The Welfare Cost of Fighting Financial Systemic Risk*

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[**PRELIMINARY AND INCOMPLETE COMMENTS ARE WELCOME**]

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Abstract

One central reason for regulating the banking sector is to prevent systemic financial crises. Specifically, the Basel Accords constrain the proportion of risky assets banks can hold in their portfolios. However, this is likely to have a negative effect on economic growth, as highly productive investment opportunities also tend to be very risky.

This paper provides a model which allows us to evaluate the key trade-off inherent to this type of banking regulation, between economic stability and fostering economic growth, and also provide a quantitative welfare assessment of the proposed regulatory measures. It shows that for reasonable parameters this type of banking regulation has a positive effect on stability but a negative on economic development and growth.

Keywords: Overlapping Generations, Competitive Equilibrium, Economic Growth, Banking Regulation.

JEL Codes: D50, D94, E44, G28.

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1 Introduction

Recent work like Barth, Caprio and Levine (2001) has examined the impact of banking regulation on the development of the financial system. Their findings suggest that regulatory and supervisory practices that force accurate information disclosure, empower private-sector corporate control of banks, and foster incentives for private agents to exert corporate control work best to promote bank performance and stability. Furthermore, work by King and Levine (1993a,b) and Levine and Zervos (1998) have found that well-functioning financial markets are positively associated with economic growth. Together, these results suggest a link between banking regulation and economic growth.

But historically, reducing systemic risk has been the main rationale of the banking regulation. Although as pointed out by Dow (2000) and by DeBandt and Hartmann (2000) there is no general accepted definition of systemic risk. However one can think of systemic risk as the problem of simultaneous failure of many banks. There are two traditional views of systemic risk crises. One is that they are random events, unrelated to changes in the real economy, we refer to this argument as the "sunspot" view.[1] An alternative to the "sunspot" view is that banking crises are a natural outgrowth of the business cycle. An economic downturn will reduce the value of bank assets, raising the possibility that banks are unable to meet their commitments. If depositors receive information about an impending downturn in the cycle, they will anticipate financial difficulties in the banking sector and try to withdraw their funds.[2]

Gorton (1988) conducted an empirical study on United States data during the national banking Era to differentiate between the "sunspot" view and the business-cycle view of systemic risk. He found evidence consistent with the view that banking panics are related to the business cycle. Calomiris and Gorton (1991) considered a broad range of evidence and concluded that the data do not support the "sunspot" view. Based on this findings, we will assume in this research that business-cycle is at the origin of systemic risk.

[1] The modern version of this view developed by Diamond and Dydvig (1983) and others, is that bank runs are self-fulfilling prophecies.

[2] Hellwig (1995,1997,1998) has pointed out that crises of the entire banking systems usually occur in conjunction with macroeconomic shocks, like interest rate and exchange rate shocks or recessions. Allen and Gale (1998) developed models of banking crises caused by aggregate risk, which can be interpreted as business cycle.

This paper is a general equilibrium analysis of banking regulation that is consistent with the business cycle view of the origins of financial systemic risk. It investigates the effects of some type of banking regulation on economic stability and growth in a simple extension of the overlapping generations model of capital accumulation. In our setup, each young individual has access to one unit of two types of a Cobb-Douglas production technology, a risky high productive and a risk-free less productive technology. The outcome of the risky production process is stochastic, i.i.d.[3] These technologies serve to produce two different types of goods, which are used to produce a final good via a CES technology. Young, an individual is called entrepreneur, when old he becomes a lender. When the entrepreneur chooses the risky technology, he faces an idiosyncratic risk. Lacking an initial endowment, and needing resources to implement their technology, entrepreneurs borrow capital from the old through a competitive banking sector. These banks transfer resources from the old to the entrepreneur by borrowing from the old at the equilibrium rental rate and lending to the entrepreneur using optimal lending contracts with terms contingent.

Although they are many types of banking regulation, we focus on the capital requirements which is the main part of the Basel 1 and 2 agreements on banking regulation.[4] We model this capital requirement regulation as a constraint on the portfolio of banks like Blum (1998). In fact, when suitably implemented, equity capital reduces the incentive for excessive risk taking, so acts as a restriction on asset holding.[5]

We first show in the case of deterministic environment that competitive equilibrium can achieve the first-best allocation and that regulation hampers economic growth and maintains the economy in a lower level of production than in the free banking economy. In presence of aggregate uncertainty, the introduction of capital adequacy regulation has a positive impact on stability. In these cases our main result is that the relation between banking regulation and growth depends on two opposing effects. The first effect, which we call demand effect, supports a negative association between banking regulation and

[3]The risky technology can be assimilated to the sector of new technologies.

[4]As pointed out in BIS(2003), the Basel 2 accord consists of three pillars: (1) minimum capital requirement, (2) supervisory review of capital adequacy, and (3) public disclosure.

[5]According to Gale (2004), capital adequacy requirements are justified on two grounds. On the one hand, equity capital reduces the incentives for excessive risk taking. on the other, it provides a buffer that offsets a shortfall in the realized value of assets and allows the orderly disposal of assets in the event of bankruptcy.

growth. A tighter regulation leads to less capital for risky highly productive technology, so in average the overall production level goes down. Then growth is low but less volatile. The second effect, which we call the supply effect, is a general equilibrium effect that always works in the direction of a positive association between banking regulation and growth. For a given aggregate capital stock, tighter regulation implies a lower interest rate, which, given the total resources available, translates into higher income for entrepreneurs. In other words, regulation prompts a redistribution away from lenders towards entrepreneurs.

The rest of the paper is organized as follows. The model is described in section 2. In the third section, we investigate the growth effect of regulation in a deterministic environment. In section 4, we introduce aggregate risk and in section 5, we introduce possibility of default .Concluding remarks are contained in section 6.

2 The Model

2.1 Environment

The economy consists of individuals and banks. Individuals live for two periods. When young an individual is called an entrepreneur, and when old he becomes a lender. We assume that people do not die early, the size of the population is not allowed to grow. Therefore, we normalize the size of each generation to 1. We denote by z_t an independently and identically distributed random variable according to the discrete probability distribution $Prob(z_t = z_e) = \pi_e$, where $e \in \{h, l\}$ with $z_h \geq z_l$. We also denote by $\overline{z}$ and σ_z^2 the mean and variance of z_t respectively. This random variable describes the state of nature in this economy. Finally we set $\pi_h = \pi$ so $\pi_l = 1 - \pi$.

The member of the initial old generation is endowed with an equal share of the aggregate capital stock k_0. The individual of generation $t \geq 1$ has as endowment two types of technology when young but can implement only one, and no endowment when old. Each member of generation t has preferences over consumption streams given by

$$U(c_t, c_{t+1}) = E[u(c_t) + \beta u(c_{t+1})], \tag{1}$$

where u is strictly increasing, strictly concave, twice continuous differentiable and satisfies the Inada conditions, and β is a time preference parameter. All individuals are assumed to be selfish and have no bequest motives. The initial old generation has preferences $U(c_1^0) = u(c_1^0)$.

2.2 Production and investment

There are two types of technology, a high return risky technology $y_{1t} = z_t f(k_{1t})$, and a low return safe technology $y_{2t} = f(k_{2t})$,where k denotes physical capital. These technologies serve to produce two intermediate goods. We assume that f is C^2 and satisfies $f(0) = 0$, $f' > 0$, $f'' < 0$, $\lim_{k \longrightarrow 0} f'(k) = \infty$, and $\lim_{k \longrightarrow \infty} f'(k) = 0$. These assumptions on f are one way of providing a positive revenue to entrepreneurs. The random variable z_t determines the quality of the risky investment. To make the return of the risky technology higher than the one of the risk-free technology we assume that $\overline{z} > 1$.

There is a large number of competitive firms, which produce output using these two intermediate goods as inputs according to the production function

$$Y_t = F\left(Y_{1t}, Y_{2t}\right) = \left[\gamma Y_{1t}^{\sigma} + (1-\gamma) Y_{2t}^{\sigma}\right]^{\frac{1}{\sigma}} . \tag{2}$$

Where Y_{1t} denotes the risky intermediate input and Y_{2t} denotes the risk-free intermediate input at time t. Competition drives firms to remunerate each factor at its marginal productivity. Assuming a CES production function for the final good is one way of taking in account the fact that in every economy when one sector is in crisis, other sectors may also be in trouble.[6] Let us use the final good as the numeraire, and denote by p_{1t} the relative price of the risky intermediate good and by p_{2t} the relative price of the risk-free intermediate good. The optimality of firms yields $p_{1t} = F_{1t}$ and $p_{2t} = F_{2t}$; where $F_{1t} = \frac{\partial F(Y_{1t}, Y_{2t})}{\partial Y_{1t}}$ and $F_{2t} = \frac{\partial F(Y_{1t}, Y_{2t})}{\partial Y_{2t}}$.

Capital is, durable, non depreciable, and is the only way for young agents to save. One unit of consumption placed into investment in period t yields one unit of capital in period $t+1$. Final good is perishable, one can not store it.

2.3 The bank's problem

We assume that the banking sector is competitive, so some banks can specialize in the risky technology and some in the risk-free technology. This assumption also drives banks

[6] For example, the crises of the sector of NewTech reduced the growth level in United States. This was followed by a decrease in the value of buildings. Buildings remain unchanged but their value goes down.

to maximize the expected utility of entrepreneurs. The old generation invests in the bank which pays the highest interest rate. This drives all banks to provide the same interest rate to each lender (old).

We can now present the banks' problem. They behave as follows. They collect savings from the old cohort (with a promise to give them some level of consumption in the next period) and lend to entrepreneurs. Lending contracts are set according to the type of technology : $\Lambda_1 : (k_{1t}, \tau_1(z_t))$ for the risky technology and $\Lambda_2 : (k_{2t}, \tau_2(z_t)$ for the risk-free. The timing of the bank is provided in figure 1.

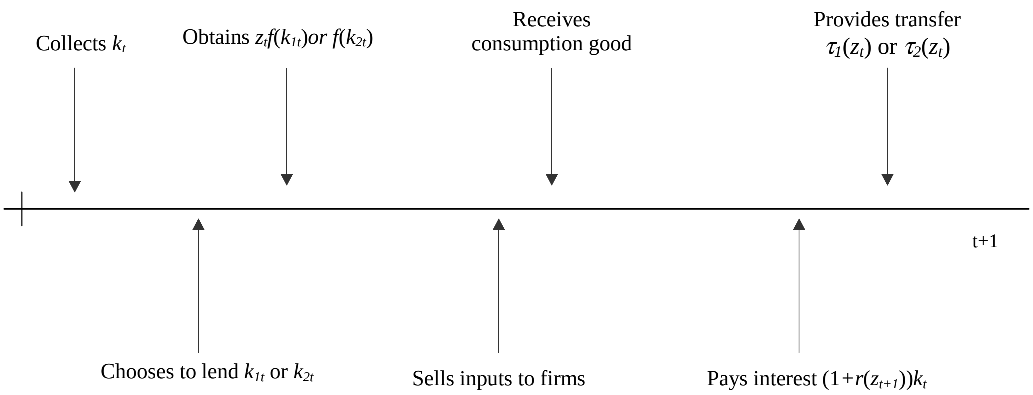

Figure 1. *Timing of banks*

Optimal contract for those operating the risky technology is (k_{1t}, τ_{1t}). It solves the optimization problem,

$$\max_{(k_{1t}, \tau_1(z_t))} E_t \left[v(\tau_1(z_t), r(z_{t+1})) \right] \tag{3}$$

subject to the zero profit constraint, $\tau_1(z_t) + r_t(z_t)k_{1t} = z_t f(k_{1t}) p_1(z_t)$.

In case of regulation there is an additional constraint set to bank specialized in risky technology. That bank is forced to provide at least a given share, say $1-\theta$ of its portfolio of capital for entrepreneurs operating the risk-free technology. Therefore, the regulation constraint has the form $\frac{n_t k_{1t}}{n_t k_{1t} + (1-n_t)k_{2t}} \leq \theta$.

Optimal contract for those operating the risky technology is $(k_{2t}, \tau_2(z_t))$. It solves the optimization problem,

$$\max_{(k_{1t}, \tau_2(z_t))} E_t \left[v(\tau_2(z_t), r(z_{t+1})) \right] \tag{4}$$

subject to the zero profit constraint, $\tau_2(z_t) + r_t(z_t)k_{2t} = p_2(z_t) f(k_{2t})$.

2.4 Individual's problem

Period t begins with a stock of capital k_t owned by old. They give them to banks, which rent them to entrepreneurs. Entrepreneurs produce intermediate goods and give these goods to banks. Banks then sell the intermediate goods to firms, which produce the consumption good. After the production takes place, old agents sell their capital and obtain the consumption good. An old agent then has $(1 + r_t)$ units of consumption for each unit of capital he had at the beginning of the period. He consumes all his goods and exits the economy. Figure 2 describes the timing of event of an individual born at time t.

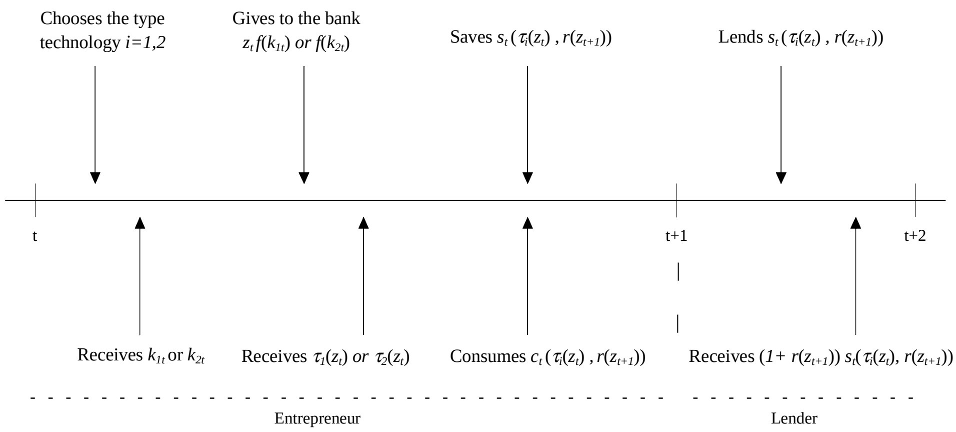

Figure 2. Timing of events of an individual born at time t

An entrepreneur chooses between two types of technology and then chooses his level of saving. Therefore, the decision problem of an entrepreneur can be split into two steps. First step (choice of technology); if $E_t\left[v(\tau_{1t}(z_t), r_{t+1}(z_{t+1}))\right] \geq E_t\left[v(\tau_{2t}(z_t), r_{t+1}(z_{t+1}))\right]$ the entrepreneur chooses the risky technology and if not, the risk-free. i.e., the agent chooses the technology, which provides him the higher utility. Second step (intertemporal consumption); after the entrepreneur has chosen the technology, the production takes place and the entrepreneur saves according to his transfer and the expected lending interest rate. To ease the derivation of our model we will make some further assumptions when there is a need. The first assumption is on the utility function.

Assumption 1. *We assume that u is a power utility function of the form*

$$u(c) = \frac{c^{1-\rho}-1}{1-\rho}. \tag{5}$$

With this assumption $v(\tau_i(z_t), r(z_{t+1}))$ is a strictly increasing, strictly concave and a linear translation of a log separable function of $\tau_i(z_t)$.[7] More precisely we have

$$s(\tau_i(z_t), r_{t+1}(z_{t+1})) = b(r(z_{t+1}))\tau_i(z_t) \tag{6}$$

and then

$$v(\tau_i(z_t), r(z_{t+1})) = G\left(r(z_{t+1})\right)\tau_i(z_t)^{1-\rho} - \Psi. \tag{7}$$

where

$$\begin{aligned} b(r(z_{t+1})) &= \frac{1}{1+\beta^{\frac{-1}{\rho}} E_t\left[(1+r(z_{t+1}))^{\frac{\rho-1}{\rho}}\right]}; \Psi = \frac{1+\beta}{1-\rho}, \text{and} \\ G\left(r(z_{t+1})\right) &= \frac{\left[1-b(r(z_{t+1}))\right]^{1-\rho} + \beta E_t\left(\left[(1+r(z_{t+1}))b(r(z_{t+1}))\right]^{1-\rho}\right)}{1-\rho} \end{aligned} \tag{8}$$

Recall that

$$v(\tau_{it}, r\left(z_{t+1}\right)) = u\left[\tau_{it} - s\left(\tau_{it}, r\left(z_{t+1}\right)\right)\right] + \beta E_t\left(u\left[(1+r\left(z_{t+1}\right))s\left(\tau_{it}, r\left(z_{t+1}\right)\right)\right]\right),$$

where the optimal saving function $s\left(\tau_{it}, r\left(z_{t+1}\right)\right)$ is

$$s\left(\tau_{it}, r\left(z_{t+1}\right)\right) = \arg\max_s \left\{u\left[\tau_{it} - s\right] + \beta E_t\left(u\left[(1+r\left(z_{t+1}\right))s\right]\right)\right\}. \tag{9}$$

We complete the general equilibrium framework of the model by assuming that markets of inputs and of consumption goods open at any time t.

As the benchmark we investigate the properties of the banking regulation in a deterministic environment i.e. $z_h = z_l = \overline{z}$

3 Competitive Equilibrium in a Deterministic Environment

In this section, all economic variables are determined with certainty. Therefore, we will omit z_t in front of variables. In the remainder of this section we characterize the evolution of this economy in an unregulated banking environment, and then explore how the nature of financial contracts and the paths of variables such as capital and output change in response to the introduction of the regulation.

[7] This result is provided by Castro et al. (2003).

3.1 Unregulated Banking

Definition 1. *Given* k_0 *units of capital in period* $t = 0$*, a sequential market equilibrium is the consumption level of the initial old generation* c_0^o*, consumption allocation for entrepreneurs who choose the risky technology* $\{c_{1t}^y, c_{1t}^o\}_{t=0}^{\infty}$ *, consumption allocation for those who choose the risk-free technology* $\{c_{2t}^y, c_{2t}^o\}_{t=0}^{\infty}$ *, aggregate capital* $\{k_{t+1}\}_{t=0}^{\infty}$ *, proportion of entrepreneurs implementing the risky technology* $\{n_t\}_{t=0}^{\infty}$ *, contracts* $\{(k_{1t}, \tau_{1t})\}_{t=0}^{\infty}$ *, for the risky technology and* $\{(k_{2t}, \tau_{2t})\}_{t=0}^{\infty}$ *for the risk-free one, an allocation* $\{Y_t, Y_{1t}, Y_{2t}\}_{t=0}^{\infty}$ *for firms and sequences of prices* $\{r_t, p_{1t}, p_{2t}\}_{t=0}^{\infty}$ *, such that for all* $t \geq 0$ *:* [8]

1. *consumers optimize, i.e.* $c_0 = k_0(1 + r_0)$ *; for* $i = 1, 2$ *and* $t > 0$*,*

$$c_{it}^y = \tau_{it} - s(\tau_{it}, r_{t+1}) \text{ and } c_{it}^o = (1 + r_{t+1})s(\tau_{it}, r_{t+1}) \ ;$$

2. *contracts are optimal, i.e. for all* $t \geq 0$*, they solve bank's problem* ;
3. *ex-ante, entrepreneurs are indifferent between technologies, i.e.* $v(\tau_{1t}, r_{t+1}) = v(\tau_{2t}, r_{t+1})$;
4. *final goods firms optimize, i.e.* $\{Y_t, Y_{1t}, Y_{2t}\}_{t=0}^{\infty}$ *solves firm's problem ;*
5. *the aggregate capital stock equals supply at all* $t \geq 0$ *, i.e.*

$$n_{t+1}k_{1t+1} + (1 - n_{t+1})k_{2t+1} = n_t s_{1t} + (1 - n_t)s_{2t};$$

6. *the risky intermediate goods market clears at all* $t \geq 0$ *, i.e.* $Y_{1t} = n_t\overline{z}f(k_{1t})$;
7. *the risk-free intermediate goods market clears at all* $t \geq 0$ *, i.e.* $Y_{2t} = (1-n_t)f(k_{2t})$;
8. *and the final goods market clears at all time* $t \geq 0$*, i.e.*[9]

$$Y_t = n_t c_{1t}^y + (1 - n_t)c_{2t}^y + n_{t-1}c_{1t-1}^o + (1 - n_{t-1})c_{2t-1}^o.$$

We now derive the evolution of the economy in absence of regulation. Before providing the equilibrium value of some key endogenous variables, we first characterize optimal contracts.

[8] Where c^y and c^o denote the comsumption when young respectively when old.

[9] This condition is redundant. In fact, according to the Walras law when $n-1$ markets are in equilibrium the remaining one is in equilibrium.

Lemma 1. *Optimal contracts offered by banks to entrepreneurs are*

$$\Lambda_1 : \left(k_{1t} = f'^{-1}\left(\frac{r_t}{\overline{z}p_{1t}}\right); \tau_{1t} = \overline{z}p_{1t}\left[f\left(k_{1t}\right) - f'\left(k_{1t}\right)k_{1t}\right]\right),$$

to those implementing the risky technology and

$$\Lambda_2 : \left(k_{2t} = f'^{-1}\left(\frac{r_t}{p_{2t}}\right); \tau_{2t} = p_{2t}\left[f\left(k_{2t}\right) - f'\left(k_{2t}\right)k_{2t}\right]\right). \tag{10}$$

to those implementing the risk-free.

Proof. See appendix. ■

The above optimal contracts show that the demand of capital for the risky technology is a decreasing function of the interest rate but an increasing function of the average productivity shocks and of the price of the risky intermediate good. The same results hold about the demand of capital for the risk-free technology. The only difference is that it is not a function of the productivity shocks.

Assumption 2. *To obtain a closed form solution we will assume in the rest of this paper that the technology used to produce inputs has a Cobb-Douglas form i.e.* $f(x) = x^{\alpha}$.

With Assumption 2, at equilibrium each type of entrepreneurs receives the same level of capital at any date t. In fact, from lemma 1, $r_t = \overline{z}p_{1t}f'\left(k_{1t}\right) = p_{2t}f'\left(k_{2t}\right)$. This equation can be rewritten as a relation between input prices. i.e.,

$$\frac{\overline{z}p_{1t}}{p_{2t}} = \frac{f'\left(k_{2t}\right)}{f'\left(k_{1t}\right)}. \tag{11}$$

On the other hand the monotonicity of $v(\tau_t, r_{t+1})$ in its first argument yields that the indifference condition between technologies is given by $\tau_{1t} = \tau_{2t}$. Substituting (11) in the indifferent condition yields $\frac{f'(k_{2t})}{f'(k_{1t})} = \frac{f(k_{2t})-f'(k_{2t})k_{2t}}{f(k_{1t})-f'(k_{1t})k_{1t}}$. Using assumption 2, the above equation is equivalent to $\left[\frac{k_{1t}}{k_{2t}}\right]^{\alpha-1} = \left[\frac{k_{1t}}{k_{2t}}\right]^{\alpha}$ *i.e.* $k_{1t} = k_{2t}$.

Lemma 2. *At any time t, the equilibrium proportion of entrepreneurs implementing the risky technology is a constant and is given by* $n^* = \left[1 + \left(\frac{1-\gamma}{\gamma\overline{z}^{\sigma}}\right)^{\frac{1}{1-\sigma}}\right]^{-1}$.

Proof. From the fact that competitive equilibrium yields the same level of capital to each type of entrepreneurs, (11) yields $\frac{\overline{z}p_{1t}}{p_{2t}} = 1$. But this is just a relation between prices. To obtain n_t we must go further and provide an expression of prices in function of n_t. For that purpose we use the market clearing conditions for intermediate goods; i.e., $Y_{1t} = n_t \bar{z} k_{1t}^{\alpha}$, and $Y_{2t} = (1-n_t)k_{2t}^{\alpha}$. Recall that $F_{1t} = p_{1t}$ and $F_{2t} = p_{2t}$. We derive from $F(Y_{1t}, Y_{2t}) = (\gamma Y_{1t}^{\sigma} + (1-\gamma) Y_{2t}^{\sigma})^{\frac{1}{\sigma}}$ that $\frac{\bar{z}p_{1t}}{p_{2t}} = \frac{\bar{z}F_1}{F_2} = \frac{\bar{z}^{\sigma}\gamma}{1-\gamma}\left(\frac{n_t}{1-n_t}\right)^{\sigma-1}$. Substituting the above equality in $\frac{\bar{z}p_{1t}}{p_{2t}} = 1$ yields $n_t = \left[1 + \left(\frac{1-\gamma}{\gamma\bar{z}^{\sigma}}\right)^{\frac{1}{1-\sigma}}\right]^{-1}$ ■

This result shows that the share of the portfolio of banks used for the high return technology in the entire economy is not a function of time, so we omit t on n_t in the rest of this section. It shows also that this share increases with the productivity, increases with the share of the risky input in the production process γ, and with the complementarity of inputs σ.[10] Therefore, when the elasticity of substitution increases the share of portfolio of banks allocated to the risky technology increases and when this tends to infinity this share tends to 1.

In the case where $\sigma < 1$. i.e. the elasticity of substitution of inputs in the technology of the production of the final good is different to infinity, n^* is strictly less than one. This is an interesting result, because empirically in economies without capital adequacy requirement or restriction on asset holding, the amount of safe assets holds by banks is strictly positive, respectively the capital hold by bank.[11]

Direct calculation shows that prices p_{1t} and p_{2t} are time invariant. In fact, they are just function of n which is a constant. This result on prices was awaited because the prices of inputs are function of their relative rarity and the complementarity of inputs in the process of production. This result holds in the rest of the paper.

Finally in the unregulated banking, our economy evolves exactly as a standard *OLG* model with constant productivity

$$\phi(\bar{z}) = \left((\gamma\bar{z}^{\sigma})^{\frac{1+\sigma}{1-\sigma}} + (1-\gamma)^{\frac{1+\sigma}{1-\sigma}}\right)\left((\gamma\bar{z}^{\sigma})^{\frac{1}{1-\sigma}} + (1-\gamma)^{\frac{1}{1-\sigma}}\right)^{-\sigma}. \tag{12}$$

Lemma 3. *The portfolio composition of bank in competitive equilibrium is efficient.*

Proof. See appendix. ■

[10] When $\sigma = 0$ (case of the Cobb-Douglas technology) i.e. $F(Y_1, Y_2) = Y_1^{\gamma} Y_2^{1-\gamma}$; $n^* = \gamma$. In this case n is just equal to the share of input 1 in the production process. It is then not a function of the productivity of inputs. When $\sigma = -\infty$ (case of the Leontief technology) i.e. $F(Y_1, Y_2) = \min(Y_1, Y_2)$, $n^* = \frac{1}{2}$.

[11] As pointed out by Alexander (2004), in the 1970s and early 1980s, most countries did not have minimum capital requirements for banks.

This result was eagerly awaited. In fact, this competitive equilibrium yields the same level of transfer, the same level of capital per entrepreneur and also a deterministic interest rate for old. Therefore, ex-ante and ex-post entrepreneurs are indifferent. It is then a competitive equilibrium with a representative agent and a representative bank, there is no way to have a market failure which can provide a rationale for a planner intervention in order to achieve a better portfolio of assets.

3.2 Regulated Banking

3.2.1 Optimal contracts and preliminaries

Since the competitive equilibrium portfolio of banks is efficient any regulation of the banking system will be welfare-reducing. But what will be its amplitude and its effect on the evolution of major macroeconomic indicators.[12] To assess those effects let consider a capital adequacy regulation which works well, i.e. it is equivalent to an asset holding regulation.

The regulated banks' problem is unchanged for those implementing the risk-free technology. But it is impossible to a bank to be specialized in the risky technology. Therefore, in this economy the former risky bank will now deal with both technologies. It determines the optimal contracts for entrepreneurs by solving the following problem,

$$\max_{\left(\widehat{k}_{1t},\widehat{\tau}_{1t},\widehat{k}_{2t},\widehat{\tau}_{2t}\right)} v(\widehat{\tau}_{1t}, r_{t+1}) \tag{13}$$

subject to,

$$n_t\widehat{\tau}_{1t} + (1-n_t)\widehat{\tau}_{2t} + r_t\left(n_t\widehat{k}_{1t} + (1-n_t)\widehat{k}_{2t}\right) = n_t\bar{z}p_{1t}\widehat{k}_{1t}^{\alpha} + (1-n_t)p_{2t}\widehat{k}_{2t}^{\alpha}, \tag{14}$$

$$v(\widehat{\tau}_{2t}, r_{t+1}) \geq v(\tau_{2t}, r_{t+1}), \tag{15}$$

$$\frac{n_t\widehat{k}_{1t}}{n_t\widehat{k}_{1t} + (1-n_t)\widehat{k}_{2t}} \geq \theta. \tag{16}$$

Equations (14) is the zero-profit condition for intermediaries, while (15) is the participation constraint for those implementing the risk-free technology, and (16) is the regulatory constraint, which states that the bank portfolio cannot have up to a given proportion of capital allocated to the risky technology.

[12] Bernanke and Getler (1985) states that most of the original regulation was imposed on macroeconomic gounds.

Definition 2. *Given k_0 units of capital in period $t = 0$, a sequential market equilibrium is the consumption level of the initial old generation c_0^o, consumption allocation for entrepreneurs who choose the risky technology $\{c_{1t}^y, c_{1t}^o\}_{t=0}^{\infty}$, consumption allocation for those who choose the general bank but operating the risk-free technology $\{\widehat{c}_{2t}^y, \widehat{c}_{2t}^o\}_{t=0}^{\infty}$, consumption allocation for entrepreneurs who choose the risk-free technology $\{c_{2t}^y, c_{2t}^o\}_{t=0}^{\infty}$,aggregate capital $\{k_{t+1}\}_{t=0}^{\infty}$, proportion of entrepreneurs in the risky bank implementing the risky technology $\{n_t\}_{t=0}^{\infty}$, proportion of entrepreneurs who choose the general bank $\{m_t\}_{t=0}^{\infty}$, contracts $\left\{(\widehat{k}_{1t}, \widehat{\tau}_{1t})\right\}_{t=0}^{\infty}$, for those operating the risky technology, $\left\{(\widehat{k}_{2t}, \widehat{\tau}_{2t})\right\}_{t=0}^{\infty}$ for entrepreneurs implementing the risk-free in the general bank and $\{(k_{2t}, \tau_{2t})\}_{t=0}^{\infty}$ for those operating the risk-free technology in the risk-free bank, an allocation $\{Y_t, Y_{1t}, Y_{2t}\}_{t=0}^{\infty}$ for firms and sequences of prices $\{r_t, p_{1t}, p_{2t}\}_{t=0}^{\infty}$, such that for all $t \geq 0$:* [13]

1. *consumers optimize, i.e.* $c_0 = k_0(1 + r_0)$ *; for $i = 1, 2$ and $t > 0$,*

$$c_{it}^y = \tau_{it} - s(\tau_{it}, r_{t+1}) \text{ and } c_{it}^o = (1 + r_{t+1})s(\tau_{it}, r_{t+1}) \ ;$$

2. *contracts are optimal, i.e. for all $t \geq 0$, they solve bank's problem* ;

3. *ex-ante, entrepreneurs operating the risk-free technology are indifferent between banks, i.e.* $v(\widehat{\tau}_{2t}, r_{t+1}) = v(\tau_{2t}, r_{t+1})$;

4. *ex-ante, entrepreneurs in the risky bank are indifferent between technologies, i.e.* $v(\widehat{\tau}_{1t}, r_{t+1}) = v(\widehat{\tau}_{2t}, r_{t+1})$;

5. *final goods firms optimize, i.e.* $\{Y_t, Y_{1t}, Y_{2t}\}_{t=0}^{\infty}$ *solves firm's problem* ;

6. *the aggregate capital stock equals supply at all $t \geq 0$, i.e.*

$$m_{t+1}n_{t+1}\widehat{k}_{1t+1}+m_{t+1}(1-n_{t+1})\widehat{k}_{2t+1}+(1-m_{t+1})k_{2t+1} = m_t n_t \widehat{s}_{1t}+m_t(1-n_t)\widehat{s}_{2t}+(1-m_t)s_{2t};$$

7. *the risky intermediate goods market clears at all $t \geq 0$,i.e.* $Y_{1t} = m_t n_t \overline{z}\widehat{k}_{1t}^{\alpha}$;

8. *the risk-free intermediate goods market clears at all $t \geq 0$, i.e.* $Y_{2t} = m_t(1-n_t)\widehat{k}_{2t}^{\alpha} + (1 - m_t)k_{2t}^{\alpha}$;

[13] Where c^y and c^o denote the comsumption when young respectively when old.

9. *and the final goods market clears at all time* $t \geq 0, i.e.$[14]

$$Y_t = m_t n_t \widehat{c}^y_{1t} + m_t(1-n_t)\widehat{c}^y_{2t} + (1-m_t)c^y_{2t} + m_t n_t \widehat{c}^o_{1t-1} + m_t(1-n_t)\widehat{c}^o_{2t-1} + (1-m_t)c^o_{2t-1}.$$

We now characterize this new equilibrium. This characterization depends on the value of θ. In fact we have two different type of adjustment depending on the interval where θ belongs to.

If $\theta \in (n^*, 1)$

In this case, the equilibrium allocation verify the following property: $\widehat{k}_{1t} = \widehat{k}_{2t} = k_{2t}$. The proportion of entrepreneurs in the general bank implementing the risky technology $n_t = \theta$, and the proportion of people in the general bank is $m_t = \frac{n^*}{\theta}$. This allocation yields the same welfare to entrepreneurs than the unregulated economy. Therefore, an important feature of this model is that the introduction of regulation drives entrepreneurs to move from the risk-free bank to the risky bank. They move till the transfer in the general bank equalized the one of the risk-free. This will be done without deterioration of the welfare till this way of adjustment is not possible. In fact the maximum proportion of entrepreneur is the risky bank cannot exceed 1. From $m_t = \frac{n^*}{\theta}$, we obtain that this way of adjustment is possible only if $\theta \geq n^*$.[15]

If $\theta \in (0, n^*)$

In this case banks and entrepreneurs can not adjust their self and obtain the first-best solution. The following lemma provides the optimal contracts of the risky bank in this case.

Lemma 4. *Optimal contracts in case of regulated banking are,*

$$\Lambda_1 : \left(\widehat{k}_{1t} = \theta(1-n_t)\left[\frac{\alpha B_t}{r_t}\right]^{\frac{1}{1-\alpha}}; \widehat{\tau}_{1t} = (1-\alpha)\frac{(1-n_t)}{n_t}\left[n_t B_t^{\frac{1}{1-\alpha}} - p_{2t}^{\frac{1}{1-\alpha}}\right]\left[\frac{\alpha}{r_t}\right]^{\frac{\alpha}{1-\alpha}} \right);$$

for entrepreneurs using the risky technology and

$$\Lambda_2 : \left(\widehat{k}_{2t} = n_t(1-\theta)\left[\frac{\alpha B_t}{r_t}\right]^{\frac{1}{1-\alpha}}; \widehat{\tau}_{2t} = (1-\alpha)p_{2t}\left[\frac{\alpha p_{2t}}{r_t}\right]^{\frac{\alpha}{1-\alpha}} \right)$$

[14] This condition is redundant. In fact, according to the Walras law when $n-1$ markets are in equilibrium the remaining one is in equilibrium.

[15] $m = \frac{n^*}{\theta}$, when $\theta = n^*$, $m = 1$.

for entrepreneurs using the risk-free technology. Where

$$B_t = \overline{z} p_{1t} \theta^{\alpha} (1 - n_t)^{\alpha - 1} + p_{2t} (1 - \theta)^{\alpha} n_t^{\alpha - 1}$$

Proof. See appendix ∎

Lemma 5. When $\theta \in (n^*, 1)$, the proportion of entrepreneur implementing the risky technology is constant. When $\theta \in (0, n^*)$, $\frac{\partial n}{\partial \theta} = \frac{-\alpha n(1-n)}{(1-\alpha)\theta(1-\theta)} \frac{\left[\theta - \sigma\left(1-(1-\theta)^{\alpha}(1-n)^{1-\alpha}\right)\right]}{\left[n-\sigma(1-(1-\theta)^{\alpha}(1-n)^{1-\alpha})\right]}$, so *the equilibrium proportion of entrepreneurs using the risky technology is a decreasing continuous function of* θ *if* $\sigma \leq 0$.

Proof See appendix

Intuitively, in the case of regulation, entrepreneurs anticipate a diminution of the price of the type 2 input, and since the transfer obtained by those implementing the risk-free technology is an increasing function of its price, the anticipated transfer is lower. Entrepreneurs move then from this type of bank to the new risky bank. They will move till both transfers equalize. When $\theta \in (n^*, 1)$, it is possible to achieve by this type of adjustment the solution of the unregulated banking, so in this case the proportion of people implementing the risky technology is a constant. When $\theta \in (0, n^*)$ the above way of adjustment is not enough. Any entrepreneur will join the new risky bank and as the regulation becomes tight many entrepreneurs will like to implement the type 2 inputs.

3.2.2 Implications for growth and development

In this subsection we investigate the growth and the development implications of the introduction of regulation.

Proposition 1. When the supply of capital is given, the equilibrium aggregate output increases with θ.

Proof See appendix

Regulation has a negative effect on the production of the risky input and a positive effect on the production of the risk-free input. On the aggregate output, regulation yields then two opposite effects, but the negative effect is always dominant.

Proposition 2. In the case of a logarithmic utility, the output growth is an increasing function of θ.

Proof See appendix

This result holds also in any situation where the regulation has a negative impact on the saving rate, and for many reasonable values of ρ.

We now turn to comparing the dynamics of two economies differing only in terms of θ. Let θ_c and θ_d be the maximum shares of the portfolio a bank is allowed to use for the risky technology in the two economies, and suppose $\theta_c > \theta_d$. Start with date $t = 0$. Since the capital stock is given, the supply of capital by the old generation is completely inelastic at k_0. The difference capital adequacy requirement is reflected in different demand schedules. The result is that the interest rate r_0, is lower in the economy with θ_d, and the transfer received by entrepreneurs τ_0 is lower. The economic intuition is straightforward. Since the amount of capital allocated to the production of the high return intermediate good is fixed and lower in economy d, and given that the demand is function of the productivity, demand of capital is low and the supply is the same, so the interest rate will adjust i.e. r_0 is lower.

In the other hand the production of the risky intermediate good will be lower while the production of the risk-free intermediate good will be higher. The proposition 1 shows that the dominant effect is the one on the risky input, this ends up by a lower level of the production i.e. $Y(\theta_d) < Y(\theta_c)$. This also has an effect of the prices. $p_2(\theta_d) < p_2(\theta_c)$, and given that the transfer of entrepreneur implementing the risk-free technology is an increasing function of p_2, the number of people implementing this technology will decrease in economy d. As a result the capital per-entrepreneur in economy d will be higher than the one of economy c. The converse results hold for those operating the risky technology.

At any date t the demand schedule of capital will be higher the higher the parameter θ. A larger value of τ_0 implies an outward shift of the supply curve at $t = 1$. This results in a higher or lower value of k_1 depending on the relative slopes of demand and supply, as well as on the magnitudes of their shifts.

If the income effect deriving from an interest rate change dominates the substitution effect in a way that causes the slope of the supply curve to be greater than the slope of the demand curve (in absolute value), then, for all $t \geq 1$, $k_t(\theta_d) < k_t(\theta_c)$. This occurs, for example, if the utility is logarithmic. In such cases, the income and the substitution effects exactly offset each other. As a result, savings are a constant proportion of transfers. This implies that at all t the supply of capital is inelastic.

Finally, this means that for any pair (θ_c, θ_d) with $\theta_c > \theta_d$, $\tau_0(\theta_c) > \tau_0(\theta_d)$. It will also be the case that $k_1(\theta_d) < k_1(\theta_c)$, $r_1(\theta_d) < r_1(\theta_c)$, and $\tau_1(\theta_c) > \tau_1(\theta_d)$. By repeating the same argument at any date t, we conclude that capital accumulation is higher in the economy with less capital adequacy requirement.

For a more general result i.e. a power utility function with $\rho \neq 1$, the complexity of equilibrium equations makes the properties of equilibrium variables difficult to derive analytically. We now investigate the growth implications of regulation by computing a typical example of our model. In particular, consider the following parameters value $\rho = 0.5$. We take the capital share of income α to be equal to 0.34. The productivity is $\overline{z} = 1.1$. The coefficient of complementarity $\sigma = 0.95$. The maximum share of the portfolio for risky asset are $\theta_c = n^*$ and $\theta_d = 0.5$. $\gamma = 0.5, \beta = 0.98$, $k_0 = 0.045$. The results of this example are available in figure 4.

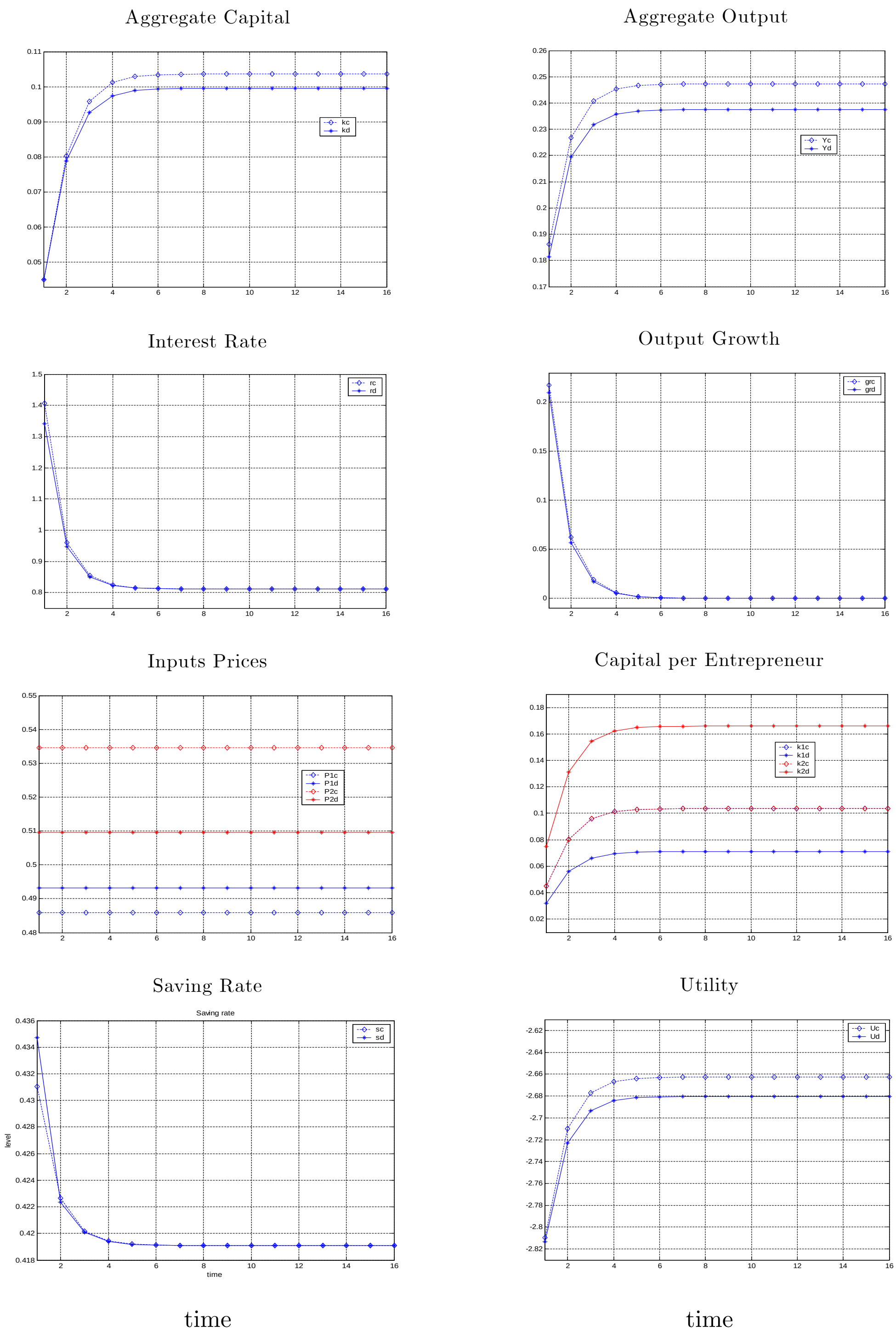

Figure 3.Comparative Dynamic in a Deterministic Environment

As awaited same in the case of $\rho \neq 1$, the result on economic growth and economic development stills holds. i.e. the asset holding restriction hampers economic growth and maintains the regulated economy is a steady state with lower production. i.e. it has a negative impact on economic development.

4 Competitive Equilibrium under Uncertainty

As we said in the introduction, it has been proven that banking crises are often due to macroeconomic shocks, these shocks can not be insure by the economy and in such cases we want to investigate the impact of the banking regulation on economic growth and stability.

4.1 Preliminaries and Definition of Equilibrium

Assumption 4 *We assume that r_t is not a function of the current state of nature. In fact, the interest rate is determined in term of numeraire before the production takes place.*[16]

Each contract in this environment provides a risky payoff. The uncertainty affects optimal contracts through two ways. In one hand the technology shock affects the level of the high return technology making this activity risky but has no effect on the low return activity. In the other hand it affects prices. In fact, when the aggregate state of nature is high, there is too much high return intermediate goods in the economy, since prices depend on relative rarity of factors, its relative price will be lower. Conversely, when the state of nature is low, the relative price of the high return technology is higher. And finally the optimal contract for those implementing the low return technology is uncertain because of the price effect. This is an important feature of this model. It allows to capture the fact that shocks in one sector always induce uncertainty in other sectors through prices. With this result the participation constraint for each type of entrepreneur can be reset as $\tau_1(z_t) = \widehat{\tau}_1(z_t)$ and $\tau_2(z_t) = \widehat{\tau}_2(z_t)$ at any date t.

[16] Hellwig (1994, 1997) pointed out that de facto aggregate risks in banking are not shifted to depositors. The deposit contract is demandable and does not include clauses for macroeconomic risks: It is non-contingent and makes banks vulnerable to aggregate shocks. The fact that interest rate does not depend on the current state of nature is an important assumption. It means that old do not share risk with entrepreneur. In fact, in the real economy deposit contracts are not contingent, they work like debt contracts with a fix interest rate. We want to use this feature and assume that in any case bank has to pay the lender.

Banks provide less capital stock per entrepreneur implementing the risk-free and the risky technologies than in a deterministic world. This is a general result, in case of uncertainty, entrepreneurs demand less capital because they may invest and suffer a lost, so for two economies started with the same stock of capital, the first interest rate will be lower in the risky economy than in a risk-free one, but the supply of capital for the next period depends on the behavior of savers in front of uncertainty.

To provide the competitive equilibrium allocation let us define what we will call by equilibrium in this section.

Definition 3. *Given k_0 units of capital in period $t = 0$, a sequential market equilibrium is the consumption level of the initial old generation c_0^o, consumption allocation for entrepreneurs, who choose the risky technology $\{c_{1t}^y(z_t), c_{1t}^o(z_t)\}_{t=0}^{\infty}$, consumption allocation for those who choose the risk-free technology $\{c_{2t}^y(z_t), c_{2t}^o(z_t)\}_{t=0}^{\infty}$, aggregate capital $\{k_{t+1}(z_t)\}_{t=0}^{\infty}$, proportion of entrepreneurs who choose the risky technology, $\{n_t\}_{t=0}^{\infty}$, contracts $\{(k_{1t}, \tau_{1t}(z_t))\}_{t=0}^{\infty}$, for the risky technology and $\{(k_{2t}, \tau_{2t}(z_t))\}_{t=0}^{\infty}$ for the risk-free technology, an allocation $\{Y_t(z_t), Y_{1t}(z_t), Y_{2t}\}_{t=0}^{\infty}$ for firms and sequences of prices $\{r_t, p_{1t}(z_t), p_{2t}(z_t)\}_{t=0}^{\infty}$, such that for all $t \geq 0$;*

1. *consumers optimize, i.e., $c_0 = k_0(1 + r_0)$; for $i = 1, 2$ and $t > 0$,*

$$c_{it}^y(z_t) = \tau_{it}(z_t) - s(\tau_{it}(z_t), r_{t+1}(z_t)) \text{ and } c_{it}^o(z_t) = (1 + r_{t+1}(z_t))s(\tau_{it}(z_t), r_{t+1}(z_t))$$

2. *contracts are optimal, i.e., for all $t \geq 0$, they solve the representative bank's problem;*

3. *ex-ante, each entrepreneur is indifferent between technologies, i.e.*

$$E_t v(\tau_{1t}(z_t), r_{t+1}(z_t)) = E_t v(\tau_{2t}(z_t), r_{t+1}(z_t))$$

4. *final goods firms optimize, i.e., $\{Y_t(z_t), Y_{1t}(z_t), Y_{2t}\}_{t=0}^{\infty}$ solves the firm's problem;*

5. *the aggregate capital stock equals supply at all $t \geq 0$, i.e.,*

$$n_{t+1}(z_t)k_{1t+1}(z_t) + (1 - n_{t+1}(z_t))k_{2t+1}(z_t) = n_t s_{1t}(z_t) + (1 - n_t)s_{2t}(z_t)$$

6. *the risky intermediate goods market clears, i.e., $Y_{1t}(z_t) = n_t z_t k_{1t}^{\alpha}$,*

7. *the risk-free intermediate goods market clears, i.e.*, $Y_{2t} = (1 - n_t)k_{2t}^{\alpha}$,

8. *the final goods market clears, i.e.*,

$$Y(z_t) = n_t c_{1t}^y(z_t) + (1 - n_t)c_{2t}^y(z_t) + n_{t-1}c_{1t-1}^o + (1 - n_{t-1})c_{2t-1}^o.$$

Entrepreneurs implementing the risky technology will have more when the state of nature is high and less if not. This result is a function of the complementarity of inputs. If inputs are perfect substitutes, the prices of inputs are equal to one and can not constitute a channel through which uncertainty can be passed to others. As the complementarity of inputs increases, the price becomes the major channel of transmission of uncertainty from those implementing the risky technology to those implementing the risk-free. In the case of a Leontief technology or a Cobb -Douglas technology the prices capture all the uncertainty, so the real transfer for entrepreneur tends to be the same regardless the type of technology.[17]

At equilibrium the number of people implementing the risky technology is a constant $\widetilde{n}^*$ independent of the date, furthermore it is less that n^*. This result was awaited, the uncertainty drives entrepreneurs in average more for the risky technology. One way to do that in to increase it expected price, for that they will move from the risky to the risk-free technology. A consequent of this is that in average the price of the risky input is higher in this environment than in the deterministic one. We will then omit t and z_{t-1} in front of variables n.

4.2 On the optimality of unregulated banking

We investigate in this subsection the contrained-efficiency of the competitive equilibrium. For that we will first reset the banks problem and then define equilibrium in this case.

Like in the previous section, the regulated banks' problem is unchanged for those implementing the risk-free technology. But it is impossible to a bank to be specialized in the risky technology. Therefore, in this economy there is a new risky bank dealing with both technologies. This bank determines the optimal contracts for entrepreneurs by solving,

$$\max_{(\widehat{k}_{1t}, \widehat{\tau}_{1t}(z_t), \widehat{k}_{2t}, \widehat{\tau}_{2t}(z_t))} E_t\{v(\widehat{\tau}_{1t}(z_t), r_{t+1}(z_t))\} \tag{17}$$

[17] In these cases agent are ex-post homogemous and competitive equilibrium allocation of risk-sharing is constrained-efficient according to Allen and Gale (2003).

subject to,

$$n\widehat{\tau}_{1t}(z_t)+(1-n)\widehat{\tau}_{2t}(z_t)+r_t\widehat{k}_t = nz_tp_{1t}(z_t)\widehat{k}_{1t}^{\alpha}+(1-n)p_{2t}(z_t)\widehat{k}_{2t}^{\alpha}, \quad (18)$$

$$v(\widehat{\tau}_{2t}(z_t),r_{t+1}) \geq v(\tau_{2t}(z_t),r_{t+1}), \quad (19)$$

$$\frac{n_t\widehat{k}_{1t}}{\widehat{k}_t} \geq \theta. \text{ Where } \widehat{k}_t=n\widehat{k}_{1t}+(1-n)\widehat{k}_{2t} \quad (20)$$

Constraint (18) is the zero-profit constraint for intermediaries. (19) is the participation constraint for those implementing the risk-free technology, and (20) is the regulatory constraint, which states that the bank portfolio cannot have up to a given proportion of capital allocated to the risky technology.

Definition 4. *Given k_0 units of capital in period $t=0$, a sequential market equilibrium is the consumption level of the initial old generation c_0^o, consumption allocation for entrepreneurs who choose the risky technology $\{c_{1t}^y(z_t),c_{1t}^o(z_t)\}_{t=0}^{\infty}$, consumption allocation for those who choose the risky bank but operating the risk-free technology $\{\widehat{c}_{2t}^y(z_t),\widehat{c}_{2t}^o(z_t)\}_{t=0}^{\infty}$, consumption allocation for entrepreneurs who choose the risk-free technology $\{c_{2t}^y(z_t),c_{2t}^o(z_t)\}_{t=0}^{\infty}$,aggregate capital $\{k_{t+1}(z_t)\}_{t=0}^{\infty}$, proportion of entrepreneurs in the general bank implementing the risky technology $\{n\}_{t=0}^{\infty}$, proportion of entrepreneurs who choose the risky bank $\{m\}_{t=0}^{\infty}$, contracts $\left\{(\widehat{k}_{1t},\widehat{\tau}_{1t}(z_t))\right\}_{t=0}^{\infty}$, for those operating the risky technology, $\left\{(\widehat{k}_{2t},\widehat{\tau}_{2t}(z_t))\right\}_{t=0}^{\infty}$ for entrepreneurs implementing the risk-free in the risky bank and $\{(k_{2t},\tau_{2t}(z_t))\}_{t=0}^{\infty}$ for those operating the risk-free technology in the risk-free bank, an allocation $\{Y_t(z_t),Y_{1t}(z_t),Y_{2t}\}_{t=0}^{\infty}$ for firms and sequences of prices $\{r_t,p_{1t}(z_t),p_{2t}(z_t)\}_{t=0}^{\infty}$, such that for all $t\geq 0$:*

1. *consumers optimize, i.e. $c_0=k_0(1+r_0)$; for $i=1,2$ and $t>0$,*

   $c_{it}^y(z_t)=\tau_{it}(z_t)-s(\tau_{it}(z_t),r_{t+1}(z_t))$ and $c_{it}^o(z_t)=(1+r_{t+1}(z_t))s(\tau_{it}(z_t),r_{t+1}(z_t))$;

2. *contracts are optimal, i.e. for all $t\geq 0$, they solve bank's problem* ;

3. *ex-ante, entrepreneurs operating the risk-free technology are indifferent between banks, i.e. $E_tv(\widehat{\tau}_{2t}(z_t),r_{t+1}(z_t))=E_tv(\tau_{2t}(z_t),r_{t+1}(z_t))$;*

4. *ex-ante, entrepreneurs in the general bank are indifferent between technologies, i.e. $E_tv(\widehat{\tau}_{1t}(z_t),r_{t+1}(z_t))=E_tv(\widehat{\tau}_{2t}(z_t),r_{t+1}(z_t))$;*

5. *final goods firms optimize, i.e.* $\{Y_t(z_t), Y_{1t}(z_t), Y_{2t}\}_{t=0}^{\infty}$ *solves firm's problem* ;

6. *the aggregate capital stock equals supply at all* $t \geq 0$, *i.e.*

$$\begin{aligned} & mn\widehat{k}_{1t+1}(z_t) + m(1-n)\widehat{k}_{2t+1}(z_t) + (1-m)k_{2t+1}(z_t) \\ = \ & mn\widehat{s}_{1t}(z_t) + m(1-n)\widehat{s}_{2t}(z_t) + (1-m)s_{2t}(z_t); \end{aligned}$$

7. *the risky intermediate goods market clears at all* $t \geq 0$ *,i.e.* $Y_{1t}(z_t) = mnz_t\widehat{k}_{1t}^{\alpha}$;

8. *the risk-free intermediate goods market clears at all* $t \geq 0$ *,i.e.* $Y_{2t} = m(1-n)\widehat{k}_{2t}^{\alpha} + (1-m)k_{2t}^{\alpha}$;

9. *and the final goods market clears at all time* $t \geq 0$*, i.e.*[18]

$$Y_t(z_t) = mn\widehat{c}_{1t}^{y}(z_t) + m(1-n)\widehat{c}_{2t}^{y}(z_t) + (1-m)c_{2t}^{y}(z_t) + mn\widehat{c}_{1t-1}^{o}(z_t) + m(1-n)\widehat{c}_{2t-1}^{o}(z_t) + (1-m)c_{2t-1}^{o}(z_t).$$

Like in the deterministic environment, in presence of regulation, economy adjusts in this way; given the fact that transfer for those implementing the risk-free technology is an increasing function of the price of the risk-free input, entrepreneurs will move from the risk-free bank to the new risky bank so as to obtain the same allocation as before regulation. When this way of adjustment is possible i.e. $m < 1$, the economy walks as if there is no regulation. But when $\theta < \widetilde{n}^*$, it is impossible to entrepreneurs to obtain the competitive equilibrium allocation, and as in the previous section entrepreneurs will continue to move from the risk-free technology to the risky in order to increase the expected price of the risk-free input.

4.3 Implications of banking regulation for growth and stability

We now investigate the impact of this regulation on growth and stability. Because of the complexity of the equilibrium equations, properties of key variables are difficult to derive analytically. We now compute solutions numerically and simulate the equilibrium behavior of the economy by comparing the dynamics of an unregulated banking economy and a regulated banking. Let θ be the maximum share of the portfolio a bank can invest in the risky technology in the regulated banking economy. We start with date

[18] This condition is redundant. In fact, according to the Walras law when $n-1$ markets are in equilibrium the remaining one is in equilibrium.

$t = 0$. Since the capital stock is given, the supply of capital by the old generation is completely inelastic at k_0. The capital adequacy requirement is reflected in different demand schedules. The result is that the interest rate r_0 is lower in the economy with θ, and the expected transfers received by entrepreneurs τ_0 is lower. The economic intuition is straightforward. Since the amount of capital allocated to the production of the high return intermediate good is fixed and lower in the regulated economy, and given that the demand is function of the productivity, demand of capital is low and supply is the same, so the interest rate will adjust i.e. r_0 is lower.

In the other hand the production of the risky intermediate good will be lower while the production of the risk-free intermediate good will be higher. Ex-post if the state of the nature is high, the production in the regulated economy will be low. But if the state of the nature is low the production of the regulated economy will be higher than in the free-banking economy. There are many possible path for this economy. In average, the unregulated-banking will yield a better dynamic.

In particular, consider the above power utility function with the following parameters value $\rho = 0.5$. We take the capital share of income α to be equal to 0.34 . The productivity shocks are $z_h = 1.25$, $z_l = 0.95$ and $\pi = 0.5$, the coefficient of complementarity $\sigma = 0.95$, and $\theta = 0.5., \gamma = 0.5, \beta = 0.98, k_0 = 0.045$.

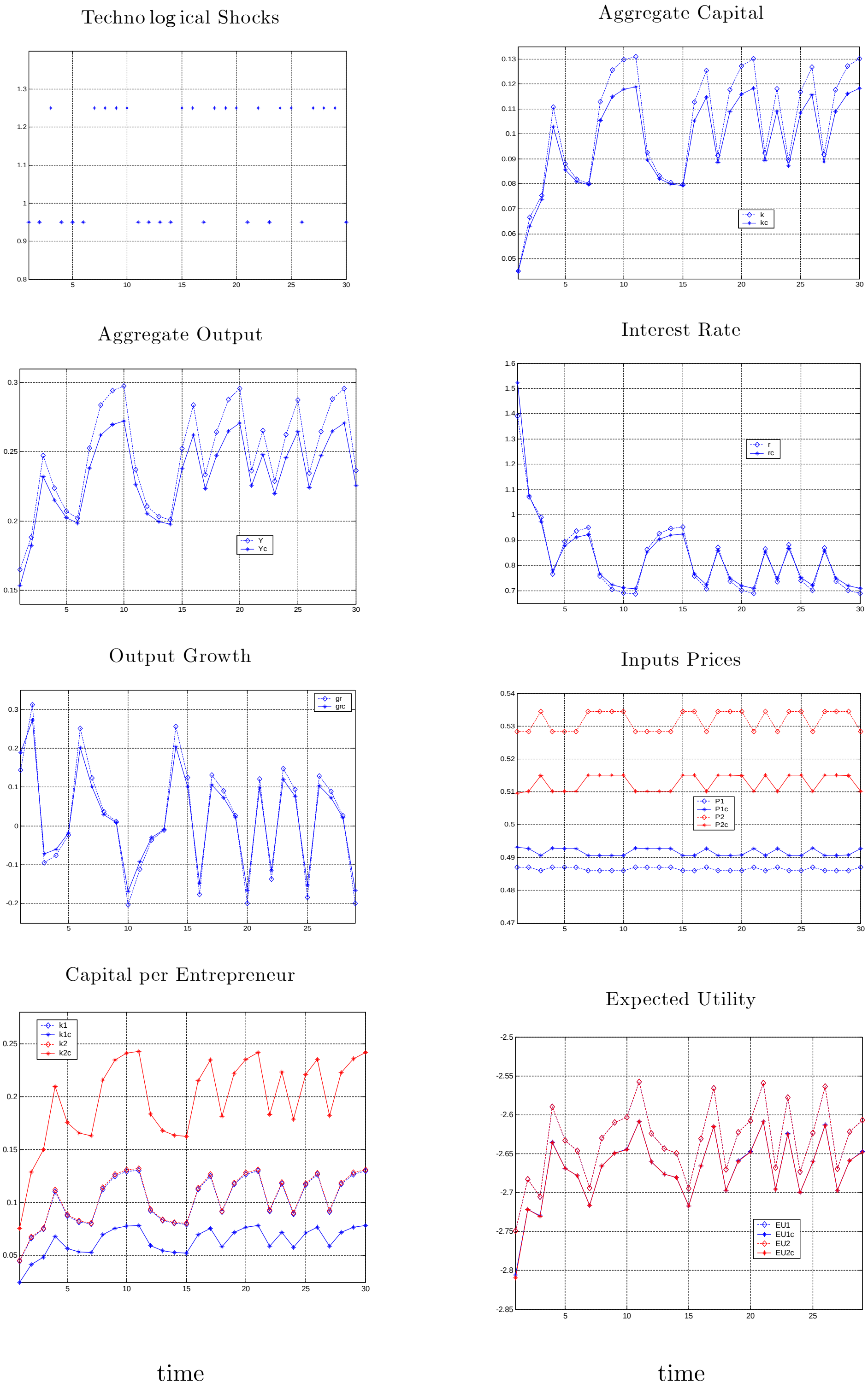

Figure 4. Comparative Dynamics in a Stochastic Environment

In this scenario, the impact of regulation on capital accumulation is difficult to handle. In fact, in this model capital converges toward the steady state, so after any shock, the model continues it recovery toward steady state. But in average they will be less capital in the economy, less proportion of entrepreneurs using the risky technology. This will end up with a more stable economy but with less amplitude of growth.

Finally in this case banking regulation does not reduce the probability of banking crises because it is due to business cycle but it continue to stabilize growth but reduces its magnitude.

5 Competitive Equilibrium with possibility of default (Incomplete)

We assume as Allen and Gale (JPE, 2000) the existence of a special state which occurs with probability 0. This is to state the point and after we will remove this assumption and see. In this case the equilibrium allocation obtained in section 4 still hold. When the state of the nature is z_h or z_l the economy continue walk as previewly. But if the special state w occurs, the risky bank in an unregulated economy will go bankrupt and the overall system will collapse. As a result, from this date to infinity entrepreneurs will just implement the risk-free technology.

The regulation forces the risky bank to finance a positive proportion of the production of the risk-free input. When the special state occurs, the new risky bank is able to pay a positive interest rate to the lender. It can be insolvent but will no go bankrupt.

6 Conclusion

In the first part of this paper we have introduced the banking regulation in the familiar two-period OLG model of capital accumulation in a deterministic environment. The level of regulation is measured by the capital adequacy requirement which is the main quantitative part of the Basel accords. In this environment, our model produces several interesting implications. First, the portfolio of bank in competitive equilibrium is efficient. Second, we identified two important channels in which banking regulation affects the growth process : (i) it constraints banks to provide less capital to high productive technology, as a result the current production is lower, (ii) banks tend to provide lower interest rate to old reducing the saving rate or at least the amount save. These two effects make the banking regulation detrimental for economic growth.

In the second part we have introduced uncertainty in the previous OLG framework. We have studied the effect of regulation on growth and other macroeconomic indicators by a numerical computation. We have simulated a typical economy and found that economic growth will be less volatile but in average the aggregate output and the output growth in the regulated economy are lower in transition and in the steady state than in the unregulated banking economy.

The third part of this paper is incomplete and

Appendix

Proof of Lemma 1.

τ_{1t} is obtained from the risky bank's problem.

$$\max_{(k_{1t},\tau_{1t})} v(\tau_{1t}, r_{t+1})$$

subject to the zero-profit constraint $\tau_{1t} + r_t k_{1t} = \overline{z} p_{1t} f(k_{1t})$.

Also τ_{2t} is obtained from the risk-free bank's problem.

$$\max_{(k_{2t},\tau_{2t})} v(\tau_{2t}, r_{t+1})$$

subject to the zero-profit constraint $\tau_{2t} + r_t k_{2t} = \overline{z} p_{2t} f(k_{2t})$.

From zero-profit conditions, transfers are given by $\tau_{1t} = p_{1t}\overline{z} f(k_{1t}) - r_t k_{1t}$ and $\tau_{2t} = p_{2t} f(k_{2t}) - r_t k_{2t}$ Then, by strict monotonicity, banks will just choose the capital to maximize transfers. The optimal capital derived from bank problem are

$$(k_{1t}) \quad : \quad \overline{z} p_{1t} f'(k_{1t}) = r_t \tag{21}$$

$$(k_{2t}) \quad : \quad p_{2t} f'(k_{2t}) = r_t \tag{22}$$

From (21) we have $k_{1t} = f'^{-1}\left(\frac{r_t}{\overline{z} p_{1t}}\right)$ and from (22) $k_{2t} = f'^{-1}\left(\frac{r_t}{p_{2t}}\right)$ Finally substituting r_t by it value yields

$$\begin{aligned} \tau_{1t} &= \overline{z} p_{1t}\left(f(k_{1t}) - f'(k_{1t}) k_{1t}\right) \\ \tau_{2t} &= p_{2t}\left(f(k_{2t}) - f'(k_{2t}) k_{2t}\right). \end{aligned}$$

Proof of Lemma 3. (Optimal allocation in a deterministic environment)

We assume a simple OLG two periods model of capital accumulation. We assume that we have a risky activity ($y_{1t} = \overline{z} f(k_{1t}), \overline{z} > 1$) and a risk-free ($y_{2t} = f(k_{2t})$). We normalize the size of the population to 1. So the number of people implementing the risky project n_t is a proportion. The agent preference over consumption is given by $u(c_t, c_{t+1}) = u(c_t) + \beta u(c_{t+1})$. The overall production is $Y_t = F(n_t y_{1t}, (1-n_t) y_{2t}) = F(n_t \overline{z}_t f(k_{1t}), (1-n_t) f(k_{2t}))$.Recall that $k_t = n_t k_{1t} + (1-n_t) k_{2t}$; So $k_{2t} = \frac{k_t - n_t k_{1t}}{1-n_t}$. i.e. $k_{2t}(n_t, k_{1t})$. We can show that $\frac{\partial k_{2t}}{\partial n_t} = \frac{k_t - k_{1t}}{(1-n_t)^2}$ and $\frac{\partial k_{2t}}{\partial k_{1t}} = \frac{-n_t}{1-n_t}$.

The planner problem is

$$\max_{(c_t^o, c_t^y, k_{t+1}, n_t, k_{1t})} \beta u(c_0^o) + \sum_{t=0}^{\infty} \left(\frac{1}{1+R}\right)^{t+1} \left[u(c_t^y) + \beta u(c_{t+1}^o)\right]$$
$$s.t \quad : \quad F(n_t \overline{z} f(k_{1t}), (1-n_t) f(k_{2t})) + k_t = k_{t+1} + c_t^y + c_t^o.$$

Substituting the resource constraint in the objective function yields,

$$\max_{(c_t^o, c_t^y, k_{t+1}, n_t, k_{1t})} \beta u(c_0^o) + \sum_{t=0}^{\infty} \frac{\left[u(F(n_t \overline{z} f(k_{1t}), (1-n_t) f(k_{2t})) + k_t - k_{t+1} - c_t^o + \beta u(c_{t+1}^o)\right]}{(1+R)^{t+1}}.$$

We are interested here to solve for n_t and k_{1t}. The related FOC are:

$$(n_t) \quad : \quad \overline{z} f(k_{1t}) F_1 + \left(\frac{\partial k_{2t}}{\partial n_t}(1-n_t) f'(k_{2t}) - f(k_{2t})\right) F_2 = 0 \tag{23}$$
$$(k_{1t}) \quad : \quad n_t \overline{z} f'(k_{1t}) F_1 + (1-n_t) \frac{\partial k_{2t}}{\partial k_{1t}} f'(k_{2t}) F_2 = 0. \tag{24}$$

Substituting $\frac{\partial k_{2t}}{\partial n_t}$ and $\frac{\partial k_{2t}}{\partial k_{1t}}$ by their value in (23) and (24) yield,

$$(n_t) \quad : \quad \overline{z} f(k_{1t}) F_1 + \left(\frac{k_t - k_{1t}}{1-n_t} f'(k_{2t}) - f(k_{2t})\right) F_2 = 0$$
$$(k_{1t}) \quad : \quad n_t \overline{z} f'(k_{1t}) F_1 - n_t f'(k_{2t}) F_2 = 0.$$

If $f(x) = x^{\alpha}$,

We obtain

$$(n_t) \quad : \quad \overline{z}_t k_{1t}^{\alpha} F_1 = \left(k_{2t}^{\alpha} - \alpha \frac{k_t - k_{1t}}{1-n_t} k_{2t}^{\alpha-1}\right) F_2, \tag{25}$$
$$(k_{1t}) \quad : \quad \overline{z}_t k_{1t}^{\alpha-1} F_1 = k_{2t}^{\alpha-1} F_2. \tag{26}$$

Dividing (25) by (26) yields,

$$k_{1t} = \left(k_{2t} - \alpha \frac{k_t - k_{1t}}{1-n_t}\right). \tag{27}$$

Substituting k_t by $n_t k_{1t} + (1-n_t) k_{2t}$ in the (27) yields, $k_{1t} = k_{2t} - \alpha \left(k_{2t} - k_{1t}\right)$ which equivalent to $(1-\alpha) k_{1t} = (1-\alpha) k_{2t}$, and if $\alpha \neq 1$,we have,

$$k_{1t} = k_{2t}. \tag{28}$$

It is optimal to give the same capital per entrepreneur for each type of project if the intermediate good production is a Cobb-Douglas.

Now we focus our attention on (26), it is equivalent to

$$\overline{z} = \frac{F_2}{F_1}. \tag{29}$$

To continue the resolution we use a more general functional form of the final good production technology, $F(A,B) = [\gamma A^\sigma + (1-\gamma)B^\sigma]^{\frac{1}{\sigma}}$ .We obtain $\frac{F_2}{F_1} = \frac{1-\gamma}{\gamma}\left(\frac{B}{A}\right)^{\sigma-1}$. With $A = n_t \overline{z} k_{1t}^\alpha$ and $B = (1-n_t)k_{1t}^\alpha$, (29) is equivalent to $\overline{z} = \frac{1-\gamma}{\gamma}\left(\frac{1-n_t}{n_t \overline{z}_t}\right)^{\sigma-1}$. Finally

$$n_t = \frac{1}{1 + \left(\frac{1-\gamma}{\gamma \overline{z}^\sigma}\right)^{\frac{1}{1-\sigma}}}$$

The planner portfolio is the same as the competitive equilibrium one.

Proof of Lemma 4.

The bank provides capital for both types of technology and the optimal capital supply must satisfy the regulatory constraint with equality. The regulatory constraint can then be reset as

$$\widehat{k}_{2t} = \frac{n_t(1-\theta)}{\theta(1-n_t)}\widehat{k}_{1t}. \tag{30}$$

Therefore, to obtain the optimal capital offered by the bank for each type of contract we just need to maximize the objective function according to $\widehat{k}_{1t}$. Furthermore we saw that the indirect utility function is a strictly increasing function of its first argument, given the zero-profit constraint and the free entry assumption of any type of banks in the economy, the general bank will provide $\tau_{2t} = \widehat{\tau}_{2t}$ to entrepreneurs implementing the risky technology.

Given that there is no uncertainty and that the indirect utility of agents are increasing function of transfer, the optimal choice of capital for the risky technology will be one which maximizes the amount of transfer provided to entrepreneurs. i.e. $\widehat{k}_{1t} \equiv \arg\max_k \{\tau_{1t}(k)\}$. Where $\tau_{1t}(k)$ is obtained by substituting $\widehat{k}_{2t}$ and $\widehat{\tau}_{2t}$ by their expressions in the zero-profit condition. Then

$$\widehat{\tau}_{1t} = \frac{B_t}{\theta^\alpha(1-n_t)^{\alpha-1}}\widehat{k}_{1t}^\alpha - \frac{r_t}{\theta}\widehat{k}_{1t} - \frac{(1-n_t)}{n_t}\widehat{\tau}_{2t}. \tag{31}$$

From the FOC, capital demand for the risky technology is given by,

$$\widehat{k}_{1t} = \theta(1-n_t)\left[\frac{\alpha B_t}{r_t}\right]^{\frac{1}{1-\alpha}}. \tag{32}$$

Given (30), and replacing $\widehat{k}_{1t}$ by its value in (32) we obtain $\widehat{k}_{2t} = (1-\theta)n_t \left[\frac{\alpha B_t}{r_t}\right]^{\frac{1}{1-\alpha}}$. Substituting for $\widehat{k}_{1t}$ and $\widehat{\tau}_{2t}$ in the zero-profit condition yields

$$\widehat{\tau}_{1t} = (1-\alpha)\frac{(1-n_t)}{n_t}\left[n_t B_t^{\frac{1}{1-\alpha}} - p_{2t}^{\frac{1}{1-\alpha}}\right]\left(\frac{\alpha}{r_t}\right)^{\frac{\alpha}{1-\alpha}}. \tag{33}$$

$\widehat{\tau}_{2t}$ is obtained from the participation constraint $\tau_{2t} = \widehat{\tau}_{2t}$, and from lemma 1 with assumption 2.

Proof of Lemma 5.

The proof when $\theta \in (n^*, 1)$ is straightforward. We now investigate when $\theta \in (0, n^*)$.

The equilibrium proportion, of entrepreneurs using the risky technology in the representative bank is obtained from the indifference condition between technologies which is, $\widehat{\tau}_{1t} = \widehat{\tau}_{2t}$. Using the optimal transfers given by lemma 3, this condition is equivalent to

$$[(1-n_t)n_t]^{1-\alpha} B_t = p_{2t} \tag{34}$$

To complete the determination of n_t, we must determine p_{2t} and B_t.

1. Computation of p_{2t} : From markets clearance conditions we have $Y_{1t} = m_t n_t \overline{z}\widehat{k}_{1t}^{\alpha}$; and $Y_{2t} = m_t(1-n_t)\widehat{k}_{2t}^{\alpha} + (1-m_t)k_{2t}^{\alpha}$. In this case we know that $m_t = 1$. Substituting for $\widehat{k}_{1t}$ and $\widehat{k}_{2t}$ in the above equations yields $Y_{1t} = n_t\overline{z}\theta^{\alpha}(1-n_t)^{\alpha}\left[\frac{\alpha B_t}{r_t}\right]^{\frac{\alpha}{1-\alpha}}$, and $Y_{2t} = (1-n_t)n_t^{\alpha}(1-\theta)^{\alpha}\left[\frac{\alpha B_t}{r_t}\right]^{\frac{\alpha}{1-\alpha}}$. Recall that $\frac{\overline{z}p_{1t}}{p_{2t}} = \frac{\overline{z}\gamma}{1-\gamma}\left(\frac{Y_{1t}}{Y_{2t}}\right)^{\sigma-1}$ and substituting Y_1 and Y_2 in the above expression yields,

$$p_{2t} = p_{1t}\frac{(1-\gamma)}{\gamma}\left[\frac{n_t\overline{z}\theta^{\alpha}(1-n_t)^{\alpha}B_t^{\alpha}}{(1-n_t)n_t^{\alpha}(1-\theta)^{\alpha}B_t^{\alpha}}\right]^{1-\sigma} \tag{35}$$

2. Computation of B_t.

$$B_t = \overline{z}p_1\theta^{\alpha}(1-n)^{\alpha-1} + p_2(1-\theta)^{\alpha}n^{\alpha-1}.$$

We will express it in function of p_{2t}. From (35) we have $p_{1t} = \frac{\gamma p_{2t}}{1-\gamma}\left[\frac{n_t^{1-\alpha}\theta^{\alpha}\overline{z}}{(1-n_t)^{1-\alpha}(1-\theta)^{\alpha}}\right]^{\sigma-1}$. Substituting p_{1t} in the expression of B_t yields

$$B_t = \left[\frac{\gamma}{1-\gamma}\frac{\overline{z}\theta^{\alpha}}{(1-n)^{1-\alpha}}\left(\frac{n^{1-\alpha}\theta^{\alpha}\overline{z}}{(1-n)^{1-\alpha}(1-\theta)^{\alpha}}\right)^{\sigma-1} + (1-\theta)^{\alpha}n^{\alpha-1}\right]p_{2t}. \tag{36}$$

We now substituting the above expression of B_t into (34) and obtain

$$\frac{\gamma\bar{z}^{\sigma}\theta^{\alpha\sigma}}{1-\gamma}n^{(1-\alpha)\sigma}(1-n)^{(1-\alpha)(1-\sigma)}(1-\theta)^{\alpha(1-\sigma)} = 1-(1-\theta)^{\alpha}(1-n)^{1-\alpha}. \tag{37}$$

This is also equivalent to $G(n) = 0$ where

$$G(x) = \frac{\gamma\bar{z}^{\sigma}\theta^{\alpha\sigma}}{1-\gamma}x^{(1-\alpha)\sigma}(1-x)^{(1-\alpha)(1-\sigma)}(1-\theta)^{\alpha(1-\sigma)} - 1 + (1-\theta)^{\alpha}(1-x)^{1-\alpha}.$$

We study the properties of $G(.)$ on $(0,1)$.

$$G(0) = \begin{cases} +\infty, \text{ if } \sigma \le 0 \\ (1-\theta)^{\alpha} - 1, \text{ if } \sigma > 0 \end{cases} \quad andG(1) = -1.$$

$$\frac{\partial G(x)}{\partial x} = (1-\alpha)(1-\theta)^{\alpha}(1-n)^{-\alpha}\left[\frac{\gamma\bar{z}^{\sigma}\theta^{\alpha\sigma}(\sigma-n)n^{\sigma}(1-n)^{\alpha\sigma}}{(1-\gamma)n^{(1+\alpha\sigma)}(1-n)^{\sigma}} - 1\right]$$

$\frac{\partial G(x)}{\partial x}$ has the sign of $\left[\frac{\gamma\bar{z}^{\sigma}\theta^{\alpha\sigma}(\sigma-n)n^{\sigma}(1-n)^{\alpha\sigma}}{(1-\gamma)n^{(1+\alpha\sigma)}(1-n)^{\sigma}} - 1\right]$
When $\sigma \le 0$, $(\sigma - n) \le 0$ and then $\frac{\partial G(x)}{\partial x} \le 0$.
Else $\frac{\gamma\bar{z}^{\sigma}\theta^{\alpha\sigma}(\sigma-n)n^{\sigma}(1-n)^{\alpha\sigma}}{(1-\gamma)n^{(1+\alpha\sigma)}(1-n)^{\sigma}} - 1 \le 0$. i.e.

$$\gamma\bar{z}^{\sigma}\theta^{\alpha\sigma}(\sigma-n)n^{\sigma}(1-n)^{\alpha\sigma} \le (1-\gamma)n^{(1+\alpha\sigma)}(1-n)^{\sigma}, \frac{\partial G(x)}{\partial x} \le 0.$$

Else $\frac{\partial G(x)}{\partial x} > 0$.
When $\sigma = 0$, the above equation yields $n = 1 - \left[\frac{1-\gamma}{(1-\theta)^{\alpha}}\right]^{\frac{1}{1-\alpha}}$.

So when $\sigma \le 0$, the above equation yield a unique solution. If not two different solutions and other equation of the model serve to select the real solution.

We derive the logarithm in respect to $\theta$of the above equation and obtain

$$\frac{\partial n}{\partial \theta} = \frac{\alpha n(1-n)}{(1-\alpha)\theta(1-\theta)}\frac{\left[-\theta + \sigma\left(1-(1-\theta)^{\alpha}(1-n)^{1-\alpha}\right)\right]}{\left[n - \sigma\left(1-(1-\theta)^{\alpha}(1-n)^{1-\alpha}\right)\right]} \tag{38}$$

It has the sign of

$$\frac{-\left[\theta - \sigma\left(1-(1-\theta)^{\alpha}(1-n)^{1-\alpha}\right)\right]}{\left[n - \sigma\left(1-(1-\theta)^{\alpha}(1-n)^{1-\alpha}\right)\right]}$$

When $\sigma \le 0$, the numerator is negative and the denominator is positive, so $\frac{\partial n}{\partial \theta} \le 0$.

When $\sigma \ge 0$, it is hard to provide a precise decision.

Proof of Proposition 1.

Assume that k_t is given. Then $Y_{1t} = n^{1-\alpha}\theta^\alpha \overline{z} k_t^\alpha$ and $Y_{2t} = (1-n)^{1-\alpha}(1-\theta)^\alpha k_t^\alpha$. Substituting Y_{1t} and Y_{2t} in $Y_t = [\gamma Y_{1t}^\sigma + (1-\gamma) Y_{2t}^\sigma]^{\frac{1}{\sigma}}$ yields

$$Y_t = \left[\gamma \left(n^{1-\alpha}\theta^\alpha \overline{z}\right)^\sigma + (1-\gamma)\left((1-n)^{1-\alpha}(1-\theta)^\alpha\right)^\sigma\right]^{\frac{1}{\sigma}} k_t^\alpha.$$

From (37) have

$$\gamma \left(n^{1-\alpha}\theta^\alpha \overline{z}\right)^\sigma = \frac{(1-\gamma)\left[1-(1-\theta)^\alpha(1-n)^{1-\alpha}\right]}{[(1-\theta)^\alpha(1-n)^{1-\alpha}]^{(1-\sigma)}}.$$

Therefore,

$$\begin{aligned} Y_t &= \left[\frac{(1-\gamma)\left[1-(1-\theta)^\alpha(1-n)^{1-\alpha}\right]}{[(1-\theta)^\alpha(1-n)^{1-\alpha}]^{(1-\sigma)}} + (1-\gamma)\left[(1-n)^{1-\alpha}(1-\theta)^\alpha\right]^\sigma\right]^{\frac{1}{\sigma}} k_t^\alpha \\ &= (1-\gamma)^{\frac{1}{\sigma}}\left[(1-\theta)^\alpha(1-n)^{1-\alpha}\right]^{\frac{(\sigma-1)}{\sigma}} k_t^\alpha \end{aligned}$$

We now use the logarithmic derivation.

$$\log(Y_t) = \log\left[(1-\gamma)^{\frac{1}{\sigma}}\right] + \frac{(\sigma-1)}{\sigma}\left[\alpha\log(1-\theta) + (1-\alpha)\log(1-n)\right] + \alpha\log(k_t),$$

so

$$\frac{\partial log(Y_t)}{\partial\theta} = \frac{(1-\sigma)}{\sigma}\left[\frac{\alpha}{(1-\theta)} + \frac{(1-\alpha)\frac{\partial n}{\partial\theta}}{(1-n)}\right]$$

Recall that $\frac{\partial n}{\partial\theta} = \frac{\alpha n(1-n)}{(1-\alpha)\theta(1-\theta)}\frac{[-\theta+\sigma C]}{[n-\sigma C]}$ where $C = 1-(1-\theta)^\alpha(1-n)^{1-\alpha}$
Therefore,

$$\begin{aligned} \frac{\partial log(Y_t)}{\partial\theta} &= \frac{(1-\sigma)}{\sigma}\left[\frac{\alpha}{(1-\theta)} + \frac{(1-\alpha)\frac{\alpha n(1-n)}{(1-\alpha)\theta(1-\theta)}\frac{[-\theta+\sigma C]}{[n-\sigma C]}}{(1-n)}\right] \\ &= \frac{(1-\sigma)\alpha}{\sigma(1-\theta)}\left[1+\frac{n}{\theta}\frac{[-\theta+\sigma C]}{[n-\sigma C]}\right] \end{aligned}$$

If $\sigma < 0$, $\frac{\partial log(Y_t)}{\partial\theta} \geq 0$ if and only if $1+\frac{n}{\theta}\frac{[-\theta+\sigma C]}{[n-\sigma C]} \leq 0$
i.e. $n\sigma \leq \theta\sigma$, because$C > 0$, and since $\sigma < 1$, it is equivalent to $n \geq \theta$.
If $\sigma > 0$, $\frac{\partial log(Y_t)}{\partial\theta} \geq 0$ if and only if $1+\frac{n}{\theta}\frac{[-\theta+\sigma C]}{[n-\sigma C]} \geq 0$
i.e. $\frac{[-\theta+\sigma C]}{[n-\sigma C]} \geq -\frac{\theta}{n}$ i.e. $n[-\theta+\sigma C] \geq -\theta[n-\sigma C]$ i.e. $n\sigma C \geq \theta\sigma C$
i.e. $n \geq \theta$ since $\sigma C > 0$.

When $\sigma = 0$,we have

$\frac{\partial log(Y_t)}{\partial\theta} = (1-\alpha)\frac{\partial n}{\partial\theta}\left[\frac{\gamma-n}{n(1-n)}\right] + \alpha\left[\frac{\gamma-\theta}{\theta(1-\theta)}\right]$ and $n = 1 - \left[\frac{1-\gamma}{(1-\theta)^\alpha}\right]^{\frac{1}{1-\alpha}}$, so

$\frac{\partial n}{\partial\theta} = -\frac{\alpha}{(1-\alpha)(1-\theta)}\left[\frac{1-\gamma}{(1-\theta)^\alpha}\right]^{\frac{1}{1-\alpha}} = -\frac{\alpha(1-n)}{(1-\alpha)(1-\theta)}$

Therefore,

$$\begin{aligned}\frac{\partial log(Y_t)}{\partial\theta} &= -\frac{\alpha(1-n)(1-\alpha)}{(1-\alpha)(1-\theta)}\left[\frac{\gamma-n}{n(1-n)}\right] + \alpha\left[\frac{\gamma-\theta}{\theta(1-\theta)}\right] \\ &= \frac{\alpha}{(1-\theta)}\left[\frac{n-\gamma}{n} + \frac{\gamma-\theta}{\theta}\right] \\ &= \frac{\alpha\gamma(n-\theta)}{(1-\theta)\theta n}\end{aligned}$$

Therefore, $\frac{\partial log(Y_t)}{\partial\theta} \geq 0 \Leftrightarrow n \geq \theta$.

Therefore for any value of σ, $\frac{\partial log(Y_t)}{\partial\theta} \geq 0 \Leftrightarrow n \geq \theta$.

We now have to verify that $n \geq \theta$.

1. When $\sigma = 0$, $n = 1 - \left[\frac{1-\gamma}{(1-\theta)^\alpha}\right]^{\frac{1}{1-\alpha}}$ and it is straightforward that $n \geq \theta$.

2. When $\sigma \neq 0$, suppose that $n < \theta$.

From $\gamma\left(n^{1-\alpha}\theta^\alpha\overline{z}\right)^\sigma = \frac{(1-\gamma)\left[1-(1-\theta)^\alpha(1-n)^{1-\alpha}\right]}{[(1-\theta)^\alpha(1-n)^{1-\alpha}]^{(1-\sigma)}}$ we have:

If $n < \theta \Longrightarrow n^{1-\alpha} \leq \theta^{1-\alpha}$, so $\gamma\left(n^{1-\alpha}\theta^\alpha\overline{z}\right)^\sigma \geq \gamma\left(n\overline{z}\right)^\sigma$

i.e. $\frac{(1-\gamma)\left[1-(1-\theta)^\alpha(1-n)^{1-\alpha}\right]}{[(1-\theta)^\alpha(1-n)^{1-\alpha}]^{(1-\sigma)}} \geq \gamma\left(n\overline{z}\right)^\sigma$.

$n < \theta \Rightarrow (1-n) < (1-\theta) \Rightarrow (1-n)^{1-\alpha} < (1-\theta)^{1-\alpha}$

$\Rightarrow (1-n)^{1-\alpha}(1-\theta)^\alpha > (1-n) \Rightarrow 1-(1-n)^{1-\alpha}(1-\theta)^\alpha < n$

$\left[(1-\theta)^\alpha(1-n)^{1-\alpha}\right]^{(1-\sigma)} > (1-n)^{(1-\sigma)}$

$\frac{1}{[(1-\theta)^\alpha(1-n)^{1-\alpha}]^{(1-\sigma)}} < \frac{1}{(1-n)^{(1-\sigma)}}$

Then

$\frac{1-(1-n)^{1-\alpha}(1-\theta)^\alpha}{[(1-\theta)^\alpha(1-n)^{1-\alpha}]^{(1-\sigma)}} < \frac{n}{(1-n)^{(1-\sigma)}}$ We then have $\frac{(1-\gamma)n}{(1-n)^{(1-\sigma)}} > \gamma\left(n\overline{z}\right)^\sigma$

i.e. $\frac{n^{(1-\sigma)}}{(1-n)^{(1-\sigma)}} > \frac{\gamma\overline{z}^\sigma}{(1-\gamma)}$ i.e. $\left[\frac{(1-n)}{n}\right]^{(1-\sigma)} < \left(\frac{(1-\gamma)}{\gamma\overline{z}^\sigma}\right)$ i.e. $\frac{1-n}{n} < \left(\frac{(1-\gamma)}{\gamma\overline{z}^\sigma}\right)^{\frac{1}{1-\sigma}}$

i.e. $\frac{1}{n} < 1 + \left(\frac{(1-\gamma)}{\gamma\overline{z}^\sigma}\right)^{\frac{1}{1-\sigma}}$ i.e. $n > \frac{1}{1+\left(\frac{(1-\gamma)}{\gamma\overline{z}^\sigma}\right)^{\frac{1}{1-\sigma}}} > \theta$, so $n > \theta$, contradiction.

Proof of Proposition 2.

From $Y_t = \left[\gamma Y_{1t}^\sigma + (1-\gamma)Y_{2t}^\sigma\right]^{\frac{1}{\sigma}}$, we obtain $Y_t = (1-\gamma)^{\frac{1}{\sigma}}\left[(1-\theta)^\alpha(1-n)^{1-\alpha}\right]^{\frac{(\sigma-1)}{\sigma}} k_t^\alpha$,

so

$\frac{Y_{t+1}}{Y_t} = \left(\frac{k_{t+1}}{k_t}\right)^{\alpha}$.

Recall that $k_{t+1} = s_t = b(r_{t+1})\tau_t.with\ \tau_t = (1-\alpha)p_{2t}\left[\frac{\alpha p_{2t}}{r_t}\right]^{\frac{\alpha}{1-\alpha}}$; and

$$k_t = nk_{1t} + (1-n)k_{2t} = n\theta(1-n)\left[\frac{\alpha B}{r_t}\right]^{\frac{1}{1-\alpha}} + (1-n)(1-\theta)n\left[\frac{\alpha B}{r_t}\right]^{\frac{1}{1-\alpha}}.$$

This yields $k_t = n(1-n)\left[\frac{\alpha B}{r_t}\right]^{\frac{1}{1-\alpha}}$.

Furthermore the indifferent condition of entrepreneurs between technologies yields $n(1-n)B^{\frac{1}{1-\alpha}} = p_{2t}^{\frac{1}{1-\alpha}}$. Therefore $k_t = \left[\frac{\alpha p_{2t}}{r_t}\right]^{\frac{1}{1-\alpha}}$, so $\tau_t = (1-\alpha)p_{2t}k_t^{\alpha}$.Then $k_{t+1} = b(r_{t+1})(1-\alpha)p_{2t}k_t^{\alpha}$, so $\frac{k_{t+1}}{k_t} = (1-\alpha)b(r_{t+1})p_{2t}k_t^{\alpha-1}$

With the logarithmic utility function we have $b(r_{t+1}) = \frac{\beta}{1+\beta}$ and then $\frac{k_{t+1}}{k_t} = (1-\alpha)\frac{\beta}{1+\beta}p_{2t}k_t^{\alpha-1}$.

- Since at t, k_t is given,$\frac{\partial\left[\frac{k_{t+1}}{k_t}\right]}{\partial\theta}$ has the sign of $\frac{\partial p_{2t}}{\partial\theta}$.

We will now focus on p_{2t}.We know that

$$p_{2t} = (1-\gamma)Y_t^{1-\sigma}Y_{2t}^{\sigma-1} = (1-\gamma)\left[\frac{Y_t}{Y_{2t}}\right]^{1-\sigma}$$

,

$$Y_t = (1-\gamma)^{\frac{1}{\sigma}}\left[(1-\theta)^{\alpha}(1-n)^{1-\alpha}\right]^{\frac{(\sigma-1)}{\sigma}}k_t^{\alpha}$$

and also that $Y_{2t} = (1-\theta)^{\alpha}(1-n)^{1-\alpha}k_t^{\alpha}$.

Therefore,

$$\frac{Y_t}{Y_{2t}} = \frac{(1-\gamma)^{\frac{1}{\sigma}}\left[(1-\theta)^{\alpha}(1-n)^{1-\alpha}\right]^{\frac{(\sigma-1)}{\sigma}}}{(1-\theta)^{\alpha}(1-n)_t^{1-\alpha}} = (1-\gamma)^{\frac{1}{\sigma}}\left[(1-\theta)^{\alpha}(1-n)^{1-\alpha}\right]^{\frac{-1}{\sigma}}$$

This yields

$$p_{2t} = (1-\gamma)^{\frac{1}{\sigma}}\left[(1-\theta)^{\alpha}(1-n)^{1-\alpha}\right]^{\frac{\sigma-1}{\sigma}} = \frac{Y_t}{k_t^{\alpha}}$$

Therefore, $\frac{\partial p_{2t}}{\partial\theta}$ has the sign of $\frac{\partial Y_t}{\partial\theta}$.

Recall that $\frac{\partial Y_t}{\partial\theta} \geq 0$.

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