Costly Capital Allocation with Credit Market Frictions as a Propagation Mechanism*

André Kurmann† — UQÀM, CIRPÉE

Nicolas Petrosky-Nadeau‡ — UQÀM, CIRPÉE

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Abstract

Empirical evidence suggests that capital separation is an important phenomenon over and beyond depreciation and that reallocation is a costly and time-consuming process. In addition, both separation and reallocation rates display substantial variation over the business cycle. We build a dynamic general equilibrium model where capital separation occurs endogenously because of credit constraints and capital (re)allocation is costly due to search frictions and capital specificity. Compared to the frictionless counterpart but also compared to models of financial frictions without costly capital reallocation, our model matches surprisingly well the persistence in U.S. output growth. Furthermore, our model implies that productive capital stocks are more volatile and more procyclical than reported in the data, which has the potential to substantially reduce the size of technology shocks inferred from the Solow residual.

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†*Contact address:* André Kurmann, Université du Québec à Montréal, Department of Economics, P.O. Box 8888, Downtown Station, Montréal (QC) H3C 3P8, Canada. *Email:* kurmann.andre@*uqam.ca*.

‡*Contact address:* Nicolas Petrosky-Nadeau, Université du Québec à Montréal, Department of Economics, P.O. Box 8888, Downtown Station, Montréal (QC) H3C 3P8, Canada. *Email:* petrosky-nadeau.nicolas@*courrier.uqam.ca*.

1 Introduction

Recent evidence from firm-level data shows that capital separation is an important phenomenon over and beyond depreciation and that investment in used capital represents up to one fourth of total investment, even though reallocation is a costly and time-consuming process. Aggregate investment flows inferred from these data are thus substantially larger than the ones reported in the national accounts (where reallocation is by definition missed) and exceed depreciation at almost all phases of the business cycle. The same firm-level evidence also shows that even for narrowly defined sectors the distribution of investment rates across individual firms is wide, with large mass at zero (firms with no investment) and a fat right tail (firms with investment spikes).

These findings strongly contrast with standard assumptions about capital accumulation in modern Dynamic General Equilibrium (DGE) models of the business cycle. Households invest in a generic stock of capital that is rented out on a period-by-period basis; separation and reallocation across firms is costless (apart from possible frictions to changing the generic capital stock); and investment flows net of depreciation are either positive or negative across all firms.

In this paper, we propose a new model that captures the main empirical characteristics of investment and capital reallocation, yet remains tractable and ready-to-use in a DGE context. The main objective is to examine whether the more realistic description of capital flows helps to generate internal amplification to small shocks and persistence in output growth.

Our model extends the standard real business cycle (RBC) benchmark along three dimensions. First, firms must post projects at a cost and search for available capital to undertake investments. The probability of a match depends on how much capital is made available by households relative to the total number of projects posted. Second, matched capital remains with the same firm until separation occurs. Separated capital looses a fraction of its value due to specificity, and reallocation to another productive unit is time-consuming due to the aforementioned search friction. Third, separation occurs in part endogenously when the firm's revenue falls short of covering factor payments, which are determined prior to the realization of an idiosyncratic productivity shock. A key assumption implied by this endogenous separation mechanism is that firms cannot borrow short term to bridge over bad realizations of the ex-post idiosyncratic shock.

This last restriction can be interpreted as a simple form of a credit constraint. It implies that capital separations vary countercyclically with business conditions, in line with the empirical evidence. As a result, exogenous shocks directly affect the *productive capital stock* rather than just *investment in capital*. For example, a temporary technology shock increases average firm profits and thus decreases the separation rate of capital from production. The subsequent larger capital stock leads to a humpshaped response of labor and output. By the same token, the increased productivity increases the number of project postings by firms relative to the capital available for investment and the drop in separations decreases the amount of ressources lost due to specificity of separated capital. Both of these factors imply that the household's marginal value of investment relative to consumption increases more than in a frictionless model. Households are therefore willing to supply more work for a given wage, thus further magnifying the effect of the positive technology shock.

Overall, the state-dependent nature of the frictions in our model generate substantial endogenous amplification and imply output dynamics that come surprisingly close to replicating the marked positive autocorrelation in U.S. output growth over short horizons. Our model thus provides an answer to Cogley and Nason (1995) who show that the RBC benchmark but also extensions with adjustment costs or time-to-build lags in investment fail to perform satisfactorily along these two key dimensions. Furthermore, we illustrate that while the costly capital matching friction already generates considerable amplification on its own, the specificity of capital and the countercyclical separation rate implied by the credit constraint are both central in obtaining persistence in output growth and lead to even further amplification.

The significance of the credit constraint in our model is especially interesting for two reasons. First, it supplements the rich microeconomic literature on the importance of credit market frictions to rationalize firm behavior.[1] Second, it contrasts with most of the existing literature on the business cycle effects of credit market frictions. For example, models of incomplete

[1] Empirically, panel data studies find that small firms with more difficult access to credit pay fewer dividends, take on more debt, and have investment rates that are more sensitive to cash flows even after controlling for future profitability. See Hubbard (1998) and Stein (2000) for surveys. Theoretically, numerous papers show how optimizing models of the firm with incomplete contract enforcement and asymmetric information in the lending process can rationalize the observed correlation of firm size and age with mean growth (negative) and survival rates (positive). See Cooley and Quadrini (2001) or Clementi and Hopenhayn (2002) for examples.

information between lenders and firms such as Bernanke and Gertler (1989) imply that the firm's ability to finance investment varies inversely with the value of its collateral and thus with the business cycle. This financial accelerator mechanism has the potential to generate amplified and persistent output effects in response to small shocks. Yet, simulations in a DGE context by Chari, Kehoe and McGrattan (2002), Dib and Christensen (2005) or Petrosky-Nadeau (2005) suggest that for plausible calibrations, credit market frictions of this type alone fail to generate *quantitatively* important business cycle fluctuations.[2] Similar to models with adjustment costs or time-to-build lags in investment, this lack of internal propagation can be traced back to the assumption of costless capital allocation, which implies that credit market frictions only affect the response of aggregate investment with respect to shocks. But investment is small relative to the productive capital stock and thus, the impact on output remains very small. Furthermore, indirect effects through expenditure induced changes in the labor supply are limited by offsetting movements in interest rates. By contrast, the credit constraint in our model affects directly the productive capital stock and not just investment; and the state-dependent nature of the different frictions implies that households are much more willing to pour ressources into capital today for higher consumption in the future. Our model therefore suggests that credit market frictions may well be very important for business cycle fluctuations, which has potentially far reaching policy implications.

Finally, the countercyclical separation and procyclical reallocation rates in our model as well as the firm-level data suggest that capital stocks used for actual production may be much more volatile and procyclical than reported in the national accounts. This has the potential to substantially reduce the size of technology shocks inferred from the Solow residual.

Our strategy to formalize costly capital allocation is inspired by the now widely employed search-and-matching approach to model labor market frictions, as pioneered by Diamond (1981) and Mortensen and Pissarides (1994). This approach abstracts from the microfoundations for market incompleteness but provides a dynamic mechanism that has proved tractable and encompasses different frictions encountered in the allocation of physical capital to productive units.

[2] For example, Petrosky-Nadeau (2005) simulates the financial accelerator model of Bernanke, Gertler and Gilchrist (1998) in a New Keynesian context. He finds that the financial accelerator contributes only about 0.05% to the response of output to shocks and fails to generate persistence in output growth.

Previously, Dell' Aricia and Garibaldi (2000), den Haan, Ramey, and Watson (2003) and Wasmer and Weil (2004) have interpreted the same matching process as the result of firms soliciting financing for their capital expenditures. While such financing frictions may be highly relevant for new enterpreneurs and small firms, they seem less obvious for large firms that account for the bulk of aggregate capital accumulation. Our interpretation in this paper is therefore more general in that we describe the allocation of physical capital by a search and matching environment that – parallel to workers and firms in the labor market – has its origins in the limited yet state-dependent availability of investment opportunites, capital suppliers and financiers. Aside from our different interpretation of the matching process, we also incorporate our model in a modern DGE framework with endogenous labor supply and intertemporal consumption/savings decisions. The advantage of doing so is that the *quantitative* implications of our model can be readily compared to the RBC benchmark that our model nests as a special case, but also to the aforementioned financial accelerator models where credit market frictions only affect investment.[3]

To our knowledge, only few papers have so far examined the business cycle implications of costly capital entry/exit together with credit market frictions. One of them is Cooley, Marimon and Quadrini (2003) who derive credit market frictions from limited contract enforcability and allow for heterogeneity in firm size. This heterogeneity makes aggregation and the computation of the equilibrium a non-trivial issue. By contrast, our modeling approach bypasses the issue of firm size by assuming constant returns to scale production and the equilibrium is solved for a loglinear approximation around the balanced growth path. This greatly facilitates computation, allows for straightforward comparison with well-known business cycle models, and leaves plenty of flexibiliy to extend our analysis to more general descriptions of the rest of the economy.

The remainder of the paper is organized as follows. Section 2 reviews the empirical evidence on investment flows and capital stocks. Section 3 presents the model. Section 4 discusses functional specifiations and calibration. Sections 5 and 6 report quantitative results and assess their robustness. Section 7 concludes.

[3]Moran (2005) and Pierrard (2005) also incorporate credit matching frictions into a business cycle context. However, they do not model endogenous capital separation and reallocation. In line with our results, their models fail to generate endogenous amplification and persistence.

2 Empirical evidence

To motivate our extension of the business cycle model, we first review the computation of investment flows and capital stocks in the U.S. national accounts. Second, we document firm-level evidence on capital flows.

2.1 NIPA investment flows and capital stocks

For the National Income and Production Accounts (NIPA), the Bureau of Economic Analysis (BEA) computes investment flows and aggregate capital stocks (called fixed reproducible tangible wealth) using a supply-side top-down approach.[4] Investment flows by asset type are measured as the real value of shipments from capital goods producing industries after subtracting inventory changes, net exports abroad as well as private and government consumption of these assets. Capital stocks for each asset are then inferred from the respective investment flows using the perpetual inventory method

$$K_{a,t} = \sum_{j=0}^{\infty} \omega_{ajt} I_{a,t-j},$$

where $K_{a,t}$ is the capital stock of asset a in period t; $I_{a,t-j}$ is the real investment flow into asset a at $t-j$; and ω_{ajt} is the weight given to vintage j of asset a. This weight embodies the depreciation (decline in value) of the asset due to wear and tear, obsolescene, accidental damage, and aging. For most asset types, depreciation schedules are assumed to decline geometrically over time.[5]

For the sample 1950-1995, the annual investment flow for private non-residential assets averages 9.4% of its capital stock. The capital stock series is very smooth. Its Hodrick-Prescott filtered standard deviation for 1950-1995 is a mere 0.08. The low variability is one of the main reason why many business cycle researchers abstract from capital when computing Solow residuals (see for example King and Rebelo, 2000).

Aside from the many problems associated with measuring and appropriately deflating shipments of capital goods, there are three reasons why NIPA's investment and capital stock series

[4] Becker, Haltiwanger, Jarmin, Klimek and Wilson (2005) provide a more detailed description of these computations and discuss the associated problems.

[5] See Katz and Herman (1997) for a description.

are problematic quantities for business cycle researchers. First, shipments of capital goods only provides information about investment flows of new assets and the BEA adjusts these series only for net transfers of used capital from consumers, government and foreign countries. Inter- and intra-industry transfers are completely missed. Second, the BEA's depreciation schedules in ω_{ajt} are supposed to reflect the service life of an asset, which implicitly assumes that capital from sales and exiting businesses is transferred costlessely to other productive units. To the extent that capital separation (i.e. exit of firms and sales) is an important phenomenon, ω_{ajt} therefore underestimates the loss in capital value due to irreversibilities, specificity and other reallocation frictions. Both the first and second point imply that total annual investment in new and used capital goods may be substantially larger than 9.4%. Third, if capital separation and reallocation vary over the cycle, the NIPA capital stock measure may be much too smooth.

2.2 Firm-level evidence on gross capital flows

To quantify the importance of gross capital flows, we need to adopt a bottom-up approach and look at firm-level data on investment expenditures and disinvestment. Conceptually, capital separation from a productive process and subsequent reallocation occurs either because a continuing firm sells property, plant and equipment (PP&E); because a firm is liquidated; or because of mergers/acquisitions. We include this latter case in our investigation since mergers/acquisitions not only represent a change of ownership but often involve important modifications to the composition and use of existing capital.[6] Here, we review results from different U.S. firm- and establishment-level surveys on these quantities. None of the surveys is entirely representative and each one of them suffers from its own shortcomings.[7] They nevertheless provide valuable evidence on the importance of gross capital flows.

One of the first studies with firm-level data is Ramey and Shapiro (1998) who use the Compustat survey to document gross flows of capital. For their full sample (1959-1995), they find that roughly 70% of gross investment flows come from expenditures in new PP&E by existing firms, about 25% come from purchases of used capital, while entry of new firms contributes only 5%. These addition rates exhibit large fluctuations and comove with the cycle. For capital

[6] See Ramey and Shapiro (1998) or Eisfeldt and Rampini (2005a) and references therein.

[7] See Becker, Haltiwanger, Jarmin, Klimek and Wilson (2005) for a detailed discussion about the shortcomings of the different surveys.

separations, in turn, retirements – which can be interpreted as the physical result of depreciation – are the most important component (71%), followed by sales (21%) and exits due to mergers and bankruptcies (9%). By contrast to additions, these rates vary countercyclically around trend, resulting in a correlation coefficient with unemployment of 0.52%.

Overall, these gross flows of capital additions and substractions average 9.7% and 7.3% of undepreciated capital stocks, respectively, which is comparable to the job creation and destruction rates reported in Davis, Haltiwanger and Schuh (1996). These flows may not be entirely comparable to the extent that depreciation and replacement of capital is a more important phenomenon than retirement of old / entry of young workers in the labor market. At the same time, Ramey and Shapiro's gross flows of capital should be considered as a lower bound because they do not take into account acquisitions and because their measure abstracts from depreciation of capital in use.[8] When taking depreciation into account, the gross flow of additions jumps up to 17.3% of total capital stocks, with investment in new capital representing 12.3%. Part of these depreciation rates probably represent accounting standards rather than actual decreases in the value-of-use. Nevertheless, it remains true that reallocation of used capital accounts for an important part of investment that is entirely missed in the NIPA tables.

The findings of Ramey and Shapiro are broadly confirmed by another study with Compustat data by Eisfeldt and Rampini (2005a). Based on a sample from 1971 to 2000, they also report that reallocation of used capital makes up 25% of gross investment, where reallocation is measured by sales of PP&E *plus* acquisitions and gross investment is defined as the sum of capital expenditures and acquisitions.[9] Sales of PP&E represent about one third of these reallocation flows, with the remaining two thirds coming from acquisitions. Both components fluctuate over

[8] This may explain why Ramey and Shapiro's ratio of new investment to total capital averages only 6.9%, which is substantially less than reported in the NIPA tables.

[9] There is some disagreement between Ramey and Shapiro (1998) and Eisfeldt and Rampini (2005a) about the treatment of acquisitions in the measure of reallocation. Ramey and Shapiro claim in their appendix that total capital expenditures (schedule v30 in Compustat) also includes the book value of the PP&E of acquired companies. Eisfeldt and Rampini, by contrast, claim that this variable excludes the net asset of businesses acquired and measure acquisitions with a separate variable (schedule v129) instead. A further difference between the two studies is that by contrast to Eisfeldt and Rampini, Ramey and Shapiro convert book values to current values, which necessitates strong assumptions about price deflators and the age of the different capital vintages in each firm.

the cycle, with a combined H-P filtered correlation coefficient with output of 0.64. Furthermore, Eisfeldt and Rampini's reallocation measure represents 1.5% of capital stocks if capital stocks are defined as total assets but 5.5% if capital stocks are defined by total PP&E. This implies that annual gross investment flows average between 6% and 22% of total capital stocks. Since total assets comprise a substantial part of non-tangible capital, the ratio of gross investment to physical capital that is relevant for traditional business cycle models such as the one presented here is likely to be substantially larger than the net investment ratio of 9.4% reported in the NIPA tables.[10]

One concern with Compustat is that it covers only corporations that file with the SEC. Other proprietorships and partnerships as well as establishments held by foreign firms not registered with the SEC are not part of their capital stock measure. Small and medium-size firms are thus underrepresented. Given that it is exactly these firms that are most likely to undergo major changes (merger/acquisition, bankruptcy, structural reorganisation), the share of separation due to sales and exits and thus the separation rate in general is likely to be larger for the economy as whole. This conjecture receives some support from Eisfeldt and Rampini (2005b) who use data from the Annual Capital Expenditure Survey (ACES) to document differences in investment behavior between large and small firms. In existence since 1993, ACES is a nationally representative firm-level survey of capital expenditures in new and used structures and equipment. The survey abstracts, however, from reallocation that is due to the acquisitions of existing firms. For the 2004 survey, investment in used PP&E averages 8.7% of total capital expenditures over all firms. As Eisfeldt and Rampini report, however, this fraction decreases considerably with firm size. For firms in the lowest asset decile, the fraction averages 28% while for firms in the highest asset decile (which make up the bulk of total investment and capital stocks), it represents only 10%. The low average compared to these larger fractions for the two extreme deciles indicate that aggregate capital flows are mostly driven by large firms. Nevertheless, these numbers still suggest that the investment flows from Compustat discussed above would be even higher if it also included small firms that are not listed with the SEC.

[10]Following the business cycle literature, we abstract from non-tangible capital here but acknowledge that it may play an important role for fluctuations and represents an interesting dimension to investigate in future research.

2.3 Capital specificity

Capital reallocation associated with sales of PP&E seems to be associated with a substantial loss in value relative to its replacement cost at the original place of use. For example, Ramey and Shapiro (2000) argue that reselling capital is a time-consuming and costly process because of thinness in used-capital markets and sectoral specificity of capital. Their argument is based on equipment level data about closures of aeronautical plants. They find that other aerospace companies are overrepresented among buyers, and that even after taking into account age-related depreciation, the average resale value of equipment is only 28% relative to replacement cost.[11]

For acquisitions of existing establishments (which account for two thirds of reallaction in Eisfeldt and Rampini's Compustat sample), this cost of reallocation may well be smaller. But even for this type of reallocation, the following computations with ACES data by Becker et al.'s (2005) corroborate Ramey and Shapiro's (2000) findings that capital reallocation is often associated with loss in value. Every year, ACES selects a new probability sample that can be used to compute the capital stock of firms that disappear, either because they cease to be active or because they continue to operate under a different firm. This series of capital separation due to exit/acquisition can then be compared with the following year's series of used capital expenditures and other additions and acquisitions. Over their 8-year sample, the thus defined absorption rate equals on average 64% of total separations. Since this measure also includes assets sold by continuing firms, the absorption of separated capital from firm death is likely to be lower.

2.4 Distribution of investment flows across firms

Studies by Caballero, Engel and Haltiwanger (1995), Doms and Dunne (1998) or Cooper, Haltiwanger and Power (1999) show based on that investment on the microlevel is lumpy, with a wide distribution across firms. At any given point in time and even for narrowly defined sectors, there is a substantial mass of firms with zero investment that coexists with firms that exhibit very high investment rates, so called investment spikes.

Becker et al. (2005) reconfirm these findings in their summary using firm-level data from the

[11] Even for machine tools, which typically have a better resale value than specialized aerospace equipment, the resale value is only about 40% relative to the replacement cost.

Annual Survey of Manufacturers (ASM). Over the sample 1972 to 2001, they find that fraction of plants with zero investment varies between 25 and 10% (with a slightly decreasing tendency). Furthermore, establishments are much more likely to have zero investment in structures (up to 62%) than equipment (as low as 5%). On the other end of the distribution, between 25 and 10% of all plants have investment rates in equipment and structures that exceed 20% of their capital stock (not counting acquisitions). In addition, the share of plants with such investment spikes is procyclical.

3 The Model

As in the frictionless RBC benchmark, our model is populated by two agents: firms that produce using capital and labor; and households who decide on optimal consumption, leisure and investments in either riskless bonds or productive capital.

We add two frictions to the model. First, firms face ex-post idiosyncratic shocks to productivity that result in endogenous separation of loss-making capital units from production. Second, the allocation of capital from households to firms involves a costly and time-consuming matching process. For the sake of simplicity, we abstract from a distinct sector for capital allocation. Instead, households act directly as capital owners. Furthermore, we assume that the same matching friction applies to all investment flows and not just to the reallocation of used capital. While the frictions involved in the allocation of new capital are certainly different than the frictions in the reallocation of used capital, this simplifiying assumption adds greatly to the tractability of our model. Furthermore, we believe that the market for investment in new capital could very well be described by a search and matching environment that – not unlike the labor market – has its origins in the limited yet state-dependent availability of investment opportunities, capital suppliers and financiers.[12]

[12] As discussed in the introduction, Dell' Aricia and Garibaldi (2000), Den Haan, Ramey, and Watson (2003) and Wasmer and Weil (2004) motivate their search-and-matching friction for investment by a process of firms soliciting lenders to finance their capital expenditures. Such financing frictions are most plausible for small, bank-financed firms. The same assumption seems, however, somewhat less applicable for larger firms with ready access to liquid capital markets.

3.1 Search and matching in the capital market

Capital is either in a productive state or in a liquid state. We define by K_{it} the productive capital stock that enters the production function of firm i in period t. Liquid capital L_t, in turn, is made up of two components: used capital that has been separated previously from other firms and new capital made available by households. As described below, we allow for the possibility that separation involves a loss of value of capital. But once this adjustment is made, our model does not distinguish between used and new capital. Hence, a negative flow of new capital simply implies that households reaffect used capital for consumption or investment in riskless bonds.

To undertake new investments, firms must post projects and search for liquid capital at cost κ per project. We denote by V_{it} the number of posted projects of firm i in period t. The amount of liquid capital allocated to firms in a given period is subject to a technology that matches the total number of projects $V_t = \int V_{it} di$ to available liquidity L_t. We describe this matching process with a function $m(L_t, V_t)$. A firm's probability to find capital is therefore given by $p(\theta_t) = \frac{m(V_t, L_t)}{V_t}$ with $\partial p(\theta_t)/\partial\theta_t > 0$, where $\theta_t = \frac{L_t}{V_t}$ may be interpreted as a measure of relative capital market liquidity. Likewise, the probability of liquid capital being matched to a firm equals $q(\theta_t) = \frac{m(V_t, L_t)}{L_t}$ with $\partial q(\theta_t)/\partial\theta_t < 0$[13] We will assume that $m(L_t, V_t)$ exhibits constant returns to scale and thus $p(\theta_t) = \theta_t q(\theta_t)$.

Capital matched to a firm in period $t-1$ enters production in period t. This relationship between firm and capital continues to hold in $t+1$ with probability $(1 - s_t)$ and so on for periods thereafter. If the relationship is terminated, which happens with probability s_t, the capital is separated and returned to the household net of depreciation δ. Both the matching probability and the separation rate are taken as exogenous by firms but depend on the state of the economy, as will be described below. Given these assumptions, firm i's total capital stock used in existing projects evolves according to the following law of motion

$$K_{it+1} = (1 - \delta)(1 - s_t)K_{it} + p(\theta_t)V_{it}.$$

[13] In addition, to ensure that $p(\theta)$ and $q(\theta)$ are between 0 and 1, we require that $m(l_t, v_t) \leq min[l_t, v_t]$

3.2 Households

The representative household chooses consumption C_t, leisure $1 - N_t$, risk-free bond holdings B_{t+1}, and the amount of liquid capital L_t destined for capital investment in order to maximize the expected discounted flow of utility $u(C_t, 1 - N_t)$. When liquid capital gets matched with a project and is transformed into productive capital, it yields a net return of ρ_{t+1} in the following period. Any liquid capital that remains unmatched yields zero return.

Given these assumptions, the optimization program of the household is described by the Bellman equation

$$
\begin{aligned}
V(U_t, K_t, B_t) = & \max_{C_t, N_t, L_t, B_{t+1}} [u(C_t, 1 - N_t) + \beta E_t V(U_{t+1}, K_{t+1}, B_{t+1})] \\
& + \Lambda_t [W_t N_t + \rho_t K_t + \phi(1 - \delta) s_t K_t + U_t + B_t + D_t - C_t - L_t - \frac{B_{t+1}}{(1 + r_t)}] \\
\text{s.t. } K_{t+1} = & (1 - \delta)(1 - s_t) K_t + q(\theta_t) L_t
\end{aligned}
$$

where $U_t = (1 - q(\theta_{t-1}))L_{t-1}$ is the quantity of unmatched liquidity in $t - 1$; D_t are firm profits transferred to households, $\phi(1-\delta)s_t K_t$ is the value of capital separated from firms and returned into the budget constraint; r_t is the risk-free rate between t and $t + 1$; and Λ_t is the shadow value of the budget constraint. The coefficient ϕ allows for the possibility that separated capital net of depreciation $(1 - \delta)s_t K_t$ suffers a loss in value due to specificity and/or costs related to separation. In particular, $\phi = 1$ implies no loss while $\phi = 0$ implies irreversibility. Also note that for now, both matching probability $q(\theta_t)$ and separation rate s_t are taken as exogenous by households.

The first-order conditions of this optimization problem are

$$(C_t) : u_C = \Lambda_t \tag{1}$$

$$(N_t) : u_N = \Lambda_t W_t \tag{2}$$

$$(B_{t+1}) : \beta E_t[\Lambda_{t+1}(1 + r_t)] = \Lambda_t \tag{3}$$

$$(L_t) : \beta E_t[V_U(U_{t+1}, K_{t+1}, B_{t+1})(1 - q(\theta_t)) + V_K(U_{t+1}, K_{t+1}, B_{t+1})q(\theta_t)] = \Lambda_t \tag{4}$$

The first three conditions are standard. The fourth condition for the household's choice of liquidity available for capital investment calls for some interpretation. It states that the discounted expected utility of the marginal unit of liquidity must equal the expected discounted return from investing in the riskless bond. With probability $(1-q(\theta_t))$ a unit of liquidity remains unmatched and is worth $V_U(U_{t+1}, K_{t+1}, B_{t+1})$ to the household, while with probability $q(\theta_t)$ it is matched with a project and turned into productive capital with marginal value $V_K(U_{t+1}, K_{t+1}, B_{t+1})$. From the above Bellman equation, we can work out these marginal values as

$$V_U(U_t, K_t, B_t) = \Lambda_t \tag{5}$$

$$V_K(U_t, K_t, B_t) = \Lambda_t[\rho_t + \phi(1-\delta)s_t] + (1-\delta)(1-s_t)\beta E_t V_K(U_{t+1}, K_{t+1}, B_{t+1}) \tag{6}$$

Note that V_K is dynamic because with probability $1 - s_t$ the investment relationship between household and firm continues into the next period.

3.3 Firms

At the beginning of each period, firm i observes exogenous aggregate technology X_t and hires labor N_{it} given the capital stock of its existing projects K_{it} to produce with technology

$$a_{it} f(X_t N_{it}, K_{it}), \tag{7}$$

with f_N, $f_K > 0$ and f_{NN}, $f_{KK} < 0$. The variable $a_{it} > 0$ denotes an idiosyncratic productivity shock to firm i that is independently distributed over time with cumulative density $F(a_{it})$ and mean $E(a_{it}) = 1$. The realization of a_{it} is assumed to take place after all input decisions and factor price equilibria are established. The ex-post nature of this shock gives rise to endogenous separation of capital, which is explained in detail in the next subsection.

Aside from the optimal amount of labor to hire, the firm needs to decide on new project postings V_{it}, which come at unit cost κ. The profit maximization problem of the firm is thus described by the following Bellman equation

$$\begin{aligned} J(K_{it}) &= \max_{N_{it}, V_{it}} \left[f(X_t N_{it}, K_{it}) - W_t N_{it} - \rho_{it} K_{it} - \kappa V_{it} + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} J(K_{it+1}) \right] \\ \text{s.t. } K_{it+1} &= (1-\delta)(1-s_t)K_{it} + p(\theta_t)V_{it}, \end{aligned}$$

where W_t and ρ_t are the wage rate and the rental rate of capital, respectively; and $\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}$ is the discount factor of future cash flows. This discount factor is a function of Λ because the firm transfers all profits to the households. Note that we dropped the idiosyncratic productivity shock a_{it} from the production function because the firm's optimal decision occurs before the realization of the shock, which is expected to equal $E(a_{it}) = 1$. Furthermore, both W_t and ρ_t are taken to be exogenous by the firm. The exogeneity of W_t is a direct consequence of our assumption of competitive labor markets. The exogeneity of ρ_{it}, in turn, implies that firms in our model do not internalize the effects of their capital stock on the marginal productivity of capital and thus on the negotiation of ρ_{it} discussed below.

The resulting first-order conditions of the optimization problem are

$$(N_{it}) : f_N(X_t N_{it}, K_{it}) = w_t \tag{8}$$

$$(V_{it}) : \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} J_K(K_{it+1}) = \frac{\kappa}{p(\theta_t)} \tag{9}$$

where $J_K(K_{it})$ is the marginal value to the firm of an additional matched project that has been transformed into capital. In addition, differentiating the firm's value function with respect to productive capital yields

$$J_K(K_{it}) = f_K(X_t N_{it}, K_{it}) - \rho_{it} + (1-\delta)(1-s_t)\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} J_k(K_{it+1}). \tag{10}$$

This equation simply states that the value to the firm of an additional unit of capital is worth today's marginal product of capital net of the rental rate plus its expected future value net of depreciation in case the project is continued.

3.4 Separations

Capital separation can occur for a variety of reasons. Since we do not want to impose in our model that all separations are due to credit constraints, we model the separation rate s_t as

$$s_t = s^x + s_t^e,$$

where s^x denotes (constant) exogenous separation in the sense of being unrelated to credit constraints; and s_t^e denotes endogenous separation due to credit constraints. To model this latter part, we assume for now that any firm with negative profits after the realization of the idiosyncratic shock a_{it} is terminated. We therefore implicitly assume that firms cannot borrow to cover for temporary revenue shortfalls. The ex-post nature of a_{it} together with the absence of borrowing represents the credit constraint in our model and will give rise to endogenous separation. By making a_{it} known to both the firm and households, we bypass, however, any agency problems that are usually emphasized in the literature on financial frictions.

Given that the firm profits after the realization of a_{it} are $D_{it} = a_{it} f(X_t N_{it}, K_{it}) - W_t N_{it} - \rho_t K_{it} - \kappa V_{it}$, the threshold value $\bar{a}_{it}$ up to which separation occurs equals $\bar{a}_{it} = (W_t N_{it} - \rho_t K_{it} - \kappa V_{it})/f(X_t N_{it}, K_{it})$ and the endogenous part of separation equals $s_t^e = F(\bar{a}_t)$. From this formula, it is clear that separation depends on the state of the economy.

It is important to realize that this separation rule is not optimal from the point of view of the household. In fact, the i.i.d. nature of idiosyncratic shock implies that the household would like to continue the relationship with some firms below the zero profit threshold, because separation entails loss of value $(1-\phi)$ and because matching capital with a new firm is costly (there is a probability of no match at which the liquid capital unit yields zero return). Only for idiosyncratic productivity shock so low that the household needs to inject money to cover for wage payments (assuming that wage claims represent senior debt) is there a point at which separating becomes more profitable than injecting money and continuing the relationship. As the discussion below reveals, such an optimal separation rule would result in an additional time-varying risk premium for the rental rate taking into account that households bear an asymmetric risk of non-repayment. We will investigate the quantitative effects of optimal separation and this risk premium in a future version of the paper.

3.5 Rental rate of capital

To determine the rental rate of capital, we assume that once matched, households and firms split the surplus of their relationship according to a Nash bargaining process. As discussed above, this bargaining process takes places before the idiosyncratic shock a_{it} is realized. The surplus is the sum of marginal benefits to each party, $S_{it} = J_K(K_{it}) + \frac{V_k(U_t,K_t) - V_U(U_t,K_t)}{\Lambda_t}$. Define η as the household's relative bargaining power. It then receives $\frac{V_k(U_t,K_t) - V_U(U_t,K_t)}{\Lambda_t} = \eta S_{it}$, while the firm's share is $J_K(K_{it}) = (1-\eta)S_{it}$. After some algebraic manipulations (see the appendix) we obtain the following expression for the rental rate

$$\rho_{it} = \eta \left[f_K(X_t N_{it}, K_{it}) + (1-\delta)(1-s_t)\frac{\kappa}{\theta_t} \right] + (1-\eta)[\delta + (1-\phi)(1-\delta)s_t]. \tag{11}$$

The first term in brackets on the right hand side is the maximum amount the firm is willing to pay per unit of capital. It equals the marginal product of capital plus the average search cost for capital expenditures that is saved by continuing the relationship into next period. The second term in brackets is the opportunity cost of the lender, which equals the fraction not lost to depreciation when capital remains liquid δ plus the value not lost to specificity when capital is separated $(1-\phi)(1-\delta)s_t$.[14]

From the optimality conditions on liquidity and bond holdings, results from the firm's problem and Nash bargaining, a relationship between the economy's risk free rate and the capital market liquidity rate θ_t can be borne out (see again the appendix for details on this derivation)

$$\beta E_t[\Lambda_{t+1}(1+r_t)] = \frac{\eta}{1-\eta}\frac{\kappa}{\theta_t}\Lambda_t. \tag{12}$$

All else being equal, an increase in the economy's risk free rate r_t implies a decrease in the capital market liquidity rate θ_t because households find it less profitable to set aside funds for capital investments.

3.6 Aggregation and equilibrium

The micro literature on firm dynamics usually assumes decreasing returns to scale production (see for example Cooley and Quadrini, 2001 or Esteban-Rossi and Wright, 2005). Here, for reasons of tractability, we follow the traditional macro literature and assume that the production function $f(\cdot)$ exhibits constant returns to scale. Under this assumption, it is straightforward to show that the capital labor ratio of all firms is the same and thus, all optimality conditions are independent of firm size and the rental rate is identical for all firms; i.e. $\rho_{it} = \rho_t$.

With the constant returns assumption, we bypass any issues that arise from firm size heterogeneity. These issues are admittedly important but taking them into account would greatly complicate aggregation and quantitative analysis of the model. In particular, it allows us to

[14] For the optimal separation rule mentioned above, the formula for the rental rate would become

$$\begin{aligned}\rho_{it} &= \eta\left[f_K(X_tN_{it},K_{it}) + (1-\delta)(1-s_t)\frac{\kappa}{\theta_t}\right] + (1-\eta)[\delta + (1-\phi)(1-\delta)s_t] \\ &\quad +(1-\eta)\left[\rho_{it}F(\bar{a}_t) - \int_{\underline{a}_{it}}^{\bar{a}_{it}} adF(a)f_K(X_tN_{it},K_{it})\right].\end{aligned}$$

The additional term represents a risk-premium that arises because households do not receive the contractual payment ρ_{it} when the firm's idiosyncratic shock drops between $\bar{a}_{it}$ (zero profit) and $\underline{a}_{it}$ (the optimal threshold value for separation).

draw direct comparisons with other representative agents models such as the frictionless RBC benchmark or the financial accelerator model of Bernanke, Gertler and Gilchrist (1998).

To compute the equilibrium, we aggregate over all capital units. The dynamics for the aggregate stock of productive capital become

$$K_{t+1} = (1-\delta)(1-s_t)K_t + m(L_t, V_t). \tag{13}$$

The aggregate equilibrium dynamics of our model are defined by the system of equations (7), (1)-(3), (8)-(13) plus aggregate profits $D_t = f(X_t N_t, K_t) - W_t N_t - \rho_t K_t - \kappa V_t$. This last equation assumes that there exists a complete insurance market for shortfalls in wage and rental payments, which are assumed to be covered by the higher than average profits of surviving capital units.

3.7 Comparison with the baseline RBC model

Before continuing to the quantitative evaluation of our model, it is useful to compare our model with the baseline RBC model (see for example King and Rebelo, 2000) in which both credit market frictions and costly capital allocation are absent. In particular, the RBC model describes a world in which the cost of project postings κ is zero and thus, firms post an infinity of projects. Moreover, all capital is returned to the household (net of depreciation) at the end of each period and is reallocated at no cost at the beginning of following period.

In terms of our model, these assumptions translate into $s_t = 1$, $q(\theta_t) = 1$ and $U_t = 0$. Furthermore, it can easily be shown that $\rho_t = f_K(X_t N_t, K_t)$: the repayment on liquidity is equal to the marginal product of capital.[15] Finally, from the law for productive capital one sees that to choose liquidity then amounts to choosing capital in the following period; i.e. $L_t = K_{t+1}$. This implies a value of matched liquidity $V_K(U_t, K_t, B_t) = \Lambda_t[\rho_t + (1-\delta)]$, and the optimality condition for the choice of liquidity becomes a standard Euler equation:

$$\beta E_t \Lambda_{t+1}[\rho_{t+1} + (1-\delta)] = \Lambda_t.$$

[15] The value of bargaining power η is irrelevant in the RBC setting as the competitive nature of the capital market rules out any positive surplus between matched firms and lenders.

4 Shocks, functional forms and calibration

4.1 Shocks

Following much of the RBC literature we assume that our model economy is perturbed by an exogenous labor augmenting shock X_t that has both a deterministic trend part $\bar{X}_t$ and a stochastic transitory part A_t. In particular $X_t \equiv A_t^{1/(1-\alpha)} \bar{X}_t$. The deterministic trend part evolves according to

$$\bar{X}_t = g\bar{X}_{t-1},$$

and the stochastic transitory part evolves according to

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A,$$

with $\varepsilon_t^A \sim (0, \sigma_A^2)$.[16]

4.2 Functional forms

For household preferences, we follow King and Rebelo's (2000) baseline specification and define the family's period utility as $u(C, 1-N) = \log C + \frac{\omega}{1-\xi}(1-N)^{1-\xi}$. For production, we assume a Cobb-Douglas function with constant returns to scale of the form $af(XN, K) = aA(\bar{X}N)^{1-\alpha}K^{\alpha}$ with $0 < \alpha < 1$. The idiosyncratic shock is assumed to follow a log-normal distribution (which guarantees $a > 0$) with variance σ_a^2 and mean equal to $-\frac{\sigma_a^2}{2}$ (so as to satisfy $E(a) = 1$). Finally, the matching technology takes the form similar to the one used in the labor literature, $m(V, L) = \chi V^{\epsilon} L^{1-\epsilon}$ with $0 < \epsilon < 1$.

4.3 Calibration

We calibrate our model to quarterly data. For the parameters that are common with the RBC benchmark, we use calibrations that are standard in the literature. The annual trend growth rate is set to 1.6%, which implies $g = 1.004$. The household's discount factor is set to $\beta = 0.99$ in order to match an average annual real yield on a riskless 3-month treasury bill of 4.95%. We

[16] Alternatively, we could have specified a stochastic technology shock that is difference stationary and supplement it with an additional transitory shock (e.g. a labor supply or a government spending shock). We check in future versions of the paper that our results are robust to such a shock process specification.

set ω such that the average fraction of hours worked equals $n = 0.214$ and $\xi = 4$, which implies a Frisch elsaticity of labor supply of 1. The rate of depreciation of capital is set to $\delta = 0.025$, which corresponds to an annual decline of productive use of capital of 10%. Finally, the value of $\alpha = 1/3$ implies an average labor share in production of two thirds.

For the remaining parameters that are proper to our capital matching model, the calibration strategy consists of matching a number of salient long-run averages from the firm-level data discussed in Section 2 and other sources. First, we choose a quarterly steady state separation rate of $s = 0.015$. Together with $\delta = 0.025$, this rate implies the following steady state *gross* investment rate (using the capital accumulation equation (13))

$$\frac{m(V, L)}{K} = [g - (1 - \delta)(1 - s)] = 0.0485,$$

which translates into a yearly investment rate of 17.45% – a value that is in more or less in line with what Ramey and Shapiro (1998) and Eisfeldt and Rampini (2005a) report from their Compustat data.

Second, we can combine the steady state equations for optimal project postings (9), optimal liquidity (12) and the rental rate (11) to obtain the following expression for average ρ (see appendix)[17]

$$\begin{aligned} \rho &= (r + \delta) + (1 - \phi)(1 - \delta)s \\ &\quad + \left(\frac{1 - q}{q}\right)\left[r - (1 - \delta)(1 - s)\left(1 - \frac{\beta}{g}\right)\right]. \end{aligned}$$

The first term in brackets represents the steady state rental rate in the RBC benchmark where no frictions are present. The second and third term are risk premia compensating for capital specificity ($\phi < 1$) and imperfect capital allocation ($q < 1$) in case of separation. Following Ramey and Shapiro (1998b), we set $\phi = 0.5$ and assume that on average, it takes 1 quarter before capital is (re-)allocated and becomes productive; i.e. $q = 0.5$. Together with our calibrations of r, g, δ and s this implies an average annualized spread of the rentral rate over the riskless rate (net of depreciation) of 3.22%. This number lies in-between the spread of the average Aaa corporate bond yield over the 3-month Treasury bill of 1.87% and the average equity risk premium for the U.S. of 7.58% (for 1951-2000).

[17] For the rental rate under optimal separation, an additional term compensating for the risk of incomplete payment of ρ would have to be added.

Third, to calibrate endogenous separation, we assume that the exogenous part of separation is constant and accounts for half of total separations; i.e. $s^x = 0.0075$. Furthermore, we set the variance of the lognormal distribution to $\sigma_a^2 = 0.25$. Together, these values pin down the zero profit threshold $\bar{a} = F^{-1}(s - s^x)$.

Fourth, we set the elasticity of the matching function $\epsilon = 0.5$ and the bargaining weight $\eta = 0.5$. Admittedly, we have little information to calibrate these two parameters (the same is true to a lesser extent about q and $\sigma_{\bar{a}}^2$). We will assess, however, the robustness of our results to alternative calibrations of these parameters.[18]

Finally, we need to calibrate the parameters of the exogenous driving processes. For the temporary technology shock process, we extract a Solow residual from the data and then subtract a linear trend with average growth rate g. Estimation of the resulting AR(1) process yields $\rho_A = 0.98$ and $\sigma_A = 0.0072$ (as in King and Rebelo, 2000).

5 Simulation results

We analyze the empirical performance of our model in two stages. First, we consider impulse response functions (IRFs) of different aggregates with respect to a permanent technology shock and with respect to a temporary government spending shock. The goal of this exercise is to graphically highlight the effects of our credit market friction with costly reallocation. Second, we report a variety of unconditional second moments. To put the different results in perspective, we compare them to the RBC benchmark, which is a special case of our model, as well as a non-monetary version of Bernanke, Gertler and Gilchrist's (1998) financial accelerator model.

The assumption of a deterministic trend in labor productivity implies that we need to normalize all aggregates by the $\bar{X}_t$ so as to obtain a stationary system that we can simulate using log-linear solution techniques. Once normalized, we compute the rational expectations solution of the log-linear system of equations with the algorithm developed by King and Watson (1998).[19]

[18] The other parameters of our matching model (κ, $p(\theta)$, θ and v) are all determined endogenously from the system of steady state equations. See the appendix for details on these calculations.

[19] We thank Bob King for providing us with the relevant Matlab code.

5.1 Impulse response functions

Figure 1 plots the IRFs of prominent macro aggregates to a persistent but temporary technology shock.

As is immediately apparent from the top-left panel, our capital matching model (solid lines) generates an amplified yet humpshaped response of output compared to the RBC benchmark (dotted lines). Whereas output in the RBC benchmark peaks upon impact and then gradually decreases in line with the technology shock, output in the capital matching model peaks only after 3 periods and the maximum response is roughly 30% higher.

Both humpshape and amplification have their origins in the state-dependent nature of the credit constraint and the capital allocation friction. Consider first the humpshaped response of output. To obtain this effect, the gradual decrease in the productivity shock after impact needs to be more than compensated with higher labor and/or capital input in the periods following the shock. In our capital matching model, the higher aggregate productivity level decreases the fraction of firms with negative profits and thus, capital separation drops precipituously (see bottom-left panel of Figure 2 below[20]). As a result, the increase in capital starting in the period after the shock is more rapid and amplified compared to the RBC benchmark, even though the response of investment is smaller and irregular (we return to explaining this response further below). As a result, labor demand shifts up even more, with the associated substitution effect in the labor market leading to an additional increase of hours.

Second, consider the amplified response of output upon impact of the shock in our capital matching model. Since the capital stoc is predetermined, this amplification must come through the labor market, and more particularly through a smaller income effect (upward shift) in labor supply. This is confirmed by the more muted response of consumption relative to the RBC benchmark. To illustrate how a larger increase in output in our model can coexist with a smaller response in *both* consumption and investment, we combine the household's budget constraint with the definition of aggregate profits. We obtain

$$Y_t = C_t + [L_t + \kappa V_t] - [\phi s_t(1-\delta)K_t + U_t]. \tag{14}$$

[20]Note the 40% deviation from steady state means that the separation rate drops from its steady value of 1.5% per quarter to 0.9%.

The first term in brackets represents the ressources devoted to investment by households and firms. The second term in brackets represents returns from current capital stocks $\phi s_t(1-\delta)K_t$ plus ressources not matched in the previous period U_t . Since s_t drops precipituously after the shock, less capital is returned from firms that the household could allocate towards consumption. In the RBC model, by contrast, all capital gets separated in every period no matter what the productive situation; i.e. the second term in brackets would equal $(1-\delta)K_t$. Furthermore, since firms are more productive, they open up new vacancies each at a cost κ (see top-left panel of Figure 2 below), which decreases profits rebated to households. Hence, the only choice for households to increase consumption is to set aside less liquidity L_t for investment. In our model, this trade-off is more costly than in the RBC benchmark as both separation and capital allocation frictions in our model decrease (from the point of view of the household) in response to expansionary shocks.

To illustrate this last point, Figure 2 displays the IRFs of the different variables related to separation and reallocation of capital.As discussed before, firms respond to their increased current and future productivity with an increase in project postings V_t. Households, in turn, decrease the amount of liquid capital L_t in order to finance consumption. The consequence of these reactions is that the capital market liqudity $\theta_t = L_t/V_t$ drops, which means that the probability of locating funds for a project $p(\theta_t)$ decreases while the probability of locating a project $q(\theta_t)$ increases. From the point of view of the household, this represents a smaller degree of capital reallocation friction as the likelihood of matching L_t with a firm and thus obtaining a positive return the following period and thereafter increases. Together with state-dependent decrease in the credit market friction (less separation and thus smaller losses associated with capital specificity and matching), this friction explains why household are less willing to trade-off investment for consumption than in the RBC benchmark, which in turn leads to smaller income effect on labor supply and thus to an amplified response of hours and output on impact of the shock.

Finally, it is interesting to compare the response of the risk-free rate with the reaction of the risky rental rate of capital ρ_t. Unsurprisingly, the risk free rate increases as households need a higher return to save for future consumption. By contrast, the rental rate drops upon impact before returning to a slightly positive response from the second period onwards. To understand this result, reconsider the equation for the rental rate in equilibrium

$$\rho_t = \eta \left[A_t f_K(N_t, K_t) + (1-\delta)(1-s_t)\frac{\kappa}{\theta_t} \right] + (1-\eta)[\delta + (1-\phi)(1-\delta)s_t].$$

Several forces are at work here. On the one hand, the firm's marginal productivity of capital increases, thus putting upward pressure on ρ_t. On the other hand, the drop in the separation rate lowers the required minimum return of the household, thus putting downward pressure on ρ_t. Finally, the decrease in θ_t and s_t have inverse effects on the expected average cost of obtaining future capital for the firm. A priori, it is not clear which of these effects prevail in general equilibrium. As it turns out, the reported drop in ρ_t on impact is not a robust result.

5.2 Unconditional second moments

5.2.1 Autocorrelation of output growth

One of the great challenges in business cycle macroeconomics is the positive autocorrelation of output growth over several quarters in the data. As Cogley and Nason (1995) document, the RBC model completely misses to generate such positive autocorrelation and researchers have proposed different theories that could potentially explain this pattern. However, the results so far have been mixed at best.[21]

Figure 1 displays the autocorrelation function for output growth for the data (green line), our model (blue line), the RBC model (dotted line) and a non-monetary version of Bernanke, Gertler and Gilchrist's (1998, BGG henceforth) financial accelerator model (solid lines with stars).[22]

As is immediately apparent, both the RBC benchmark and BGG's financial accelerator model fail to generate any autocorrelation in output growth. By contrast, our capital matching model tracks the empirical autocorrelation of output in the data much better. The fit is admittedly imperfect since the autocorrelation drops one lag too early. What is remarkable, however, is the high value for the correlation at the first lag. To our knowledge, very few parsimonious models manage to generate such high values without creating substantial positive autocorrelation at

[21] See Gilchrist and Williams (1999) or Chiang, Gomes and Schorfheide (2003) for two of the more promising attempts.

[22] See Petrosky-Nadeau (2005) for a description of the BGG model. The calibration of this model is similar to the one reported in BGG.

lags four and beyond.[23] This goes to show that credit constraints together with costly capital allocation generates substantial internal propagation. What is also interesting in our model is that neither the credit market friction nor costly capital realloction would have lead to this result.

5.2.2 Business cycle volalities and cross-correlations

Table 1 presents unconditional second moments for the growth rates of different prominent macro aggregates for quarterly U.S. data, the RBC benchmark, our capital matching model, and BGG's financial accelerator model.

There are several striking features. First, our model generates substantial internal amplification compared to the two other models. This effect is due to the somewhat more volatile dynamics of output and the markedly more volatile and more correlated capital stock dynamics. In the RBC benchmark and BGG's financial accelerator model, by contrast, capital stocks do not exhibit much volatility and are hardly correlated with output. Changes in the capital stock due to endogenous separation thus represent an important channel through which business cycle dynamics are affected. These findings could also have important consequences for the measurement of the Solow residual and the thus resulting technology shocks. In particular, technology shocks as computed in Section 4 have been criticized for their large volatility that imply a substantial probability of technological regress (see for example the discussion in King and Rebelo, 2000). A more volatile and procyclical capital stock as generated in our model has the potential to reduce the size of the technology shock, thus addressing one of the main criticism of the RBC paradigm.

A second interesting result is that our model generates somewhat less volatile real wage dynamics, thus bringing this dimension closer to the data. At the same time, our model does substantially worse in terms of investment volatility. We plan to investigate this issue further in future versions of the paper.

[23] Chiang, Gomes and Schorfheide (2003), for example, propose a model with learning-by-doing to generate persistence in output growth. While their model implies sizable persistence for lags 1 and 2, it also generates substantial persistence at lags 3 and thereafter. ***check Gilchrist and Williams***

6 Assessing the effects of the different frictions

The purpose of this section is to assess the quantitative importance of the different frictions by resimulating the model, first with the credit constraint turned off (i.e. $\sigma_a^2 = 0$ such that $s_t = s$), second without capital specificity (i.e. $\phi = 1$), and third with perfect matching of liquid capital to projects (i.e. $q = 1$).

A fair question to ask, of course, is to what extent our results are robust to the calibration of the other parameters that are specific to our capital matching model. In particular, one may wonder how our results change for different values of η, s or ϵ. It turns out, however, that the main conclusions of our model remain intact with respect to these parameters.

6.1 The effect of removing credit constraints

Consider first the case where the separation rate is constant over the cycle, i.e. $s_t = s$, because there are no idiosyncratic shocks to productivity. This corresponds to a situation where firms production never falls short of covering factor costs and thus, credit constraints never apply.

As is evident from top-left panel of Figure 4, the response of output to a technology shock is less amplified (with a peak at 1.45 instead of 1.62) and exhibits less of a humpshape. In fact, to replicate the pronounced autocorrelation of output growth at lags 1 and 2, the hump needs to be much more pronounced over the first two periods. Countercyclical credit constraints are therefore crucial to generate substantial persistence in output growth.

The main reason for this lack of persistence and the smaller amplification is the absence of a drop in capital separations on impact of the technology shock (bottom-left panel of Figure 6). The consequence is that the capital stock reacts less strongly relative to its steady state even though the response of investment is larger and coincides more or less with the one for the RBC benchmark. This smaller reaction of the capital stock reduces internal amplification and creates less of a pronounced hump in the periods after the shock.

Interestingly, removing credit constraints also reduces the response of consumption and, in turn, increases the amount of liquid capital set aside by households (see top-left panel of Figure 5 below). This suggests that the absent positive effect of countercyclical seperation on the marginal value of liquid capital is more than compensated by the marked increase of the rental rate of capital upon impact. The result of the more muted consumption response implies that the negative income effect on labor supply is also smaller. This explains, why the equilibrium response of hours is approximatively the same than the one recorded above, despite the smaller upward shift of labor demand (due to the smaller increase in capital).

6.2 The effect of removing capital specificity

The main effects of turning off capital specificity (i.e. setting $\phi = 1$) are very similar than the removal of credit constraints, albeit for different reasons. As the top-left panel of Figure 6 shows, the response of output is again smaller and displays less pronounced of a hump over the first 3 quarters.As before, the smaller and less humpshaped output response is due to the more muted response of capital. This smaller reaction comes about by the smaller increase in investment upon impact (1.25 instead of 1.75) on the one hand, and the smaller drop in separation (-11% instead of -40%, see Figure 7 below) on the other. The smaller drop in separation is not obvious to explain. It occurs because the reaction of both new projects and wages is smaller and more than compensates the larger increase in the rental rate upon impact.

As in the previous experiment with constant separation, the response of consumption is smaller, the response of liquid capital larger, and the response of hours roughly equal to the model with capital specificity. This time, the larger increase in the marginal value of liquid capital that underlies the smaller increase in consumption is the absence of capital specificity in combination with the increase in the rental rate of capital.

6.3 The effect of removing imperfect capital matching

The last experiment to consider is the removal of imperfect matching of capital with firms, i.e. $q(\theta_t) = 1$, which corresponds to a situation where firms post an infinity of new projects every period because costs are zero ($\kappa = 0$). In this case, capital markets become perfectly competitive and there is no longer any rent to share between capital providers (households) and firms. Given constant-returns-to-scale production and absence of any other product or input market imperfections, average profits are therefore zero. This implies that the separation rate is constant over the cycle, no matter the variance of the idiosyncratic productivity shock σ_a^2. The model therefore collapses to a variant of the RBC benchmark where, in addition to depreciation, a constant fraction of capital is lost due to separation and capital specificity. The following figure depicts such a scenario with average annual capital lost equal to 35% ...

7 Conclusion

to be added

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A Derivation of the equation for the rental rate of capital

To derive the repayment rule, first use the first order condition on project postings from the firm's problem together with the defintion for the marginal value to the firm of an additional unti of capital, and a result from Nash barganing that the firm's share of the total surplus is $J(K_t) = (1-\eta)S_t$, to obtain

$$(1-\eta)S_t = A_t f_k(X_t N_t, K_t) - \rho_t + (1-\delta)(1-s_t)\frac{\kappa}{p(\theta_t)} \tag{15}$$

Then, by definition $S_t = J(K_t) + \frac{V_k(U_t,K_t,B_t)-V_u(U_t,K_t,B_t)}{\Lambda_t}$, or

$$\begin{aligned}
S_t &= A_t f_k(X_t N_t, K_t) - \rho_t + (1-\delta)(1-s_t)\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} J_k(K_{t+1}) \\
&\quad + \rho_t + (1-\delta)\phi s_t + (1-\delta)(1-s_t)\beta E_t \frac{V_k(U_{t+1},K_{t+1},B_{t+1})}{\Lambda_t} - \frac{V_u(U_t,K_t,B_t)}{\Lambda_t} \\
S_t &= A_t f_k(X_t N_t, K_t) + (1-\delta)\phi s_t - 1 \\
&\quad + (1-\delta)(1-s_t)\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}[J_k(K_{t+1}) + \frac{V_k(U_{t+1},K_{t+1},B_{t+1}) - V_u(U_{t+1},K_{t+1},B_{t+1})}{\Lambda_{t+1}}] \\
&\quad + (1-\delta)(1-s_t)\beta E_t \frac{V_u(U_{t+1},K_{t+1},B_{t+1})}{\Lambda_t}.
\end{aligned}$$

From the first order condition for the amount of liquid capital destined to capital investments and the household's share of the total surplus, $\frac{V_k(U_t,K_t,B_t)-V_u(U_t,K_t,B_t)}{\Lambda_t} = \eta S_t$, $\beta E_t \frac{V_u(U_{t+1},K_{t+1},B_{t+1})}{\Lambda_t}$ can be written as $\left[1 - q(\theta_t)\eta\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} S_{t+1}\right]$, and

$$\begin{aligned}
S_t &= A_t f_k(X_t N_t, K_t) + (1-\delta)\phi s_t - 1 + (1-\delta)(1-s_t)\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} S_{t+1} \\
&\quad + (1-\delta)(1-s_t)\left[1 - \eta q(\theta_t)\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} S_{t+1}\right] \\
S_t &= A_t f_k(X_t N_t, K_t) + (1-\delta)\phi s_t - 1 + (1-\delta)(1-s_t)\frac{\kappa}{p(\theta_t)(1-\eta)} \\
&\quad + (1-\delta)(1-s_t)\left[1 - \eta q(\theta_t)\frac{\kappa}{p(\theta_t)(1-\eta)}\right] \\
(1-\eta)S_t &= (1-\eta)\left[A_t f_k(X_t N_t, K_t) + (1-\delta)\phi s_t - 1\right] + (1-\delta)(1-s_t)\frac{\kappa}{p(\theta_t)} \\
&\quad + (1-\delta)(1-s_t)\left[(1-\eta) - \eta q(\theta_t)\frac{\kappa}{p(\theta_t)}\right]
\end{aligned} \tag{16}$$

Equating (15) and (16), and recalling that $\frac{p(\theta_t)}{q(\theta_t)} = \theta_t$, yields the repayment rule

$$\rho_t = \eta A_t f_k(X_t N_t, K_t) + (1-\eta)\left[\delta + (1-\delta)(1-\phi)s_t\right] + \eta(1-\delta)(1-s_t)\frac{\kappa}{\theta_t}$$

B Steady state system of equations

After normalizing by the deterministic trend to the labor augmenting technological growth, the system of equations at a steady state is

$$\frac{1}{c} = \lambda \tag{17}$$

$$\omega(1-n)^{-\xi} = \lambda w \tag{18}$$

$$1 = \frac{\beta}{g} + \frac{\eta}{1-\eta}\frac{\kappa}{\theta} \tag{19}$$

$$r = \frac{g}{\beta} - 1 \tag{20}$$

$$w = (1-\alpha)\frac{y}{n} \tag{21}$$

$$\frac{\kappa}{p(\theta)} = \frac{\beta}{g}\left\{\alpha\frac{y}{k} - \rho + (1-\delta)(1-s)\frac{\kappa}{p(\theta)}]\right\} \tag{22}$$

$$\rho = \eta\alpha\frac{y}{k} + (1-\eta)[\delta + s(1-\phi)(1-\delta)] + \eta(1-\delta)(1-s)\frac{\kappa}{\theta} \tag{23}$$

$$gk = (1-\delta)(1-s)k + q(\theta)l \tag{24}$$

$$y = c + [l + v\kappa] - [(1-\delta)\phi sk + u] \tag{25}$$

$$u = [1-q(\theta)]l \tag{26}$$

$$d = y - wn - v\kappa \tag{27}$$

$$\theta = \frac{l}{v} \tag{28}$$

$$p(\theta) = \theta^{1-\epsilon} \tag{29}$$

$$q(\theta) = \theta^{-\epsilon} \tag{30}$$

$$y = An^{1-\alpha}k^{\alpha} \tag{31}$$

$$s = F(\bar{a}) + s^x \tag{32}$$

$$\bar{a} = (wn + \rho k + v\kappa)/y \tag{33}$$

C Computing the steady state

We first derive the repayment as a function of parameters and steady state separation rate

$$\rho = (r+\delta) + (1-\delta)(1-\phi)s + \frac{1-q(\theta)}{q(\theta)}\left[r - \left(1-\frac{\beta}{g}\right)(1-\delta)(1-s)\right].$$

Equation (19) determines the value $\frac{\kappa}{\theta} = \frac{1-\eta}{\eta}\left(1-\frac{\beta}{g}\right)$ such that the ratio of capital to output is

$$\frac{k}{y} = \frac{\alpha}{\left[\frac{\kappa q(\theta)}{\theta}\left(\frac{g}{\beta} - (1-\delta)(1-s)\right) + \rho\right]}.$$

Using the production function the steady state capital stock is then simply

$$k = \left(\frac{y}{Ak}\right)^{\frac{1}{\alpha-1}} n.$$

Equations (31) and (21) give us the level of output and steady state wage. Liquid capital is then computed using the law of motion of capital, (24),

$$l = \frac{k[g-(1-\delta)(1-s)]}{q(\theta)},$$

and unmatched liquid capital is simply $u = (1-q(\theta))\frac{l}{g}$.

Using a log-normal distribution for the idiosyncratic shocks a, with $E(a) = 1$, by the properties of this distribution a has a mean of $-\frac{\sigma^2}{2}$. Thus the cutoff threshold $\overline{a}$ is given by $\overline{a} = F^{-1}(s^e)$, where s^e is the proportion of separations occurring endogenously. The elasticity of separations to the cutoff, φ, is given as

$$\varphi = \frac{\overline{a} f(\overline{a})}{s^e}.$$

To compute the remaining steady state values, by equation (33) $v\kappa = \overline{a}y - wn - \rho k$, profits are $d = y - wn - \rho k - v\kappa$, and consumption, using the ressource constraint, is $c = y - [l + v\kappa] + [(1-\delta)\phi s k + u]$. Finally, the Lagrange multiplier is given by (17) and the weight in the utililty function on leisure by $\omega = \lambda w(1-n)^{\chi}$.

D Log-Linear system

$$-\hat{c}_t = \hat{\lambda}_t \tag{34}$$

$$\hat{n}_t = \frac{1-n}{\xi n}[\hat{\lambda}_t + \hat{w}_t] \tag{35}$$

$$\frac{\beta}{g}E_t[\hat{\lambda}_{t+1}] = \frac{\beta}{g}[\hat{\lambda}_t + \hat{v}_t] + \frac{\eta\kappa}{(1-\eta)\theta}\hat{\theta}_t \tag{36}$$

$$\widehat{w}_t = \widehat{y}_t - \widehat{n}_t \tag{37}$$

$$\hat{\lambda}_{t+1} + \frac{\beta p(\theta)}{\kappa g}\left[\alpha\frac{y}{k}(\widehat{y}_{t+1} - \widehat{k}_{t+1}) - \rho\hat{\rho}_{t+1} - (1-\delta)(1-s)\frac{\kappa}{p(\theta)}p\widehat{(\theta_{t+1})} - s(1-\delta)\frac{\kappa}{p(\theta)}\hat{s}_{t+1}\right] \tag{38}$$

$$= \hat{\lambda}_t - p\widehat{(\theta_t)} \tag{39}$$

$$\rho\hat{\rho}_t = \eta\alpha\frac{y}{k}(\widehat{y}_t - \widehat{k}_t) - (1-\delta)(1-s)\eta\frac{\kappa}{\theta}\hat{\theta}_t - s(1-\delta)\eta\frac{\kappa}{\theta}\hat{s}_t \tag{40}$$

$$gk\hat{k}_{t+1} = (1-\delta)(1-s)k\hat{k}_t - (1-\delta)sk\hat{s}_t + lq(\theta)[\hat{l}_t + q(\hat{\theta}_t)] \tag{41}$$

$$y\widehat{y}_t = c\hat{c}_t + l\hat{l}_t + v\kappa\widehat{v}_t - (1-\delta)\phi sk\widehat{s}_t - u\widehat{u}_t \tag{42}$$

$$\hat{u}_{t+1} = \hat{l}_t - \frac{q(\theta)}{1-q(\theta)}q(\hat{\theta}_t) \tag{43}$$

$$d\hat{d}_t = y\hat{y}_t - wn[\hat{w}_t + \hat{n}_t] - \rho k[\hat{\rho}_t + \hat{k}_t] - \kappa v\hat{v}_t \tag{44}$$

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + \hat{R}_t \tag{45}$$

$$\hat{\theta}_t = \hat{l}_t - \hat{v}_t \tag{46}$$

$$p(\hat{\theta}_t) = (1-\epsilon)\hat{\theta}_t \tag{47}$$

$$q(\hat{\theta}_t) = -\epsilon\hat{\theta}_t \tag{48}$$

$$\hat{y}_t = \hat{a}_t + (1-\alpha)\hat{n}_t + \alpha\hat{k}_t \tag{49}$$

$$\hat{s}_t = \frac{s^a}{s}\hat{s}_t \tag{50}$$

$$\hat{s}^a_t = \varphi\widehat{\overline{a}}_t \tag{51}$$

$$\overline{a}y[\widehat{\overline{a}}_t + \hat{y}_t] = \rho k(\hat{\rho}_{t+1} + \hat{k}_t) + wn(\hat{w}_t + \hat{n}_t) + v\kappa\hat{v}_t \tag{52}$$