of this paper, we focus on a simple instrument rule that minimizes an assumed loss function of the monetary authority.[1] Indeed, two loss functions are considered. The first one is defined over inflation and output gap variability. The other loss function allows us to examine the implications of allowing the monetary authority to put a weight on the variability of interest rate movements.

We consider two classes of simple instrument rules. First, we look at Taylor-type rules where the policy instrument reacts to a set of contemporaneous and lagged variables. Second, we consider the class of Inflation Forecast Based (IFB) rules. For this class of rules, we treat as unknown the horizon at which monetary authority should look at inflation to set its policy instrument, as in Batini and Nelson (2001). Thus, this horizon is determined jointly with the other parameters in the monetary policy rule.

We calculate an optimal parameterization for both the IFB and the Taylor-type rules given the assumed preferences of the monetary authority. For each type of rule, we use stochastic simulations to calculate the variability and the persistence of inflation, of the output gap and of the policy instrument. In addition, we consider several important questions. We report the average and median horizon over which inflation returns to its targeted level. As well, we consider the implications of interest rate smoothing (in the preferences of the policy maker) for the persistence and variability of inflation, the output gap, and interest rates. In addition, we calculate confidence bands around inflation outcomes, which provides useful information about an appropriate width for the inflation target band in our modeled economy. Finally, we examine how sensitive the optimized simple instrument rule is to a change in the distribution of shocks used for its determination.

This paper is organized as follows. In section 2, we provide a brief description of ToTEM. In section 3, we present the two loss functions considered in

[1] There is already a Part 2 planned for this paper where we will look at implementing a specific targeting rule as advocated by Svensson (1999,2003).

this paper, as well as the two classes of simple instrument monetary policy rules tested here. We discuss our results in section 4 and perform a sensitivity analysis in section 5. Section 6 concludes.

2 A description of ToTEM

ToTEM is a fairly standard Open-Economy DSGE model. However, since we intend to use ToTEM both to perform economic projections and in policy analysis experiments, much emphasis is being placed on getting the model to fit the historical data and to replicate several stylized facts of the Canadian economy. Therefore, the structure of ToTEM is more detailed than the Open Economy DSGE models typically seen in the literature.

What follows is a brief non-technical description of the model. More details of the model can be found in Binette et al. (2005). We also discuss briefly how the parameters of the model have been estimated.

2.1 A non technical description of the model[2]

The production side of ToTEM is as follows. There are four types of final goods produced by domestic firms: consumption, investment, government and non-commodity export goods. To produce these goods, firms use a CES technology that combines capital with labour services, imported intermediate goods, and commodities. As well, there is a commodity sector. The commodities are produced by domestic firms by combining labour services with capital goods and a fixed factor that we refer to as land. All firms are allowed to vary their utilization rate, but this comes at a cost in terms of foregone output. The firms also face adjustment costs on the level of employment and on the change in investment, also in terms of foregone output.

There is assumed to be a continuum of monopolistically competitive importers. Each of these importers buy their goods on the world market which

[2] Figures 1 and 2 provide a graphical representation of the production and demand sides of TOTEM.

they pay for in the foreign currency. They then sell their goods to a perfectly competitive domestic distributor that bundles them together to produce a homogenous imported good. This good is then sold and used as an input by domestic firms. Following Smets and Wouters (2002), we assume that the price of the imported good is rigid in the domestic currency. Each importing firm signs a contract with the distributor of the homogenous imported good. Therefore, any exchange rate movements are initially absorbed by the importers profit margins, which leads to incomplete pass-through of exchange rate movements in the short-run. In the long-run, relative purchasing power parity holds and the pass-through to the price of imported goods is complete. As for the imported good sector, each final good producer has some degree of heterogeneity for their own good with respect to the other goods within their sector. This allows a firm to fix its price for more than one period. These firms sell their final good to a distributor and, similar to what is typical in the DSGE literature, the Calvo pricing framework applies to both the imported good and final good sectors.

The demand side of ToTEM can be summarized as follows. Domestic households buy the final consumption goods as well as bonds from the (domestic) government and foreigners. They earn (after-tax) labour income from the labour services that they provide to the domestic firms and income from their holding of domestic and foreign bonds in the form of interest payments. They also receive transfers from the government. The government buys the final government goods from the domestic firms with tax revenues and distributes transfers to the domestic households. These expenditures are financed with the tax revenues from labour income and indirect taxes. We assume that the government targets a desired level for the debt-to-GDP ratio, with some smoothing, and uses the tax rate on labour income as the policy instrument. Finally, foreigners buy and sell bonds and exports of the final non-commodity export goods and commodities. They also sell intermediate imported goods to the domestic importers.

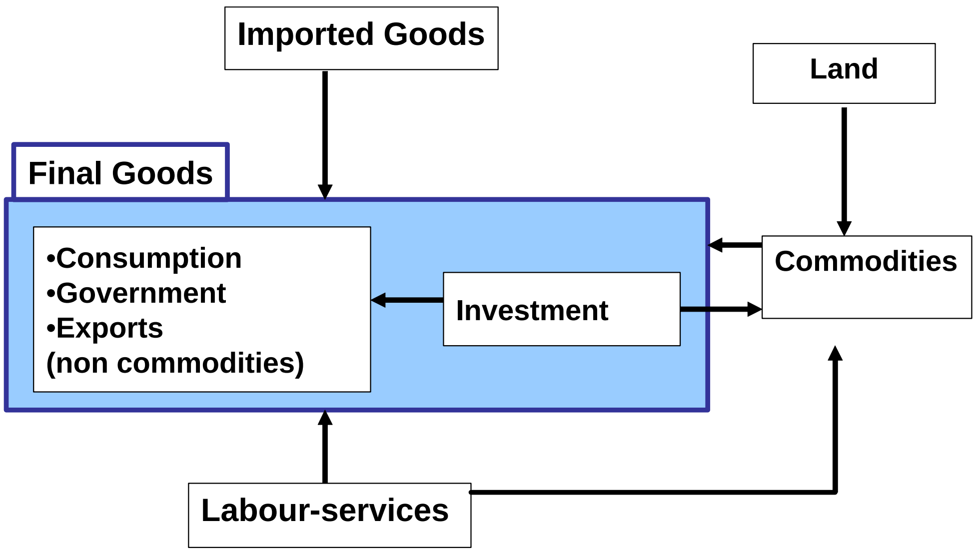

Figure 1: The production side of TOTEM.

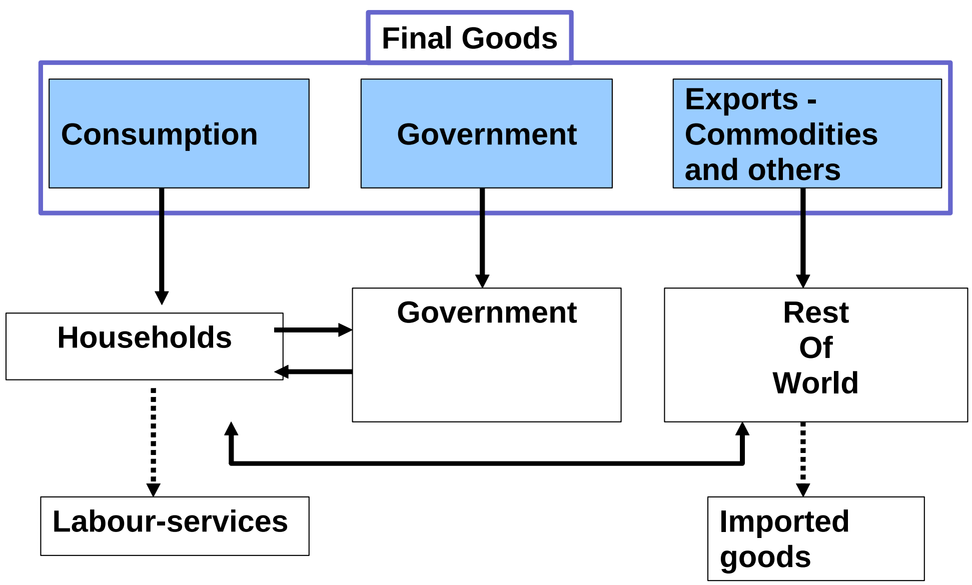

Figure 2: The demand side of TOTEM.

Finally, the foreign variables in ToTEM are presently generated with a semi-structural model.[3] This model is exogenous with respect to the core of ToTEM in the sense that there is no feedback from domestic variables to the foreign variables. This is consistent with the fact that Canada is a small open-economy. The foreign variables that enter in ToTEM are output and the output gap, price level, inflation rate, interest rates (real and nominal) and real commodity prices. This model allows us to perform temporary shocks to foreign demand, inflation and monetary policy as well as permanent shocks to foreign output and inflation (i.e. a change in the target inflation rate). We can also perform temporary and permanent shocks to real commodity prices.

2.2 Estimation of ToTEM

The parameters of TOTEM have been estimated with the Generalized Method of Moments (GMM).[4] The GMM estimation of TOTEM was done using 27 key moments. Figure 3 presents some of the moments used for estimation. The panels in this figure compare selected cross correlations as produced by TOTEM with their historical counterpart. Ninety percent confidence bands are also presented around the historical correlation.

The estimated policy rule is given by :

$$R_t = 0.8R_{t-1} + (1 - 0.8)R^* + (1 - 0.8)[2.5\ (\pi_{t+2} - \pi_t^*)] \qquad (1)$$

with R and π being respectively the policy instrument and the inflation rate, and where * denotes steady-state values. The parameters of (1) have been estimated simultaneously with the other parameters of the model.

[3] Work is still ongoing to improve the foreign side of TOTEM.

[4] We are currently in the process of estimating the model with the Bayesian approach in addition to the GMM approach.

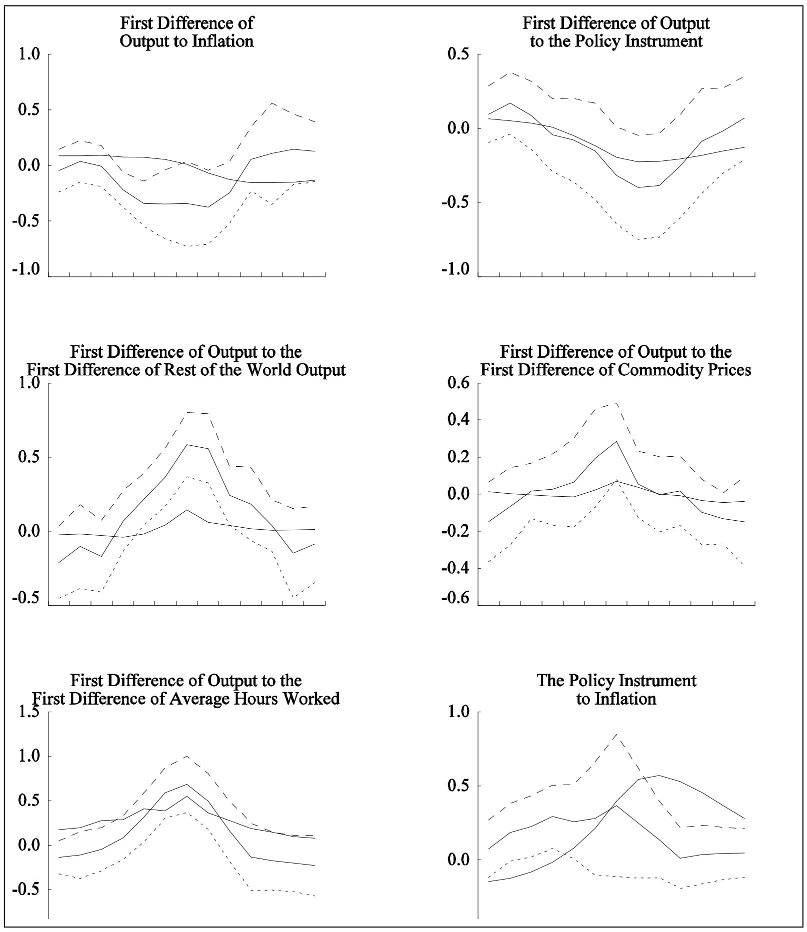

Figure 3: Cross correlations

3 The Central Bank's problem and the types of policy rules considered

In this paper, we choose a monetary policy rule that minimizes an assumed loss function of the monetary authority. This section describes this loss function of the monetary authority as well as the type of monetary policy rules that we consider.

3.1 Loss function

We assume that the monetary authority has preferences over inflation stability with some concern for output stabilization. We also allow for the possibility that the authority cares about the volatility in the movements of its instrument. We formalize these preferences of the central bank with the following contemporanous loss function :

$$L_t = (\pi_t - \pi^*)^2 + \lambda_y (y_t - y_t^*)^2 + \lambda_{\Delta R} (\Delta R_t)^2 \tag{2}$$

with $\pi_{t,}$ $y_t, \Delta R_t$ being respectively the (year-over-year) inflation rate in

period t, the log-level of real output in period t and the change in the level of the policy instrument between period *t-1* and period t. A * denotes steady-state values.[5] The parameters λ_y and $\lambda_{\Delta R}$ are respectively the relative weight on output fluctuations and the movements of the policy instrument in the preferences of the monetary authority. The problem of monetary policy in period t is to set its instrument in order to minimize the current and expected values for the period-losses. More formally, we write the intertemporal loss-function for the monetary authority as :

[5]It is important to note that the equilibrium output definition here is conceptually related to the natural rate hypothesis, which implies the long run neutrality of monetary policy. Therefore, there is no inflation bias resulting from the period loss function used here. See Svensson (1999) for a discussion about this.

$$
\begin{aligned}
\mathcal{L}_t &= E_t\left[(1-\beta)\sum_{i=0}^{\infty}\beta^i L_{t+i}\right] \\
&\equiv (1-\beta)\ E_t\sum_{i=0}^{\infty}\beta^i[(\pi_{t+i}-\pi^*)^2+\lambda_y(y_{t+i}-y^*_{t+i})^2 \\
&\qquad\qquad\qquad +\lambda_{\Delta R}(\Delta R_{t+i})^2]
\end{aligned} \tag{3}
$$

with β being the rate at which the central bank discounts future losses and E_t the conditional expectations operator, based on information available in period t. As is common in the literature and for operational purposes, we will work in this paper with the unconditional version of (3) which is given by the following representation :

$$
\bar{\mathcal{L}} = \sigma^2_\pi + \lambda_y\sigma^2_{ygap} + \lambda_{\Delta R}\sigma^2_{\Delta R} \tag{4}
$$

where σ^2_π, σ^2_{ygap} and $\sigma^2_{\Delta R}$ are the unconditional variances of the deviations of the inflation rate from its targeted level, of the output gap, and of the movements of the policy instrument, respectively. It can be shown that $lim_{\beta\rightarrow 1}\mathcal{L}_t = \bar{\mathcal{L}}$.[6]

In this study, the first loss function of the monetary authority we consider is given by $\bar{\mathcal{L}}_1$ with $\lambda_y = 1$ and $\lambda_{\Delta R} = 0$ in (4). This is equivalent to saying that the monetary authority only cares about the inflation rate and the output gap. It does not put any weight on smoothing its policy instrument. Since we do not have any prior about the relative weight given to output and inflation by the monetary authority, we give the same weight to inflation and the output gap.

We also consider an alternative loss function where the monetary authority penalizes volatility in the policy instrument, but relatively less so than inflation deviation from the targeted level or output from its equilibrium. In this alternative loss-function, we set $\lambda_y = 1$ and $\lambda_{\Delta R} = 0.5$. We follow Rudebush and Svensson (1999), Batini and Nelson (2001) and Batini, Harrison and Millard (2003) for the choice of $\lambda_{\Delta R}$.

[6] See Svensson (1999, 2003).

Some studies have justified this concern for the policy instrument by the fact that the policy makers may also care about the effect of the volatility of its instrument on financial stability (Cukierman, 1990), or by the possibility of hitting the lower nominal bound (Rotemberg and Woodford, 1997; Woodford, 1997; Woodford, 1999). In our case, we believe that a reasonable and sufficient justification is that in reality, in any given period, the monetary authority (and other agents) are uncertain about the nature and the persistence of the shocks at play in the economy. Therefore, the monetary authority might want to avoid large policy mistakes by smoothing its reaction to new developments.

3.2 The class of policy rules considered

We consider two types of simple instrument rules for this study : the Taylor-type rules and the Inflation Forecast-Based Rules (IFB). In the Taylor-type rules, the instrument usually reacts to a lagged value of itself and to contemporaneous gaps between inflation and output relative to their targeted values. For the IFB-type rules, the instrument reacts to expected deviations of inflation from its targeted level at some horizon, usually in addition to the contemporaneous value of the output gap and lagged interest rates. In this type of rule, the forecast of inflation enters as a feedback variable for the setting of the policy instrument.

The inevitable lags between monetary policy actions and their effect on inflation make the IFB-type rule appealing. It illustrates more naturally than the Taylor-type rules the problem faced by policymakers in inflation targeting countries. As discussed by Batini and Haldane (1999), since this type of rule includes forecasts of inflation, these rules may be preferable to Taylor-type rules given that they implicitly allow for the use of all relevant information, including judgement of the forecaster. Maclean et al. finds that the IFB rules outperform the Taylor-type rules in QPM in terms of the volatility of inflation, output and interest rates.[7,8] However, these authors mentioned that the IFB rules may be less robust across models than the Taylor-type rules since IFB rules include a model consistent forecast of inflation, which could be affected by a change to the model.[9]

A generic form that nests the simple instrument rules considered in this study is given by[10]:

$$R_t = \theta_R R_{t-1} + (1-\theta_R)R^* + (1-\theta_R)[\theta_\pi(\pi^q_{t+h} - \pi^*) + \theta_y YGAP_t]. \quad (5)$$

In this equation, π^q_{t+h} is the quarterly inflation rate (annualized) and h is the feedback horizon as described in Batini and Nelson (2001). For the Taylor-type rule, h will be set to 0, while for IFB rules this feedback horizon will come from the optimization exercise.

4 Optimized Simple Instrument Rules

In this section, we first describe the approach used to determine the optimal rule within the class of *simple instrument rules*. We then report the optimized

[7]QPM stands for *Quarterly Projection Model*. It is a model of similar size as TOTEM. It includes two components, which are referred to as Steady-State QPM and Dynamic QPM. Steady-State QPM describes the long-run behaviour of successive generations of utility maximizing households and profit maximizing firms, given the choices of the fiscal authority and the links to the rest of the world. The dynamic model traces the adjustment path of the economy to this long-run equilibria. The dynamics of QPM are driven by multiperiod contracts, costly adjustment and a mix of backward-looking and model consistent expectations. The latter comes from the assumption of incomplete knowledge by agents about the true structure of the economy. Please see Coletti et al. (1996) for more information about QPM.

[8]The analysis in Maclean et al. was done without the incorporation of judgement.

[9]Amano et al. documents that IFB rules might not be so sensitive to changes in the behaviour of economic agents. However, it seems that a change in the level of credibility of monetary authority could affect the coefficients and the horizon of the optimal IFB rule.

[10]We are presently working on improving the foreign part of TOTEM. Once this becomes satisfactory, we will investigate if the addition of variables related to the open-economy feature of TOTEM would enter as a determinant in the reaction functions considered in this paper, following the work of Batini, Harrison and Millard (2003).

rule for both the Taylor-type and the IFB rules for each of the two loss functions (i.e. the variations of (5) described previously). We complete this section by presenting our preferred rule and defending our choice and discuss about the implications of using this rule on the Optimal Policy Horizons (OPH).

4.1 Methodology used to determine optimal simple instrument rules

We simulate the model with the historical distribution of shocks to find the optimal values of the coefficients in equation (5). We use 27 different shocks in this simulation.[11] These shocks can be grouped into demand shocks, productivity shocks, wage and price mark-up shocks, fiscal policy shocks and foreign shocks.[12] We then optimize the vector of coefficients (Θ) of the reaction function (and the horizon in the case of IFB-type rules) such that it minimizes the central bank's loss function:

$$\Theta = \arg\min_{\Theta} \bar{\mathcal{L}} \tag{6}$$

We perform the minimization of the loss function with a grid search over all the coefficients and the feedback horizon.

With these optimized parameters, we simulate again the model over the historical distribution of shocks and calculate some key statistics such as the standard deviations of inflation, the output gap and the nominal interest rate. We also use the standard deviation of inflation to give us an idea of the bands that would surround the target inflation rate.

[11] For these stochastic simulations, the draws were generated from the covariance matrix of the shocks estimated over the period 1992-2005.

[12] We do not consider shocks to the inflation target nor domestic monetary policy shocks for this exercise given that the goal of the project is to establish the monetary policy rule.

4.2 Optimized parameters for the different instrument rules

Table 1 reports the optimal parameters for the Taylor-type and the IFB-type rules under the two loss functions described in section 3.1. It also reports the coefficients of the estimated rule over history. It should be noted that for IFB rules the feedback horizon (h) is optimized, while for the Taylor-type rule it is fixed to zero and fixed to two for the estimated rule.

Table 1 : Results for the simple instrument rules

Rule	θ_R	θ_π	θ_y	h	Loss
Estimated[a]	0.8	2.5	0	2	4.85
A: $\bar{L}_1 = \sigma^2_\pi + \sigma^2_{ygap}$					
Taylor	0	6.5	0.1	-	4.21
IFB	0	7	0.1	1	4.16
B: $\bar{L}_2 = \sigma^2_\pi + \sigma^2_{ygap} + 0.5\sigma^2_{\Delta R}$					
Taylor	0.85	6.5	0.1	-	4.40
IFB	0.95	20	0.35	2	4.21

[a]The results of the estimated rule are included as a benchmark.

Table 1 displays some interesting features. First, we can see that the IFB-type rule performs slightly better than the Taylor-type rule for both loss functions, which means that the monetary authority gains to react to expected inflation. This is consistent with the results in Armour, Fung and Maclean (2002) who also conclude that the Canadian monetary authority gains in adopting an IFB rule over a Taylor-type rule in the context of QPM. However, the gain in favour of the IFB-type rule was more important in QPM than it is for ToTEM. Another difference is the feedback horizon that was

6 to 7 quarters (*h*=*6,7*) in QPM, which is longer than the optimal feedback horizon that we are finding.[13] This seems to be due to the fact that the actual version of ToTEM is more forward-looking and generates less inflation persistence than QPM.

Second, when the loss function does not incorporate interest rate variability, the optimal policy rule prescribes putting no weight on the lagged interest rate. However, when the monetary authority puts a weight on interest rate variability in its loss function, the coefficient on the lagged interest rate (θ_R) increases considerably. This coefficient increases from 0.0 to 0.85 for the Taylor-type rule and to 0.95 for the IFB rule.

Third, the optimal feedback horizon for the IFB rule is longer when the monetary authority cares about interest variability, although there is a difference of only one quarter between the values of the two loss functions. With preferences defined over interest rate variability as well as over inflation and output, the policy maker can not react as quickly as before to offset the deviations of inflation from its target. Hence, it has to be more forward-looking than in the case with no concern for interest rate smoothing. Being more forward-looking allows the monetary authority to smooth the response of its instrument following a shock.

Fourth, the fact that the estimated rule indicates a large smoothing coefficient ($\theta_R = 0.8$) is consistent with the idea that the monetary authority had some concerns about interest rate volatility for financial stability reasons or that it wanted to reduce the probability of hitting the zero-bound of interest rate. It could also be that uncertainty about the nature of the shocks that have hit the economy over history was limiting the reaction of the monetary authority to these shocks.

Table 2 reports the standard deviation and persistence of inflation, of the output gap and of the nominal interest rate for each policy rules. As expected, the variability of inflation under an IFB rule is larger than under a Taylor rule because the Taylor rule reacts more quickly to every departure

[13] For QPM, the optimal feedback horizon reported here is in terms of the year-over-year inflation rate. For ToTEM, the optimal feedback horizons (h) reported in Table 1 are in terms of the quarterly inflation rate. However, if they were expressed in terms of the year-over-year inflation rate, they would add about two quarters to the value of h reported in Table 1.

of inflation from target, whereas an IFB rule will react only if this departure is expected to be persistent. On the other hand, this allows IFB rules to do a better job at keeping the output gap variability low relative to the Taylor rule, although the difference is small. Overall, the latter dominates in the case of the loss function that has a weight on interest rate movements ($\bar{L}_2$) whereas the performance of the IFB rule is almost the same as the Taylor-type rule in the case of the loss function $\bar{L}_1$(see Table 1).

Table 2 : Variability and persistence of inflation, output gap and interest rate under different rules

Rule	σ_π	ρ_π	σ_{ygap}	ρ_{ygap}	σ_R	ρ_R	$\sigma_{\Delta R}$
Estimated	0.47	0.77	2.15	0.99	0.48	0.95	0.15
A: $\bar{L}_1 = \sigma_\pi^2 + \sigma_{ygap}^2$							
Taylor	0.31	0.70	2.03	0.99	2.35	0.67	1.92
IFB	0.35	0.72	2.01	0.99	2.10	0.62	1.83
B: $\bar{L}_2 = \sigma_\pi^2 + \sigma_{ygap}^2 + 0.5\sigma_{\Delta R}^2$							
Taylor	0.28	0.68	2.06	0.99	0.92	0.93	0.35
IFB	0.36	0.71	2.02	0.99	0.66	0.95	0.21

When the rules are optimized such that the monetary authority puts a weight on interest rate volatility in the loss function, the standard deviation of interest rates decreases significantly (about 140 basis points for both rules). What is particular in this model is that reducing interest rate volatility is not very costly in terms of the variability of inflation and the output gap. This is best seen for the case of the IFB rule where the standard deviation of inflation and of the output gap are barely affected by the inclusion of the objective of stabilizing the policy rate in the preferences of the monetary authority. The same conclusion holds for the Taylor-type rule. This result is linked to the rational expectations feature of ToTEM combined with the fact that agents have a very low discount rate for the future, which leads them to be indifferent between an immediate increase of 100 basis points in the policy rate and four increases of 25 basis points each.

Based on the results of this section, we tend to prefer the IFB rule with a smoothing parameter of 0.95. This is the rule that optimizes the second loss function ($\bar{L}_2$). This choice is based on the results in Table 2, which shows that this specification significantly reduces interest rate volatility with only marginal effects on the variability of inflation and output gap. For both loss functions, the IFB specification performs modestly better than the Taylor specification, which means that the monetary authority gains to be forward-looking.[14] Using the rule consistent with minimizing $\bar{L}_2$ does not cause $\bar{L}_1$ to increase greatly. Thus, since it results in substantial reduction in interest rate variability, we think that this rule is preferable.

4.3 The Optimal Policy Horizon

For countries that target inflation, it is important to know how long it takes for inflation to be back on target. This is what Batini and Nelson refer to as the Optimal Policy Horizon (OPH henceforth). This OPH is largely dependant on the specific realization of shocks prevailing at the time the monetary authority must decide on its policy instrument. If the shocks that involve tensions between the objectives of the monetary authority (i.e. the variables in the preference function) are dominant, the return of the inflation rate to its targeted value would take more quarters than in the case where the dominant shocks do not imply any tension between these objectives.

Given that the number of quarters it takes for inflation to go back to its targeted value is dependent on the realization of shocks, we ran another set of stochastic simulations with 500 draws from the historical distribution of the structural shocks. For each draw, we have calculated the number of quarters inflation takes to be back on target. Using the outcomes of these simulations, we can calculate the average and median OPH as well as the

[14] However, as said before, forward-looking rules are potentially less robust to model misspecification than Taylor-type rules since they include a model consistent forecast of the inflation rate.

empirical distribution. However, given that for some shocks the inflation rate returns only asymptotically to its targeted value, we need to impose a practical criterion to assume that inflation is back to the target. We follow Batini and Nelson and use their proposed absolute criterion, where the year-over-year inflation rate is deemed to be back to its targeted level when it falls inside a band with a width of 0.1 percentage points on each side of the target. We also use a more restrictive version of this criterion where the width of this band is 0.05 percentage points on both sides of the targeted rate.

Figure 4 shows the histogram of the OPHs for our proposed policy rule (i.e. the IFB rule associated with loss function $\bar{L}_2$) for each criteria we consider. For the first criterion, where the width is 0.1 p.p. on each side of the targeted rate, the average and median OPH is 5 quarters. Also, using this criterion, year-over-year inflation is back to the target within 4 to 7 quarters 95% of the time. For the more restrictive criteria (with a width 0.05 p.p. on each side of the targeted value), the average and median OPH is 6 quarters and inflation is back to the target within 4 to 11 quarters 95% of the time.

The existence of these intervals for the return of inflation to the target reflects the fact that the central bank does not react the same way to each shock. This is because each realization of shocks implies different tensions between the variables in the preferences of the monetary authority. To better illustrate this, Figure 5 compares the behaviour of the inflation rate following a temporary demand (consumption) shock and a temporary wage mark-up shock.[15]For each of these shocks, the size has been scaled such that they generate the same initial increase in the inflation rate. From this figure, we can see that the return of inflation to the targeted rate is more rapid when the economy is hit by demand-type shocks than by mark-up-type shocks.[16]

[15]In the latter case, this shock comes from the assumption in TOTEM that households have labour skills with some degree of heterogeneity with respect to the other households. Therefore, they have some market power over their labour services. This shock can alternatively be seen as a temporary labour supply shock.

[16]In these two experiments, we used the optimal IFB rule associated with $\bar{L}_2$. However, the results would not be qualitatively different with another policy rule.

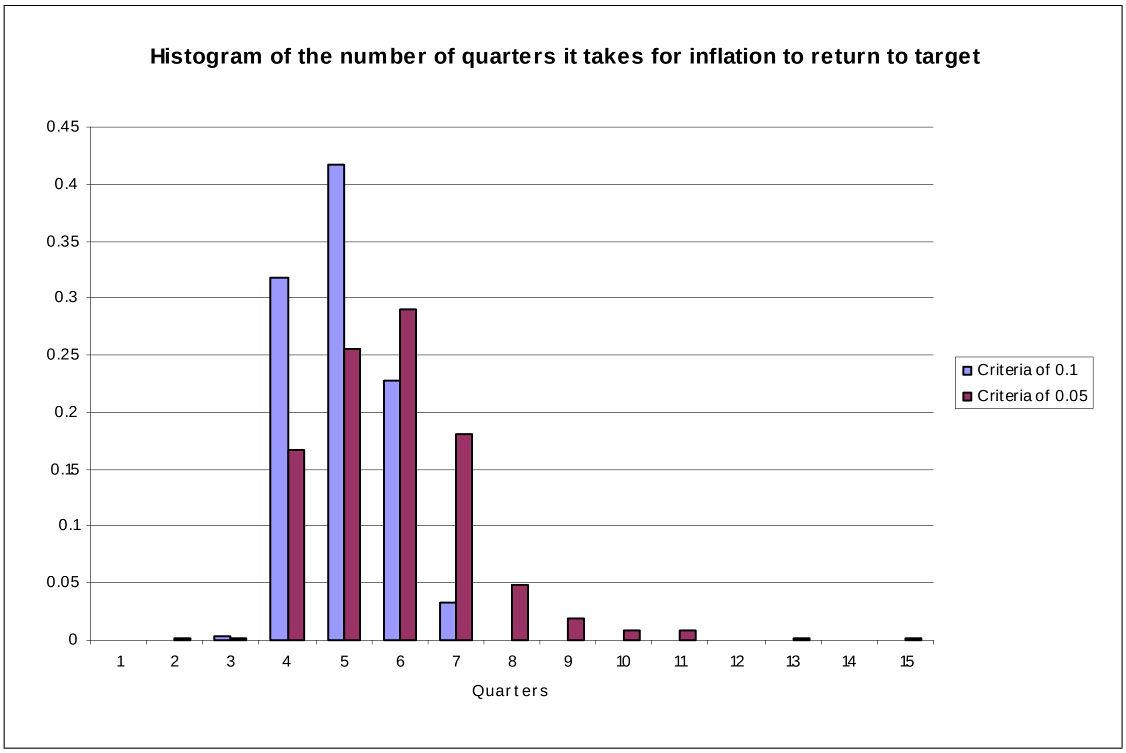

Figure 4: Histogram of the number of quarters it takes for inflation to return to target for the optimized rule (or the OPH).

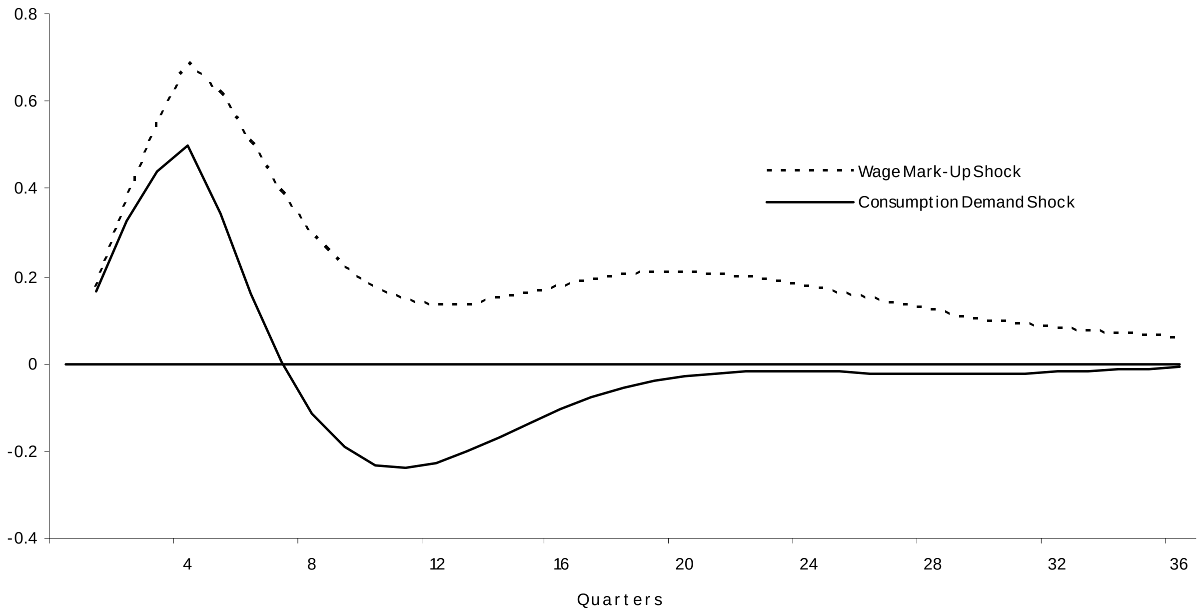

Figure 5: Response of year-over-year inflation to a wage mark-up shock and to a consumption demand shock.

4.4 Confidence Bands around the inflation rate

The stochastic simulations performed in the previous sections provide the moments of key endogenous variables. Among other things, they can be used to calculate confidence bands around the inflation rate. These confidence bands can be used to get some information regarding the appropriate width of the inflation target band. However, before we go further, we want to limit their usefulness given that the standard deviations of inflation and of the other endogenous variables do not incorporate the uncertainty related to the model (both the parameters and the model's structure).

Table 3 presents the 90, 95 and 99 per cent confidence bands around the year-over-year inflation rate for each of the policy rules associated with the loss function $\bar{L}_2$. We can see that these confidence bands are all contained within the actual inflation target band in Canada of 1-3%. As well, we see that the probability for the inflation rate to be inside the actual target band for Canada is 99.9 per cent in the case of the optimal Taylor rule and 99.5 per cent in the case of the optimal IFB rule. As an indication, the core inflation rate has been outside the official target band only once since 1991, or alternatively, it has been inside 98% of the time.

Table 3: Confidence intervals surrounding the outcome for inflation

	90%	95%	99%	Prob.($\pi \in [1, 3]$)
Taylor	1.5-2.5	1.4-2.6	1.3-2.7	99.9%
IFB	1.4-2.6	1.3-2.7	1.1-2.9	99.5%

5 The robustness of simple instrument rules to the realization of shocks

As raised earlier in this paper, one of the advantage of *simple* instrument rules over the more complicated *fully* optimal instrument rule (i.e. coming from the optimal control problem of the monetary authority) is that they are potentially less affected by a change in the distribution of shocks given that they contain less variables and coefficients.[17] However, they are not totally immune to a change in the distribution of shocks. This possible feature can have non trivial implications in a model used to produce economic projections and monetary policy recommendations given that the realization of shocks at any given period will likely differ from the historical distribution used to parameterize the rule. Svensson (2003), uses this argument to justify the use of targeting rules since these are independent of shocks, which ensures an optimal reaction of the central bank for any given realization of shock.

The distribution of shocks used to parameterize the optimized rule found in section 4.2 was based on a mixture that includes shocks that cause tensions between the output and inflation stabilization objectives and shocks that do not cause such tensions. The presence of these tensions slows the reaction of the policy rate to an inflation (output) deviation from its target since this reaction would cause output (inflation) to deviate from its targeted level. Reacting too aggressively to one gap could result in a significant increase in the other gap, which could then affect the preferences of the authority in a negative way. Therefore, the use of the IFB rule found in section 4.2 (benchmark rule henceforth) in a situation where there is only one possible type of shock (that either causes or does not cause these tensions) would result in a policy reaction that is not optimal to this realization. Indeed, if it is a situation where only the shocks that cause such tensions are present, the interest movements prescribed by the benchmark IFB rule would be too aggressive, which could negatively affect the utility of the central bank. Conversely, using this benchmark rule in a world where the only type of shock at play does not cause such tensions would imply a reaction of the central bank that is not aggressive enough, which would also affect its utility negatively. Therefore, the optimized simple instrument rule found in section

[17]This applies to the case where this rule is written such that the policy instrument reacts to a set of state or predetermined variables.

4.2 most likely will not provide the optimal reaction to the specific realization of shocks at play in any period where the central bank has to decide about policy action.

To better illustrate this point, we conducted the experiment where we found the optimal IFB rules that would be obtained in a world where the economy is submitted to only one type of shock[18]. We consider four of these worlds and their corresponding optimal rule is reported in Table 4. The four shocks (worlds) considered are : temporary demand (consumption), exchange rate and wage mark-up shocks and a permanent commodity price shock. The results confirm that the optimal IFB rule is highly affected by the nature of the shocks at play. Figures 6 and 7 show two sets of responses of some key variables for two of these worlds, one where there is only temporary exchange rate shocks and one where there is only temporary wage mark-up shocks. The first set of impulse responses (solid lines) assumes that the central bank reacts with the IFB rule that is optimal for this specific shock (the rule given in Table 4). The second set of IRFs (dashed lines) assumes that the monetary authority reacts according to the IFB rule that was optimized over the historical distribution of shocks as reported in Table 1, Section 4.

In the world with only temporary positive exchange rate shocks (the shock shown here is an initial unexplained depreciation), there is no tension between the inflation and the output stabilization objectives. In this shock, both variables move such that they imply a positive deviation from their targeted levels. The direction of the movement of the policy rate necessary to eliminate each of these two gaps individually is the same. Therefore, the optimal reaction of the policy instrument in such a world is more aggressive than in a world where both demand-type and mark-up type shocks exist. In Figure 6, we see that the deviations of inflation and output from there equilibrium values are significantly less important with the policy rule optimized for this specific type of shocks compared to the benchmark rule, which was optimized using the historical distribution of shocks.

We now consider a world of only wage mark-up shocks. In such a world, there are always tensions between the inflation and output stabilization objectives. In this world, the increase in the policy instrument necessary to eliminate a positive inflation gap (which brings down the value of $\bar{L}_2$), will

[18]Assuming that the preference of the monetary authority is given by $\bar{L}_2$.

result in a negative output gap (which increases the value of $\bar{L}_2$). Therefore, the required reaction of the policy instrument to this realization of shock will be less aggressive in comparison to a situation where there is the possible presence of shocks that do not cause such tensions. This is reflected in Table 4 where we see that the short run coefficient of the inflation deviation from its target[19] is 10 times smaller than for the benchmark rule, while the change in the short run coefficient on the output gap[20] is modestly affected. Therefore, we see that the tensions between the objectives of inflation and output stabilization reduces the required short run response of the monetary authority to the inflation gap relative to the output gap. This is illustrated in Figure 7 where we see that the use of the optimized rule in a world where only wage mark-up shocks are present means that the monetary authority must tolerate a greater variability of inflation in order to lower the variability of the output gap. We see that the reaction of the policy rate is less aggressive in this case than under the benchmark rule. Furthermore, given the actual calibration of the model, the real interest rate must be negative for a while in order to eventually bring the inflation rate back to its target[21].

Table 4 : Optimal rules based on the loss function $\bar{L}_2$ for different shocks

	θ_R	θ_π	θ_y	h
Demand Shocks				
Consumption demand shock	0.75	2	0.3	0
Exchange Rate shock	0.75	18	0	0
Relative Price Shocks				
Wage shock	0.95	2	0.3	4
Commodity Price shock	0.9	20	0	3
Optimal IFB rule for all of the shocks used	0.95	20	0.35	2

[19] Given by $(1-\theta_R)\cdot\theta_\pi$.

[20] Given by $(1-\theta_R)\cdot\theta_y$.

[21] Eventually, as shown in Figure 7, the real interest rate becomes positive in order to eliminate the inflation deviation from the target. Indeed, the cumulative deviation of the real interest rate from its equilibrium value has to be positive in order to bring inflation back to the target.

The results in this section confirm the fact that simple instrument rules are not immune to a change in the distribution of structural shocks. As said previously, this is important for a model used to peform projection scenarios given that for each exercise, the economy is affected by a realization of shocks that does not likely reflect the historical combination of shocks used to find the optimized simple rule. The results reported here support the need for policy decision strategies (in a model) optimal under any particular realization of shocks. As well, it reiterates the need for a central bank to commit to an objective rather than to a rule.

These two arguments suggest that we should consider developing a specific targeting rule, as advocated by Svensson (1999, 2003, among others). This author argues that targeting rules are more realistic representations of the framework used by targeting central banks to arrive at a decision about their policy instrument. A specific targeting rule establishes an explicit operational condition that forecasts of the targeted variables must fulfill. This operational condition is the first order condition of the optimization problem of the central bank. The central bank then commits to move its policy instrument such that this condition is met. This type of rule is immune to changes in the distribution of shock and will provide the optimal reaction of the monetary authority to any given realization of shocks since the forecasts of the targeted variables will change accordingly. As well, another advantage of this approach is that it makes use of all the relevant information about the determinants of the targeted variables, including the judgement of the monetary authority. However, even if this type of rules would propose an optimal reaction for any given realization of shocks, they are not immune to the uncertainty related to the nature of the shocks and to the model. This is an important issue since in the real world, the staff in central banks have to deal with these uncertainties on a regular basis. Therefore, the optimized simple instrument rule that we found in this study is still useful since it is potentially more robust across models than a specific targeting rule.

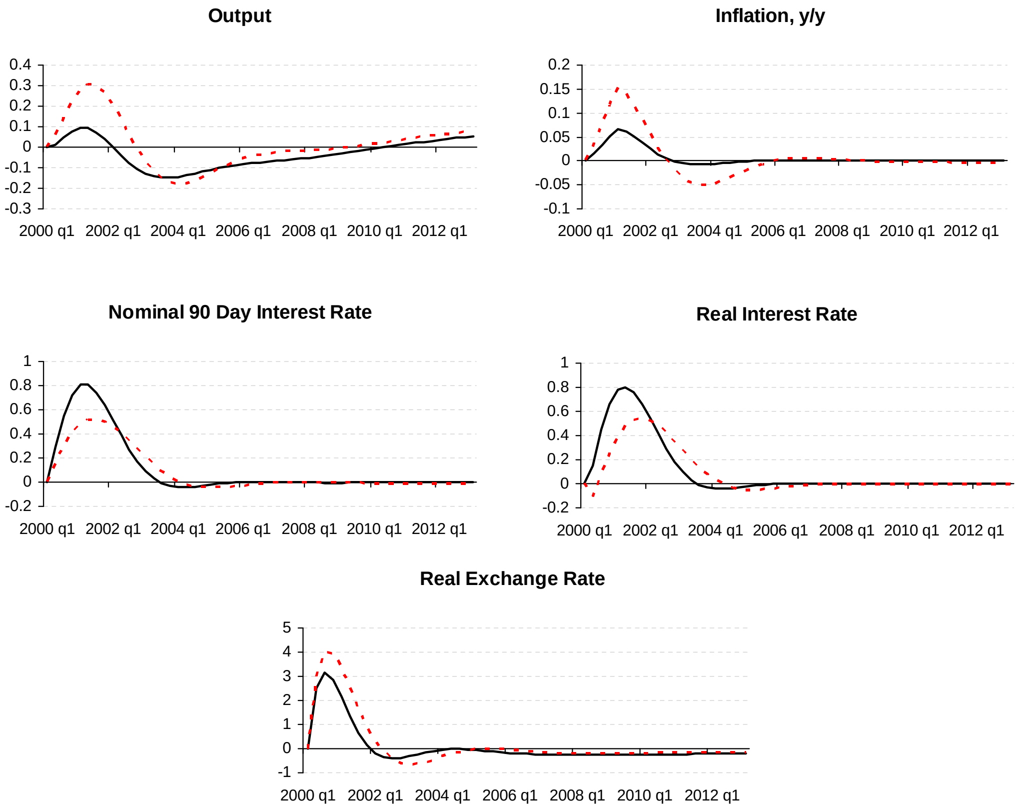

Figure 6: Exchange Rate Shock. The black line shows the reponse to an exchange rate shock using the policy rule optimized for a world with only exchange rate shocks. The dotted line shows the response to an exchange rate shock with the benchmark policy rule (from section 4.2).

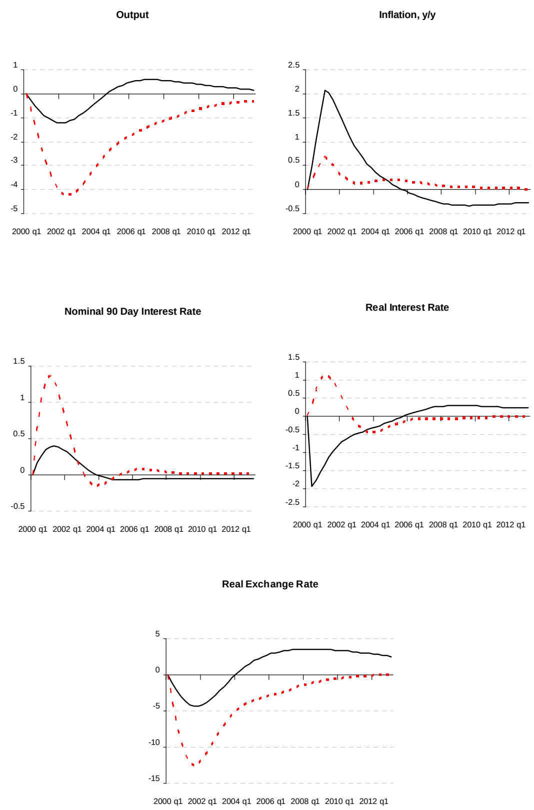

Figure 7: Wage Mark-up Shock. The black line shows the reponse to a wage mark-up shock using the policy rule optimized in a world with only wage mark-up shocks. The dotted line shows the response using the benchmark policy rule (from section 4.2).

6 Conclusion

The main objective of this paper is to determine the optimal rule to be used in the actual version of ToTEM, which is an open-economy DSGE model under development at the Bank of Canada. For this study, we considered simple instrument rules, more precisely Taylor-type and Inflation Forecast Based (IFB) rules. A fully optimal instrument rule (i.e. the one that comes from solving the optimal control problem of the monetary authority) or a targeting rule as advocated by Svensson would have been preferable. However, these rules are likely not robust to changes in the model and, therefore, at this stage of the project we considered only the class of simple instrument rules.

For each type of rule, we determined its optimal parameterization based on two representations of the assumed preferences of the monetary authority. In the case of the IFB rule, following Batini and Nelson (2001), we also determined the optimal feedback horizon, which is the horizon at which the policymaker should look at expected inflation when setting its instrument. Based on the results in this paper, our preferred rule is an IFB rule with an optimal feedback horizon of 2 quarters (for quarterly inflation), which has a high coefficient on the expected deviations of inflation from the target, a small weight on the contemporaneous output gap and a significant weight on lagged interest rates. This rule is based on a loss-function where the preferences of the monetary authority are defined over inflation and output gap variability as well as over the variability of interest rate movements. Our results show that allowing the central bank to be concerned by the volatility of interest rate movements significantly reduces the variance of the policy instrument compared to the case where the policymaker puts no weight on the volatility of its instrument in its preferences. This is achieved with no perceptible increases in the variance and the persistence of inflation and the output gap.

Another interesting result is the uncertainty regarding the number of quarters it takes for the year-over-year inflation rate to return to its targeted value after our modeled economy (that includes the preferred rule) is hit by a combination of shocks that reflects what was seen over history. The width of the confidence bands around the number of quarters it takes for inflation rate to be back at its targeted level reflects the fact that the relative importance of shocks that generate tensions between the objectives of the monetary authority likely changes in every period. The more important (dominant) these shocks are, the more time it takes for inflation to go back to its target. We also showed that our proposed simple IFB rule is not immune to a change in the distribution of shocks. This result is particularly important for a model that could be used to produce economic projections and monetary policy recommendations given that, in any period, the economy is hit by a realization of shocks that most likely differs from the historical distribution used to parameterize this optimized rule.

The results discussed in this paper are based on a model that is still under development and a parameterization that is preliminary. Changes to ToTEM will obviously affect the results quantitatively, and possibly qualitatively. As well, they are derived under the assumption that there is no uncertainty regarding the model and the nature of the shocks at play in any period. However, the work in this paper provides useful guides to help us to frame our intuition about issues that are relevant for policymakers. For instance, the confidence bands around inflation outcomes could provide an indication about the appropriate width of the inflation target band. As well, the horizon over which inflation returns to the targeted rate could help policymakers frame their communication with the public when explaining their policy actions or their view about the economy. Future work with improved versions of ToTEM will be devoted to looking at developing optimal simple instrument rules, as well as (fully) optimal instrument and targeting rules and will look at the same issues investigated in the present paper.

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