“Multivariate estimation for operational risk with judicious use of Extreme Value Theory”

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Disclaimer:
The opinions expressed are those of the author and do not necessarily represent the views of the Office of the Comptroller of the Currency (OCC) or the Department of Treasury.
Outline

• Part I: Statistical Framework
  ➢ Unit of measure
  ➢ Aggregation of Units
  ➢ Modeling dependence structure

• Part II: Data

• Part III: Statistical Methods
  ➢ Fitting the tail
  ➢ Summary of Results
  ➢ Conclusions
Motivation

- Basel II recognizes diversification benefits **within** op risk.
  - Diversification benefits = (Simple Sum) - (Capital estimated with dependence structure)
- No convergence to date on how to estimate diversification benefits.
  - E.g., U.S. experience: diversification benefits range widely -- from 20% to 70%.
  - Most banks use *ad hoc* correlation factors among Event Types (ETs) /Business Lines (BLs) (10%, 20%) and then apply correlation matrix approach.
  - Copula approach.
"A bank's risk measurement system must be sufficiently 'granular' to capture the major drivers of operational risk affecting the shape of the tail of the loss estimates."  (Basel II paragraph 669(c))

S27. The bank must employ a unit of measure that is appropriate for the bank’s range of business activities and the variety of operational loss events to which it is exposed, and that does not combine business activities or operational loss events with different risk profiles within the same loss distribution.
(Proposed U.S. Supervisory Guidance on AMA, p219, February 2007)
Identifying the appropriate unit of measure: Standard Basel II Matrix

<table>
<thead>
<tr>
<th>Event Type</th>
<th>BL1 Corporate Finance</th>
<th>BL2 Trading &amp; Sales</th>
<th>BL3 Retail Banking</th>
<th>BL4 Commercial Banking</th>
<th>BL5 Payment &amp; Settlement</th>
<th>BL6 Agency Services</th>
<th>BL7 Asset Management</th>
<th>BL8 Retail Brokerage</th>
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<tbody>
<tr>
<td>ET 1</td>
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El-Gamal, Inanoglu and Stengel 2007
Identifying the appropriate unit of measure

<table>
<thead>
<tr>
<th>People</th>
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<th>Systems</th>
<th>External</th>
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<tbody>
<tr>
<td>People</td>
<td>Processes</td>
<td>Systems</td>
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<table>
<thead>
<tr>
<th>ET</th>
<th>BL</th>
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<tbody>
<tr>
<td>Internal Fraud</td>
<td>External Fraud</td>
</tr>
<tr>
<td>Employment Practices &amp; Workplace Safety</td>
<td>Clients, Products &amp; Business Practices</td>
</tr>
<tr>
<td>Damage to Physical Assets</td>
<td>Business Disruption &amp; System Failures</td>
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<tr>
<td>Execution, Delivery &amp; Process Management</td>
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El-Gamal, Inanoglu and Stengel 2007
Identifying the appropriate unit of measure: Standard Basel II Matrix

<table>
<thead>
<tr>
<th>Business Line</th>
<th>Event Type</th>
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<th>ET 2</th>
<th>ET 3</th>
<th>ET 4</th>
<th>ET 5</th>
<th>ET 6</th>
<th>ET 7</th>
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<tr>
<td></td>
<td>Internal Fraud</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
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<td>Trading &amp; Sales</td>
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<td><img src="image16" alt="Graph" /></td>
<td><img src="image17" alt="Graph" /></td>
<td><img src="image18" alt="Graph" /></td>
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<td><img src="image34" alt="Graph" /></td>
<td><img src="image35" alt="Graph" /></td>
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<td>BL5</td>
<td>Payment &amp; Settlement</td>
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<td>BL7</td>
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<td><img src="image51" alt="Graph" /></td>
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<td><img src="image63" alt="Graph" /></td>
</tr>
</tbody>
</table>

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Range of U.S. Banks’ Unit of Measure

- Single Unit (Pooling all data in one unit)
  - Allocation Issue
    (See paragraph Basel II 666(b))

- 1 dimensional
  - Either by BL or ET
    e.g. 5 ETs (Excluding BSDF and DPA)

- 2 dimensional BL/ET

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Aggregation of units:

A.K.A. Correlation/Dependence

“Risk measures for different operational risk estimates must be added for purposes of calculating the regulatory minimum capital requirement. However, the bank may be permitted to use internally determined correlations in operational risk losses across individual operational risk estimates, provided it can demonstrate to the satisfaction of the national supervisor…”

(Basel II paragraph 669(d))
Aggregation: A brief detour on correlation…

- Recall that in market risk, correlations are described as “equal-time correlations”.
- E.g. for two assets:

<table>
<thead>
<tr>
<th>TIME</th>
<th>ASSET 1</th>
<th>ASSET 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1$</td>
<td>$r_{11}$</td>
<td>$r_{21}$</td>
</tr>
<tr>
<td>$t_2$</td>
<td>$r_{12}$</td>
<td>$r_{22}$</td>
</tr>
<tr>
<td>$t_3$</td>
<td>$r_{13}$</td>
<td>$r_{23}$</td>
</tr>
<tr>
<td></td>
<td>$\cdots$</td>
<td>$\cdots$</td>
</tr>
<tr>
<td>$t_n$</td>
<td>$r_{1n}$</td>
<td>$r_{2n}$</td>
</tr>
</tbody>
</table>

- Correlation of op risk loss severities in the market-risk manner is “ill-defined”.

<table>
<thead>
<tr>
<th>TIME</th>
<th>ET4</th>
<th>ET7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1$</td>
<td>N/A</td>
<td>$L_{71}$</td>
</tr>
<tr>
<td>$t_2$</td>
<td>N/A</td>
<td>$L_{72,1}, L_{72,2}$</td>
</tr>
<tr>
<td>$t_3$</td>
<td>N/A</td>
<td>$L_{73}$</td>
</tr>
<tr>
<td>$t_k$</td>
<td>$L_{4k}$</td>
<td>$L_{7k}$</td>
</tr>
<tr>
<td>$t_n$</td>
<td>N/A</td>
<td>$L_{7n}$</td>
</tr>
</tbody>
</table>

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Dependence Structure: A technical problem

- In market risk, correlation boils down to correlating the returns of assets.

- In op risk, the modeler has to decide on what to correlate, i.e.:
  - Frequencies (minimal effect on capital)
  - Severities (ill-defined)
  - Aggregate Losses

- Some previous work which touched base with this fact:
  - Powojowski, and et al (2002)
  - Frachot, and et al (2005)
Modeling Dependence Structure

- Limitations of linear correlation
- Interest in “joint extreme values”
- Facilitate multivariate modeling

Scatter Plots

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Dependence Structure at Various Quantiles

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Modeling Dependence Structure: Copula Approach

• Sklar’s Theorem:

\[ F(x) = C_F(F_1(x_1), \ldots, F_d(x_d)), \quad x \in \mathbb{R}^d \]

- Multivariate Distribution Function
- Copula Function
- Marginal Distribution Functions

• Choice of copula: t-copula

\[ C_F = t_{\nu, P}(t_{\nu}^{-1}(u_1), \ldots, t_{\nu}^{-1}(u_d)), \quad u \in [0, 1]^d \]

- Multivariate Student t

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Our Data

• Of the banks that participated in the 2004 LDCE, we will focus on one institution only.

• Data Range:
  ➢ Aggregated Weekly Data (2001-1 to 2004-9) for losses
  ➢ 196 weekly observations for each of 5 Event Types

|----------------|----------------|----------------------------------------|---------------------------------------|-----------------------------------------|

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Exploratory Data Analysis

MEAN EXCESS Plots

ET1
Threshold

ET2
Threshold

ET3
Threshold

ET4
Threshold

ET7
Threshold

EXPONENTIAL QQ-Plots

Ordered Data

Ordered Data

Ordered Data

Ordered Data
Fitting the Tail

Threshold choice: Dividing the body from the tail

Previous Literature

• Hill Plot (Graphical Method)
• Optimization in terms of bias-variance trade-off
• Combined likelihood function (Cibele, et al. 2004)
Combined Likelihood Function

\[ L(x \mid \mu, \sigma, \lambda, \gamma, \tau) = \begin{cases} 
    f_{LN}(x \mid \mu, \sigma) & \text{if } x \leq T_{\lambda} \\
    (1 - F_{LN}(T_{\lambda} \mid \mu, \sigma)) \times f_{GPD}(x - T_{\lambda} \mid \gamma, \tau) & \text{if } x \geq T_{\lambda}
\end{cases} \]

- Treat threshold as a parameter.

- Estimate the threshold in addition to distribution parameters within the MLE process.
Fitting Severities (ET4)

Body Histogram & Fitted Lognormal

- Lognormal
- Kernel density

Tail Histogram & Fitted GPD

- GPD, $\gamma=0.713$
- Kernel density

Minimal (-) Log Likelihood

Min at Percentile = 0.835

Normal QQ-Plot for Log(Body)

Quantiles of Input Sample

Ordered Data

GPD QQ-Plot for Exceedances

Quantiles of Input Sample; $\gamma=0.713$
Fitting Severities (ET3)

Minimized (-) Log Likelihood

Histogram and Fitted Lognormal

Normal QQ-Plot for Log(All Losses)
Results:
Value at Risks and Expected Shortfalls*

- Aggregation of “units” using t-copula

<table>
<thead>
<tr>
<th>Model</th>
<th>VaR(_{95%})</th>
<th>VaR(_{99%})</th>
<th>VaR(_{99.9%})</th>
<th>ES(_{95%})</th>
<th>ES(_{99%})</th>
<th>ES(_{99.9%})</th>
</tr>
</thead>
<tbody>
<tr>
<td>All data, lognormal only</td>
<td>0.160</td>
<td>0.208</td>
<td>0.327</td>
<td>0.192</td>
<td>0.259</td>
<td>0.424</td>
</tr>
<tr>
<td>Without 3 outliers, with EVT</td>
<td>0.481</td>
<td>0.731</td>
<td>1.721</td>
<td>0.678</td>
<td>1.148</td>
<td>2.869</td>
</tr>
<tr>
<td>Without 4 outliers, with EVT</td>
<td>0.283</td>
<td>0.354</td>
<td>0.551</td>
<td>0.335</td>
<td>0.442</td>
<td>0.763</td>
</tr>
<tr>
<td>Without 5 outliers, with EVT</td>
<td>0.230</td>
<td>0.271</td>
<td>0.367</td>
<td>0.258</td>
<td>0.312</td>
<td>0.461</td>
</tr>
</tbody>
</table>

*in $ US billions,
Results:  
Diversification Effects

• Assuming perfect correlation for VaR_{99.9\%} across “units of measure”

<table>
<thead>
<tr>
<th>Model</th>
<th>t-copula VaR_{99.9%}</th>
<th>“Simple Sum” VaR_{99.9%}</th>
<th>Diversification Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>All data, lognormal only</td>
<td>0.327</td>
<td>0.362</td>
<td>10.7%</td>
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<tr>
<td>All data, with EVT</td>
<td>28.407</td>
<td>28.535</td>
<td>0.5%</td>
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<tr>
<td>Without 3 outliers, with EVT</td>
<td>1.721</td>
<td>1.761</td>
<td>2.3%</td>
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<tr>
<td>Without 4 outliers, with EVT</td>
<td>0.551</td>
<td>0.590</td>
<td>7.1%</td>
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<tr>
<td>Without 5 outliers, with EVT</td>
<td>0.367</td>
<td>0.399</td>
<td>8.9%</td>
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</tbody>
</table>
Conclusions and Future Work:

- Use EVT judiciously.
- Complement the analysis with Bayesian Methods for outliers.

El-Gamal, Inanoglu and Stengel 2007
Conclusions continued:

• Use test

“The bank’s internal operational risk measurement system must be closely integrated into the day-to-day risk management processes of the bank. Its output must be an integral part of the process of monitoring and controlling the bank’s operational risk profile…”

(Basel II paragraph 666(b))