Yet Another Tractable Approximation for Robust Optimization

Erick Delage, Amir Ardestani-Jaafari
Dept. of Management Sciences
HEC MONTRĒAL

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INTRODUCTION

Inventory problem:

$$\min_{u}. \quad \sum_{t=1}^{T} c_t u_t + K_t 1_{\{u_t>0\}} + \max \left\{ \begin{array}{c} h_t(x_1 + \sum_{j=1}^{t}(u_j - w_j)), \\ -p_t(x_1 + \sum_{j=1}^{t}(u_j - w_j)) \end{array} \right\}$$

- Objective: Minimizing ordering and shortage/holding cost
- Decision variable (u_t): Stock ordered for the beginning of the t-th period
- Source of uncertainty (w_t): Demand during the t-th period
- Questions: How can we address uncertainty with we have only limited information about demand?

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Robust Inventory Problem

Let w be supported on uncertainty set $\mathcal{W}$ and let us be interested in minimizing the worst-case cost over this set

$$\min_{u}. \quad \max_{w\in\mathcal{W}} \sum_{t=1}^{T} c_t u_t + K_t 1_{\{u_t>0\}} + \max\left\{ \begin{array}{c} h_t(x_1 + \sum_{j=1}^{t}(u_j - w_j)), \\ -p_t(x_1 + \sum_{j=1}^{t}(u_j - w_j)) \end{array} \right\}$$

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Robust Inventory Problem

- Stage-wise Robust Counterpart (Bertsimas and Thiele, 2006):

$$\min_{u,y} \quad \sum_{t=1}^{T}(c_t u_t + K_t 1_{\{u_t>0\}} + y_t)$$

$$\text{s. t.} \quad y_t \geq h_t(x_1 + \sum_{j=1}^{t}(u_j - w_j)) \;\; \forall k, t, \;\; \forall w \in \mathcal{W}$$

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- Affinely Adjustable Robust Counterpart (Ben-Tal et al., 2005, 2009, 2011):

$$\min_{u,\lambda,\gamma} \quad \max_{w\in\mathcal{W}} \sum_{t=1}^{T}(c_t u_t + K_t 1_{\{u_t>0\}} + \lambda_t^T w + \psi_t)$$

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OUR INNER MILP MODEL

We focus on the following inner problem (with budgeted uncertainty set):

$$\begin{aligned} \max_{\zeta \in \Re^m}. \quad & \sum_{t=1}^{T} \max_k \alpha_{tk}(x_1 + \sum_{j=1}^{t}(u_j - (\bar{w}_j + \zeta_j \hat{w}_j))) \\ \text{s. t.} \quad & \|\zeta\|_\infty \leq 1 \ , \ \|\zeta\|_1 \leq \Gamma \end{aligned}$$

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$$\begin{aligned} \max_{\zeta \in \Re^m, z} \quad & \sum_{t=1}^{T} \sum_{k} \alpha_{tk} \left(\overbrace{x_1 + \sum_{j=1}^{t} (u_j - \bar{w}_j)}^{\bar{x}_{t+1}(u)} + \sum_{j=1}^{t} \zeta_j \hat{w}_j \right) z_{tk} \\ \text{s. t.} \quad & \|\zeta\|_\infty \leq 1 \ , \ \ \|\zeta\|_1 \leq \Gamma \\ & z_{tk} \in \{0,1\} \, , \forall \, t, k \, , \sum_{k} z_{tk} = 1 \, , \forall \, t \ \ , \end{aligned}$$

After introducing:

- Positive ζ^+ and negative ζ^- parts of $\zeta := \zeta^+ - \zeta^-$
- The linearization $\Delta^+_{tk} := \zeta^+ z_{tk}$, $\Delta^-_{tk} := \zeta^- z_{tk}$

we obtain an equivalent mixed-integer linear program.

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we obtain an equivalent mixed-integer linear program.

Our Inner MILP Model

$$\max_{\zeta\in\Re^m,z,\Delta} \quad \sum_{t=1}^{T}\sum_{k}\alpha_{tk}\left(\bar{x}_{t+1}(u)z_{tk}+\sum_{j=1}^{t}\hat{w}_j(\Delta^+_{jtk}-\Delta^-_{jtk})\right)$$

$$\text{s. t.} \quad \zeta^+\geq 0 \ , \ \zeta^-\geq 0 \ , \ \zeta^+ + \zeta^- \leq 1 \ , \ \sum_j \zeta^+_j + \zeta^-_j = \Gamma$$

$$z_{tk}\in\{0,1\} \ , \ \sum_k z_{tk} = 1 \, , \forall t$$

$$\Delta^+\geq 0 \ , \ \Delta^-\geq 0 \ , \ \Delta^+_{jtk}+\Delta^-_{jtk}\leq z_{tk} \, , \forall j,t,k$$

$$\sum_j \Delta^+_{jtk}+\Delta^-_{jtk} = \Gamma z_{tk} \, , \forall t,k$$

$$\sum_k \Delta^+_{jtk} = \zeta^+_j \ , \ \sum_k \Delta^-_{jtk} = \zeta^-_j \, , \forall j,t$$

This MILP is bounded above by its LP relaxation

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LP Based Robust Counterpart

After taking the dual we obtain the MILP based robust counterpart (MILP-RC):

$$
\begin{aligned}
\min_{\delta,\boldsymbol{u},\gamma,\lambda^+,\lambda^-,\psi,\theta,\bar{x}}. \quad & \sum_{t=1}^{T}(c_t u_t + K_t 1_{\{u_t>0\}} + \gamma_t + \delta_t) + \Gamma\nu \\
\text{s. t.} \quad & \nu + \delta \geq \sum_t \lambda_t^+, \nu + \delta \geq \sum_t \lambda_t^-, \\
& (\psi_{t,k})_j + \theta_{t,k} \geq -(\lambda_t^+)_j + \alpha_{t,k}\hat{w}_j \;\; \forall t, \forall j \leq t, \;\; \forall k, \\
& (\psi_{t,k})_j + \theta_{t,k} \geq -(\lambda_t^-)_j - \alpha_{t,k}\hat{w}_j \;\; \forall t, \forall j \leq t, \;\; \forall k, \\
& (\psi_{t,k})_j + \theta_{t,k} \geq -(\lambda_t^+)_j \;\; \forall t, \forall j > t, \;\; \forall k, \\
& (\psi_{t,k})_j + \theta_{t,k} \geq -(\lambda_t^-)_j \;\; \forall t, \forall j > t, \;\; \forall k, \\
& \gamma_t \geq \mathbf{1}^T\psi_{t,k} + \Gamma\theta_{tk} - \alpha_{t,k}\bar{x}_{t+1} \;\; \forall t, \;\; \forall k, \\
& \psi_{t,k} \geq 0, \theta_{t,k} \geq 0 \;\; \forall t, \forall j \leq t, \;\; \forall k.
\end{aligned}
$$

Relation to AARC

Theorem

The MILP-RC model is equivalent to applying the AARC method to the following lifted version of the robust inventory problem

$$\min_{u} . \quad \max_{(\zeta^+,\zeta^-)\in\mathcal{U}(\Gamma)} \sum_{t=1}^{T} c_t u_t + K_t 1_{\{u_t>0\}}$$

$$+ \max \left\{ \begin{array}{c} h_t(\bar{x}_{t+1}(u) + \sum_{j=1}^{t} \hat{w}_j(\zeta_j^+ - \zeta_j^-)) \,, \\ -p_t(\bar{x}_{t+1}(u) + \sum_{j=1}^{t} \hat{w}_j(\zeta_j^+ - \zeta_j^-)) \end{array} \right\},$$

where

$$\mathcal{U}(\Gamma) := \left\{ (\zeta^+,\zeta^-) \in \Re^d \times \Re^d \,\middle|\, \begin{array}{l} \zeta^+ \geq 0 \,,\ \zeta^- \geq 0 \\ \zeta^+ + \zeta^- \leq 1 \\ \sum_j \zeta_j^+ + \zeta_j^- \leq \Gamma \end{array} \right\}$$

Quality of Approximation

Theorem

- *For any feasible policy u, the following relation between robust cost of approximations is implied*

$$f_{true}(u) \leq f_{MILP-RC}(u) \leq f_{AARC}(u) \leq f_{BT-RC}(u)$$

- *MILP-RC approximation is exact when $\Gamma = 0, \Gamma = 1$ or $\Gamma = T$.*

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EXACTNESS OF APPROXIMATION FOR $\Gamma = 1$

When $\Gamma = 1$, the relaxed feasible set reduces to

$$\zeta^+ \geq 0 \ , \ \zeta^- \geq 0 \ , \ \zeta^+ + \zeta^- \leq 1 \ , \ \sum_j \zeta_j^+ + \zeta_j^- = \Gamma$$

$$\sum_k z_{tk} = 1 \, , \forall \, t$$

$$\Delta^+ \geq 0 \ , \ \Delta^- \geq 0 \ , \ \Delta_{jtk}^+ + \Delta_{jtk}^- \leq z_{tk} \, , \ \forall \, j, t, k$$

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$$\sum_k \Delta_{j,T,k}^+ = \zeta_j^+ \;,\; \sum_k \Delta_{j,T,k}^- = \zeta_j^- \,, \forall j$$

$$\sum_k \Delta_{j,t,k}^+ = \sum_k \Delta_{j,t+1,k}^+ \;,\; \sum_k \Delta_{j,t,k}^- = \sum_k \Delta_{j,t+1,k}^- \,, \forall j, t < T$$

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$$\sum_k \Delta^+_{j,T,k} - \zeta_j^+ = 0 \ , \ \sum_k \Delta^-_{j,T,k} - \zeta_j^- = 0 \, , \forall j$$

$$\sum_k \Delta^+_{j,t,k} - \sum_k \Delta^+_{j,t+1,k} = 0 \ , \ \sum_k \Delta^-_{j,t,k} - \sum_k \Delta^-_{j,t+1,k} = 0 \, , \forall j, t < T$$

These constraints describe a polyhedron with integer vertices.

Robust Multi-Item Newsvendor (integer Γ)

- The following robust multi-item newsvendor problem can also be solved exactly using an MILP based reformulation

$$\max_{x}. \quad \min_{\zeta\in\mathcal{Z}(\Gamma)} \quad \sum_j (r_j - c_j)x_j + \min\left\{ \begin{array}{c} -(r_j - s_j)(x_j - \bar{w}_j - \hat{w}_j\zeta_j)\,, \\ -p_j(\bar{w}_j + \hat{w}_j\zeta_j - x_j) \end{array} \right\}$$

- The following distributionally robust problem can also be solved exactly using MILP based reformulation

$$\max_{x}. \quad \min_{F\in\mathcal{D}} \quad \mathbb{E}_F\left[\sum_j (r_j - c_j)x_j + \min\left\{ \begin{array}{c} -(r_j - s_j)(x_j - \bar{w}_j - \hat{w}_j\zeta_j)\,, \\ -p_j(\bar{w}_j + \hat{w}_j\zeta_j - x_j) \end{array} \right\}\right]$$

where

$$\mathcal{D} = \left\{ F \in \mathcal{M} \;\middle|\; \begin{array}{l} \mathbb{P}_F(\zeta \in \mathcal{Z}(\Gamma)) = 1 \\ \mathbb{E}_F[\zeta] = \mu \\ \mathbb{E}_F[(\zeta - \mu)^+] \le r^+ \\ \mathbb{E}_F[(\mu - \zeta)^+] \le r^- \end{array} \right\}.$$

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RANDOM PROBLEM INSTANCES I

We generated 1000 synthetic inventory problems (10 periods) and compared the proportions of problems for which a given sub-optimality level was achieved

		Proportion of instances		
Γ	Sub-opt.	BT-RC	AARC	MILP-RC
1	$\leq$0.0001%	0.0%	14.1%	100%
	$\leq$1%	0.0%	26.1%	100%
	$\leq$10%	19.9%	88.7%	100%
		Largest gap		
		56.7%	24.9%	0%

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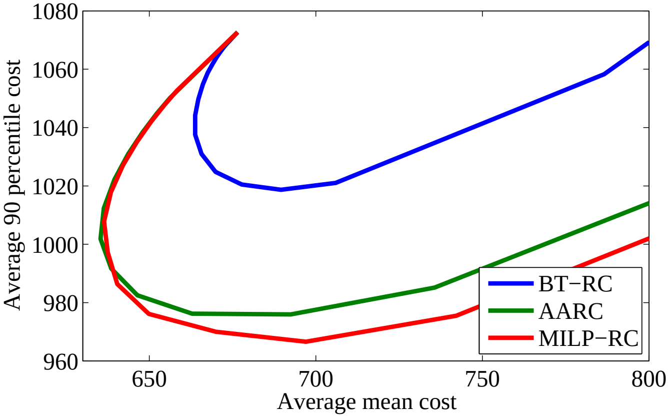
We generated 1000 synthetic inventory problems (10 periods) and compared the proportions of problems for which a given sub-optimality level was achieved

		Proportion of instances		
Γ	Sub-opt.	BT-RC	AARC	MILP-RC
3	$\leq$0.0001%	0.0%	0.4%	52.6%
	$\leq$1%	0.2%	6.2%	90.4%
	$\leq$10%	15.7%	91.1%	100.0%
		Largest gap		
		54.6%	23.0%	4.6%
5	$\leq$0.0001%	0.0%	0.2%	57.3%
	$\leq$1%	0.0%	9.3%	96.6%
	$\leq$10%	16.5%	98.6%	100.0%
		Largest gap		
		52.2%	14.9%	2.6%

Random Problem Instances II

Average expected cost versus average 90th percentile achieved when adjusting the level of conservativeness Γ in an inventory problem with 100 periods.



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Conclusions

- We proposed yet another tractable approximation for robust inventory problem that is based on a mixed-integer linear program reformulation
- We show a strong connection with AARC methods and special cases for which approximation is exact
- Numerical study suggests that MILP-RC is often very close to optimal and can help achieve better trade-off between returns and risks.

Questions & Comments ...

... Thank you!