Robust Optimization of a Class of Bi-Convex Functions

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INTRODUCTION

- Inventory problem:

$$\begin{aligned} \min_{u,v,x} \quad & \sum_{t=1}^{T} (c_t u_t + K_t v_t + \max(h_t x_t, -p_t x_t)) \\ \text{s. t.} \quad & x_t = x_0 + \sum_{j=1}^{t} (u_j - \zeta_j) \ \ \forall t \\ & 0 \leq u_t \leq M v_t \ \ , \ \ v_t \in \{0,1\} \ \ \forall t \end{aligned}$$

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- Multi-item newsvendor:

$$\max_{x \in \mathcal{X}} \; \sum_i (r_i - c_i)x_i - \max\{(r_i - s_i)(x_i - \zeta_i)\,,\, p_i(\zeta_i - x_i)\}$$

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- Multi-attribute utility theory, where $\mathcal{U}_i(y) := \min_k \alpha_k y + \beta_k$:

$$\max_{x \in \mathcal{X}} \sum_{i=1}^{N} w_i \mathcal{U}_i(\boldsymbol{c}_i(\zeta)^T \boldsymbol{x} + d_i(\zeta))$$

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Robustifying a Class of Bi-Convex Functions

Many robust optimization problem take a special form of bi-convex objective:

$$(RP) \quad \min_{x\in\mathcal{X}}. \quad \max_{\zeta\in\mathcal{Z}} \sum_i h_i(x,\zeta) \ ,$$

where h_i is piecewise-linear convex function in either variable.

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- While lots of work on bi-affine, or convex-concave functions, very little about bi-convex function
- Main reason might be that evaluating the inner problem is generally intractable. Take 3-SAT problem for example:

$$\begin{aligned} \max_{\zeta} \quad & \sum_{i=1}^{\#\text{Clauses}} \max_{k \in \text{Clause}_i} \zeta_k \qquad \geq \#\text{Clauses} \quad ??? \\ \text{s. t.} \quad & 0 \leq \zeta_k \leq 1 \\ & \zeta_k = 1 - \zeta_{k+K} \,, \ \forall k \in \{1, ..., K\} \end{aligned}$$

Recently in the Literature

Affinely Adjustable Robust Counterpart (Ben-Tal et al. (2004, 2005, 2009)):

- Suggests using affine function $h_i(x, \zeta) \approx \zeta^T \pi_t + \pi_t^0$ as long as $\zeta^T \pi_t + \pi_t^0 \geq h_i(x, \zeta) \, \forall \zeta$.

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- Reviews exact approaches
  - Vertex enumeration : possibly exponentially many
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- Reviews exact approaches
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  - Cutting plane methods : CPLEX can solve inner problem
- Presents tractable reformulation for a few very special case

Our lens : Cast Inner Problem as a MILP

- The inner problem can be formulated as

$$\max_{\zeta \in \mathcal{Z}} \sum_i \max_{z_i \in \{0,1\}^K, \mathbf{1}^T z_i = 1} \sum_k z_{i,k}(c_{i,k}(x)^T \zeta + d_{i,k}(x)) \ ,$$

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which in turn can be cast as a MILP (with $\Delta_{i,k} := \zeta z_{i,k}$)

$$\begin{aligned} \max_{\zeta \in \mathcal{Z}, z, \Delta} \quad & \sum_i \sum_k c_{i,k}(x)^T \Delta_{i,k} + d_{i,k}(x) z_{i,k} \\ \text{s. t.} \quad & \sum_k \Delta_{i,k} = \zeta \ , \ \forall i \\ & |\Delta_{i,k}| \leq M z_{i,k} \ , \ \forall i, k \\ & \sum_k z_{i,k} = 1 \ , \ \forall i \ \ \& \ \ z_{i,k} \in \{0,1\} \ , \ \forall i, k \end{aligned}$$

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- If we relax this MILP and derive its robust counterpart we get a conservative approximation of RP
- Questions:
  - Can we bound the quality of these approximations ?
  - How do these approximations compare to AARC ?
  - Can we be more sophisticated in “convexifying” the MILP ?
  - How does such conservative approximations perform in practice ?

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Our Inner MILP Model

We focus on the following inner problem (with budgeted uncertainty set):

$$\begin{aligned} \max_{\zeta \in \Re^m} \quad & \sum_{i=1}^{N} \max_k c_{i,k}^T \zeta + d_{i,k} \\ \text{s. t.} \quad & \|\zeta\|_\infty \le 1 \\ & \|\zeta\|_1 \le \Gamma \end{aligned}$$

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$$z_{ik}(\zeta^+ + \zeta^-) \leq z_{i,k}\mathbf{1} \ \ \& \ \ \sum_k z_{i,k}(\mathbf{1}^T\zeta^+ + \mathbf{1}^T\zeta^-) \leq \Gamma z_{i,k}$$

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we obtain an equivalent mixed-integer linear program.

OUR INNER MILP MODEL

Our inner problem is equivalent to:

$$
\begin{aligned}
(MILP) \quad \max_{z,\zeta^+,\zeta^-,\Delta^+,\Delta^-}. \quad & \sum_{1=1}^{N}\sum_{k} c_{i,k}(x)^T(\Delta^+_{i,k} - \Delta^-_{i,k}) + d_{i,k}(x) z_{i,k} \\
\text{s. t.} \quad & 0 \leq \zeta^+ \;\; \& \;\; 0 \leq \zeta^- \;\; \& \;\; \zeta^+_j + \zeta^-_j \leq 1 \\
& \mathbf{1}^T(\zeta^+ + \zeta^-) \leq \Gamma \\
& \sum_k z_{i,k} = 1 \, , \forall i \\
& \sum_k \Delta^+_{i,k} = \zeta^+ \;\; \& \;\; \sum_k \Delta^-_{i,k} = \zeta^- \quad , \forall i \\
& 0 \leq \Delta^+_{i,k} \;\; \& \;\; 0 \leq \Delta^-_{i,k} \;\; \& \;\; \Delta^+_{i,k} + \Delta^-_{i,k} \leq z_{i,k} \\
& \sum_j (\Delta^+_{i,k})_j + (\Delta^-_{i,k})_j \leq \Gamma z_{i,k} \\
& z_{i,k} \in \{0, 1\} \, .
\end{aligned}
$$

MILP BASED ROBUST COUNTERPART

After relaxing the binary constraints and taking the dual we obtain:

$$
\begin{aligned}
\min_{\delta,\nu,\gamma,\lambda^+,\lambda^-,\psi,\theta} \quad & \mathbf{1}^T\delta + \Gamma\nu + \mathbf{1}^T\gamma \\
\text{s. t.} \quad & \nu \geq \sum_i \lambda_i^+ - \delta \\
& \nu \geq \sum_i \lambda_i^- - \delta \\
& \gamma_i \geq \mathbf{1}^T\psi_{i,k} + \Gamma\theta_{i,k} + d_{i,k}(\boldsymbol{x}) \ \ \forall i,k \\
& \theta_{i,k} \geq -\lambda_i^+ - \psi_{i,k} + c_{i,k}(\boldsymbol{x}) \ \ \forall i,k \\
& \theta_{i,k} \geq -\lambda_i^- - \psi_{i,k} - c_{i,k}(\boldsymbol{x}) \ \ \forall i,k \\
& \delta \geq 0\,,\ \nu \geq 0\,,\ \psi \geq 0\,,\ \theta \geq 0
\end{aligned}
$$

which is for any x an upper bound on true value of inner problem and can be reintegrated to outer problem.

MILP BASED ROBUST COUNTERPART

Introducing the linear programming based robust counterpart :

$$
\begin{aligned}
(LP-RC) \quad \min_{x\in\mathcal{X},\delta,\nu,\gamma,\lambda^+,\lambda^-,\psi,\theta}. \quad & 1^T\delta + \Gamma\nu + \mathbf{1}^T\gamma \\
\text{s. t.} \quad & \nu \geq \sum_i \lambda_i^+ - \delta \\
& \nu \geq \sum_i \lambda_i^- - \delta \\
& \gamma_i \geq 1^T\psi_{i,k} + \Gamma\theta_{i,k} + d_{i,k}(\boldsymbol{x}) \;\; \forall i,k \\
& \theta_{i,k} \geq -\lambda_i^+ - \psi_{i,k} + c_{i,k}(\boldsymbol{x}) \;\; \forall i,k \\
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& \delta \geq 0\,,\; \nu \geq 0\,,\; \psi \geq 0\,,\; \theta \geq 0
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QUALITY OF LP-RC

Let η be the integrality gap for MILP, then

$$V^*_{RP} \leq V_{RP}(x_{LP\text{-}RC}) \leq V^*_{LP\text{-}RC} \leq \eta V^*_{RP} \; .$$

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$$V^*_{RP} \leq V_{RP}(x_{LP\text{-}RC}) \leq V^*_{LP\text{-}RC} \leq \eta V^*_{RP} \ .$$

Furthermore, approximation is exact (i.e. $\eta = 1$) if one of the following set of conditions is satisfied:

1. $\underline{\Gamma = 1}$
2. $\Gamma = m$ *and uncertainty is "additive",*
   i.e. for every (i, k) pair we have $c_{i,k} = \alpha_{i,k}(\boldsymbol{x}) \sum_{l<i} \beta_l(\boldsymbol{x}) \boldsymbol{e}_l$
3. $\Gamma \in \mathbb{N}$ *and uncertainty is "decomposable",*
   i.e. for every (i, k) pair we have $c_{i,k} = \alpha_{i,k}(\boldsymbol{x}) \boldsymbol{e}_i$

Example : Robust Multi-Item Newsvendor

- The following robust multi-item newsvendor problem can be solved exactly using an LP-RC

$$\max_{x \in \mathcal{X}} \min_{\zeta \in \mathcal{Z}(\Gamma_{\mathbb{N}})} \sum_i (r_i - c_i)x_i - \max \left\{ \begin{array}{c} (r_i - s_i)(x_i - \bar{w}_i - \hat{w}_i \zeta_i) \,, \\ p_i(\bar{w}_i + \hat{w}_i \zeta_i - x_i) \end{array} \right\}$$

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- The following distributionally robust problem can also be solved exactly using an LP-RC

$$\max_{x\in\mathcal{X}}. \min_{F\in\mathcal{D}} \mathbb{E}_F\left[\sum_i (r_i - c_i)x_i - \max\left\{ \begin{array}{c} (r_i - s_i)(x_i - \bar{w}_i - \hat{w}_i\zeta_i)\,, \\ p_i(\bar{w}_i + \hat{w}_i\zeta_i - x_i) \end{array} \right\}\right]$$

where

$$\mathcal{D} = \left\{ F \in \mathcal{M} \,\middle|\, \begin{array}{l} \mathbb{P}_F(\zeta \in \mathcal{Z}(\Gamma_{\mathbb{N}})) = 1 \\ \mathbb{E}_F[\zeta] = \mu \\ \mathbb{E}_F[(\zeta_i - \mu_i)^+] \leq r_i^+\,, \forall i \\ \mathbb{E}_F[(\zeta_i - \mu_i)^-] \leq r_i^-\,, \forall i \end{array} \right\}.$$

Relation to AARC scheme

Given an adversarial problem of the type

$$\max_{\zeta\in\mathcal{A}}. \sum_{1=1}^{N} \max_k c_{i,k}^T\zeta + d_{i,k} \text{ , where } \mathcal{A} = \{\zeta | \boldsymbol{A}\zeta \leq \boldsymbol{b}\} \text{ is bounded}$$

the following AARC and MILP based bounds are equivalent

$$\begin{aligned} \min_{\lambda,\gamma}. \; \max_{\zeta\in\mathcal{A}} \; & \sum_i \lambda_i^T\zeta + \gamma_i \\ \text{s. t. } \; & \lambda_i^T\zeta + \gamma_i \geq c_{i,k}^T\zeta + d_{i,k} \\ & , \forall\, i, k, \; \forall\, \zeta \in \mathcal{A} \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} \max_{z,\zeta,\Delta}. \; & \sum_{i=1}^{N}\sum_k c_{i,k}^T\Delta_{i,k} + d_{i,k}z_{i,k} \\ \text{s. t. } \; & \boldsymbol{A}\zeta \leq \boldsymbol{b} \\ & z_{i,k} \geq 0 \text{ , } \sum_k z_{i,k} = 1 \text{ , } \forall\, i \\ & \sum_k \Delta_{i,k} = \zeta \text{ , } \forall\, i \\ & \boldsymbol{A}\Delta_{i,k} \leq \boldsymbol{b}z_{i,k} \text{ , } \forall\, i, k \end{aligned}$$

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INTEGRALITY GAPS

Empirical evaluation of integrality gap and resolution time of different conservative approximations for a set of randomly generated adversarial problems

Size		AARC*	LP-RC*
N=16	CPU time	0.17 sec	0.18 sec
	Avg. gap	1.82	1.49
N=32	CPU time	10 sec	10 sec
	Avg. gap	2.61	1.96
N=64	CPU time	34 sec	70 sec
	Avg. bound	-	- 26%

*Solved with CPLEX V12.4

ROBUST INVENTORY MANAGEMENT

Comparison of average performance over a set of 1000 randomly generated inventory problem instances (T=10).

Γ		$x^*_{\text{BT-RC}}$	x^*_{AARC}	$x^*_{\text{LP-RC}}$	x^*_{Exact}
1	Approx. w.-c. cost	4404	3571	3430	3430
	Actual w.-c. cost	4000	3569	3430	3430
	Sub-opt. gap	16.6%	4.4%	0%	-
4	Approx. w.-c. cost	7830	6271	5977	5967
	Actual w.-c. cost	6999	6222	5977	5967
	Sub-opt. gap	17.5%	4.3%	0.2%	-

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- Robust optimization of functions that are non-linear & non-concave in the parameters is an important field of research
- If function decomposes as sum of piecewise affine functions, then a MILP based robust counterpart can be derived
- This is especially effective for the budgeted uncertainty set
- MILP theory can be used to justify and guide the AARC scheme

… AND A SHORT ADVERTISEMENT

- I am currently actively seeking motivated students to supervise on research questions associated to the following topics:
  - Robust optimization
  - Stochastic programming
  - Distributionally robust optimization
  - Data-driven optimization
  - Optimization under ambiguous preferences
- Feel free to contact me for more details...