

# Endogenous Borrowing Constraints and Consumption Volatility in a Small Open Economy\*

Carlos de Resende

Université de Montréal

First Draft: November 2004    This Draft: January 2005  
[PRELIMINARY; COMMENTS WELCOME]

## Abstract

Consumption volatility relative to output volatility is consistently higher in emerging economies than in developed economies. One natural explanation is that, emerging economies are more likely to face borrowing constraints and, as a consequence, find it more difficult to use international capital markets to smooth consumption. The goal of this paper is to investigate how much this mechanism alone can account for the relative consumption volatility differential between emerging and developed economies. The theoretical approach relies on a standard dynamic general equilibrium model of a small open endowment economy that is subject to an endogenous borrowing constraint. The borrowing constraint makes the small economy exactly indifferent between two options: *i*) repaying its external debt or *ii*) defaulting and having to live in financial autarky in the future. The model for the constrained economy is calibrated to match Brazilian data during the period 1980-2001. The findings suggest that the model is capable of accounting for about 25% of the observed relative consumption volatility differential.

*JEL Classification:* F32, F34, F41

*Key Words:* sovereign debt, consumption volatility, business cycles, small open economy.

---

\*The author gratefully acknowledges the financial support from CAPES (Brazilian Ministry of Education) and from the Bank of Canada, during a summer internship program.

# 1 Introduction

The purpose of this paper is to study the differences in consumption volatility observed in the data from emerging and developed small open economies. As a general rule, empirical evidence from business cycle statistics across countries suggests that the economic activity is more volatile in emerging economies than in more developed ones. In particular, the data show that output volatility is higher in the former than in the later. Considering that output volatility may be interpreted as the underlying volatility of the economy, it is not a surprise that most macroeconomic variables, including private consumption, also tend to be more volatile in emerging economies. However, and more importantly for the purposes of this paper, standard business cycle statistics show that, even if one controls for the underlying volatility of the economy, i.e. output volatility, the (relative) volatility of consumption is still higher in emerging economies than in small open developed economies.

Section 2 of this paper presents empirical evidence of consumption and output volatilities for two groups of small open economies. For a sample of five emerging economies (Argentina, Brazil, Mexico, South Korea and South Africa), and three small open developed economies (Australia, Canada and Sweden), during the period 1980:1-2001:4, the volatility of consumption relative to output volatility is, on average, 63.4% higher in the emerging economies' sub-sample. These findings are robust to the sample period as well as to the data frequency and confirm the results implied by studies containing business cycle statistics for developed economies [for example, Cooley and Prescott (1995), for the United States; Mendoza (1991), for Canada; and Correia, Neves and Rebelo (1995), for Portugal] and emerging economies [for example, Mendoza (2001), for Mexico; and Neumeyer and Perri (2004) and Aguiar and Gopinath (2004), for Argentina].

Although, it has been shown [see Neumeyer and Perri (2004) and Calvo, Leiderman and Reinhart (1993), for example] that the excess volatility of business cycles in emerging economies may have a lot to do with a possible dominant role played by external shocks that impact these economies,<sup>1</sup> in the context of a small open economy model, one natural theoretical explanation for this fact is that, perhaps, the two different groups of countries, emerging and developed economies, are subject to different external constraints in terms of their ability to borrow in the international capital markets. The obvious intuition on the relationship between borrowing constraints, including the type of constraint discussed here, and the volatility of consumption is that they may limit consumption smoothing by risk-averse agents and induce a more volatile consumption path.

If, in fact, emerging markets are different from developed economies in that they have a lower ability to use international credit markets to smooth consumption, then the data should reveal noticeable

---

<sup>1</sup>Neumeyer and Perri (2004), using Argentina as a benchmark, stress the important role that shocks to the idiosyncratic interest rate (international interest rate plus a country risk factor) may play on the business cycle volatility in emerging economies. Calvo, Leiderman and Reinhart (1993), on the other hand, suggest that external factors such as macroeconomic variables in the United States, capital flows in particular, may be very important to account for macroeconomic developments in Latin America.

differences in consumption volatility in those two groups of countries, as it seems to be the case.<sup>2</sup> This empirical evidence has one important implication for the use of theoretical models applied to the study of emerging economies. If one wants to explain the high volatility observed in their business cycles, particularly in consumption, then this external borrowing constraint has to be taken into consideration and the typical assumption of unlimited access to perfect world capital markets, which is implausible in this context, must be abandoned. That is precisely the spirit of the theoretical model discussed here.

The paper is concerned with answering the following questions: is a borrowing constraint capable of explaining the differences in consumption volatility (relative to output volatility) observed in the data from small open emerging and developed economies? How much of the observed differential can be accounted for by the borrowing constraint alone?

More specifically, in order to account for the facts, the paper proposes a dynamic general equilibrium model featuring two goods (tradable and nontradable goods) in an endowment economy that is subjected to two kinds of imperfections in international capital markets: *i*) the lack of any contingent assets (incomplete markets) and *ii*) a financial friction that may restrict international borrowing. The financial friction considered here is an endogenous borrowing constraint in the tradition of Eaton and Gersovitz (1981) [see also Kletzer (1984)] which has been recently discussed in the international macroeconomics literature [for example, Arellano (2004) and Aguiar and Gopinath (2004)]. In their paper, Eaton and Gersovitz were motivated by the apparent paradox of why sovereign governments ever choose to repay their debt even when there is no credible enforcement mechanism in the international markets. Although there is some controversy [see Bulow and Rogoff (1989)], their answer to the “paradox” is that the threat of financial autarky induces sovereign governments to make repayments on its foreign debt in order to preserve a “reputation collateral” needed for future borrowing [see also Cole and Kehoe (1995, 1998), Cole, Dow, and English (1995) and Grossman and Han (1999)]. Borrower countries know that if they default, lenders will be less willing to lend to them in the future. The potential exclusion from future borrowing is a cost to a small open economy populated by risk-averse agents because, in financial autarky, their ability to smooth consumption over time and over different states of nature is compromised. Default occurs whenever the present value of the (instantaneous) benefits of not paying

---

<sup>2</sup>The proposition that the access to international capital and credit markets is more restrict for emerging economies in comparison to, say, OECD countries does not seem very difficult to accept. It is hard to think about what type of indicator could be interpreted as a direct evidence of that. One could mention the lower credit ratings and the higher interest rates paid by emerging economies on their sovereign debt as indirect evidence that they are more likely to be credit constrained than developed economies. Events such as the Asian crisis during the late nineties, the frequent balance-of-payments crises experienced by emerging economies which usually trigger bail outs from the IMF, their not uncommon decisions to default on their external debt (the most recent been Argentina’s default in 2002), in a sense, could also be thought as indirect evidence that emerging economies are different in their access to international capital markets. Not surprisingly, those events gave enough motivation for a growing literature that deals with the specificities of emerging markets in explaining, among other things, how changes in their access to international credit may affect the domestic economies in various dimensions. This literature includes papers on currency crises [see Eichengreen, Rose and Wyplosz (1995), Kaminsky, Lizondo and Reinhart (1997) and, Frenkel and Rose (1996)], balance-of-payments crises [Kaminsky and Reinhart (1999), Calvo and Vegh (1999) and Edwards’ (2001)] and “Sudden Stops” [see Calvo (1998a), Calvo (1998b) and Calvo and Reinhart (1999)].

the due services of the external debt outweighs the (intertemporal) losses in utility that will take place during an autarky state. International lenders, aware of the potential for debt repudiation, will set in motion a defensive rule to receive back the full amount of any conceded loans, including interests at the international interest rate, in all states of nature, and will never lend funds in excess of the level of credit that leaves the borrower country exactly indifferent between defaulting and fully repaying its debt.

Although some aspects of the more volatile economic fluctuations verified in emerging economies have been already studied in the literature on emerging markets' crises, a systematic attempt to explain the differences in relative consumption volatility observed in the data from emerging and developed small open economies, using a non-*ad hoc*, endogenous borrowing constraint, has not yet been done. Using data for 1994:1-2000:2, from a similar set of countries as the used in this paper, Neumeyer and Perri (2004) present a broader set of facts about business cycle volatility, including information on relative consumption volatility. They found the average relative consumption volatility for their sample of emerging economies to be 78.2% higher than that of Canada (1.55 against 0.87), which is in line with the evidence presented in the Section 2 of this paper. However, their explanation for the facts relies on an exogenous stochastic process for the idiosyncratic international interest rate faced by the small economy. The exogenous positive shocks on the interest rate could be interpreted as a more stringent borrowing constraint that imposes additional costs to smoothing consumption through borrowing in the international capital markets, but the mechanism is not a result from the optimizing behavior from the part of lenders or borrowers.

Mendoza (2001) used an *ad hoc* borrowing constraint to explain "Sudden Stops" in capital flows to emerging economies. The constraint takes the form of a collateral, whereby the country must commit a constant (exogenous) proportion of its output before contracting any external credits. Although his model is successful in explaining the abrupt swings in capital inflows to the small emerging economy, it generates an insignificant difference in the relative volatility of consumption between the economies with and without the financial constraint.

Borrowing constraints are a way to ration out the amount of credit available to a particular economy through restriction in quantities. One could also think that, in reality, not only the quantity of credit is to be directly rationed but that the prices, i.e. the idiosyncratic interest rate that the country pays on its debt, must impose additional restrictions on the equilibrium amount of debt. One approach that allows for the interest rate on the external debt to be endogenously determined, along with the level of debt, in a model with the same kind of borrowing constraint used in this paper, is pursued by Arellano (2004) and, also, by Aguiar and Gopinath (2004). They use the same insights that motivated this paper's endogenous borrowing constraint - in their case to generate a positively sloped "supply of debt" -, in a model that allows for default to occur in equilibrium. However, these papers do not discuss how the same model would behave without the financial constraint nor try to explain the potential

differences in the relative consumption volatility in constrained and unconstrained economies.

Economists have been trying to understand why emerging economies are so vulnerable to all sorts of crises, from balance-of-payments' crises and Sudden Stops to banking crises and currency crashes. Although the profession's explanations about the underlying mechanisms of these events have improved over the last two decades, no definitive answer has yet been presented. It is likely that the road map to a more complete understanding of these phenomena includes a clear identification of the particularities, if any, that emerging economies have in comparison to the developed world. In this sense, because it explicitly proposes an explanation to an important aspect of the differences between emerging and developed economies, the paper has a clear contribution for the literature on emerging economies.

The rest of the paper is organized as follows. Section 2 discusses evidence of the differences in output and consumption volatility in small open economies, divided in "emerging" and "developed" groups. Section 3 presents the theoretical model featuring the endogenous borrowing constraint. Section 4 displays a discussion of the numerical solution of the model, its calibration and some simulation results. Section 5 concludes.

## 2 Empirical Evidence on Consumption Volatility Across Emerging and Developed Economies

Table 1 displays evidence of the higher ratio of consumption volatility to output volatility, at business cycle frequencies, in emerging economies *vis-à-vis* small open developed countries. The table was constructed from quarterly data on real output and real private consumption (as deflated by the CPI),<sup>3</sup> for five emerging economies (Argentina, Brazil, Mexico South Africa and South Korea) and three small open developed economies (Canada, Australia and Sweden). The sample of countries was selected according to data availability for a relatively long period (here, 1980:1 - 2001:4). All data, expressed in per capita values at average prices of 1995, come from the International Monetary Fund's *International Financial Statistics* (IFS/IMF) dataset,<sup>4</sup> with the exception of Brazilian and Argentinean data, which come from national sources.<sup>5</sup> The series were transformed previously to the computation of their second moment statistics, as follows. First, all the variables were expressed in logarithms. Second, a seasonal adjustment

---

<sup>3</sup>Typically, RBC statistics on consumption exclude the consumption of durable goods (since it behaves closely to investment, being more volatile) from the consumption series. We could not yet find the required information to do the same here. Probably, for the same reason, Neumeyer and Perri's (2004) similar empirical exercise also only considered total consumption. A potential problem of this procedure would arise if, for instance, durable consumption accounts for a higher proportion of the total consumption in emerging economies than in the more developed countries.

<sup>4</sup>The IMF/IFS dataset also have information about the required time series (in nominal terms, to be deflated by the CPI), for the sample period considered here, from the Euro Zone countries. However, after the introduction of the Euro, nominal series of consumption started to be presented in the new currency and since adjustments would have to be made, using an exchange rate (Euro against former national currencies), what could introduce additional volatility in the series, we decided not to include these countries in the sample of developed small open economies.

<sup>5</sup>Argentinean data came from the *Dirección Nacional de Cuentas Nacionales* (DNCN) and Brazilian data was collected from the *Instituto de Pesquisa Economica Aplicada* (IPEA) at [www.ipeadata.gov.br](http://www.ipeadata.gov.br) and from the Central Bank of Brazil. Both datasets are consistent with IFS/IMF's data, when they happen to overlap.

on the log-variables was implemented using the multiplicative ratio-to-moving-average method. Finally, a smooth trend was subtracted by using the Hodrick-Prescott filter with a smoothing parameter of 1600 for quarterly data.

From the Table 1 it seems clear that:

1. The volatility of the Gross Domestic Product (GDP), denoted as  $\sigma_y$  in Table 1, is almost twice as high in emerging economies compared with the developed economies. The average  $\sigma_y$  is 3.5% and 1.8%, respectively. The exception to the pattern is South Africa;
2. The consumption volatility ( $\sigma_c$ ) is also higher in emerging economies. The difference in average  $\sigma_c$  between the two groups amounts to 186.7% (4.3% against 1.5%). Given the results for the output volatility, this is not a surprise, since  $\sigma_y$  may be interpreted as the underlying volatility of the economy, affecting the volatility of all other variables;
3. The relative volatility of consumption tends to be higher than 1 in emerging economies and lower than 1 in developed economies, with an average that is 63.4% higher in emerging economies in comparison to developed economies (1.29 against 0.79).

Table 1.

Output and Consumption Volatility (1980.1 - 2001.4)			
Emerging Economies	$\sigma_y$ (%)	$\sigma_c$ (%)	$\sigma_c/\sigma_y$
Argentina	5.34	5.61	1.051
Brazil	2.95	3.86	1.308
Mexico	2.66	4.92	1.850
South Africa	1.82	2.38	1.308
South Korea	4.93	4.59	0.931
average	3.54	4.27	1.289
Developed Economies			
	$\sigma_y$ (%)	$\sigma_c$ (%)	$\sigma_c/\sigma_y$
Australia	1.46	1.03	0.705
Canada	2.17	1.46	0.673
Sweden	1.88	1.86	0.989
average	1.84	1.45	0.789

Although South Korea, in the “emerging’s group”, and Sweden, in the “developed group” share some similarities, the results show that, even when controlling for the underlying volatility in the economy,

consumption is more volatile in emerging economies than in more developed ones. A similar exercise, using data from 1990:1-2001:4, reveals the same overall picture: the average  $\sigma_c/\sigma_y$  is found to be 1.31 in the emerging economies' group and 0.77 in the developed economies' sub-sample (70.1% difference).

The results shown above are also consistent with those obtained by Neumeyer and Perri (2004). They used basically the same sample period in a comparison between Argentinean and Canadian business cycles' statistics<sup>6</sup> and found results in the same order of magnitude of those in Table 1. In addition, they also did comparisons between Canada and five emerging countries (the same used here, with the exception of South Africa, instead of which they used the Philippines) for the period 1994:1 - 2000:2 and, again, their results were in the same direction and magnitude of the ones in this paper.

Table 2 displays the volatilities of output and consumption, as well as their ratio, reported in Neumeyer and Perri (2004) and in other selected studies. Note that the reported relative volatility of consumption seems to be higher in small open emerging economies than in developed economies. The information on Tables 1 and 2 seems to indicate that the basic result - a higher relative consumption volatility in emerging economies in comparison with developed economies - is robust to the sample of countries, frequency of the data and sample period.

Table 2.

Examples of Output and Consumption Volatility Statistics in the Literature				
United States	$\sigma_y$ (%)	$\sigma_c$ (%)	$\sigma_c/\sigma_y$	Data
Cooley and Prescott (1995)	1.72	1.27	0.74	1954:1-1991:11
Small Open Developed Economies				
	$\sigma_y$ (%)	$\sigma_c$ (%)	$\sigma_c/\sigma_y$	Data
Canada: Mendoza (1991)	2.81	2.46	0.88	1945-1985
Portugal: Correia, Neves and Rebelo (1995)	3.78	3.17	0.84	1958-1991
Canada: Neumeyer and Perri (2004)	1.17	1.02	0.87	1994:1-2002:2
Emerging Economies				
	$\sigma_y$ (%)	$\sigma_c$ (%)	$\sigma_c/\sigma_y$	Data
Mexico: Mendoza (2001)	2.73	3.35	1.23	1980:1-1997:4
average of 5 EE: Neumeyer and Perri (2004)	2.94	4.62	1.57	1994:1-2002:2
Argentina: Aguiar and Gopinath (2004)	4.08	4.86	1.19	1983:1-2000:2

The next section discuss a possible theoretical explanation for this empirical evidence.

<sup>6</sup>Although we both used basically the same data, they adjusted the series of total consumption to include government consumption, changes in inventories and a statistical discrepancy, in order to be consistent with the only available quarterly data for Argentina previously to 1993. Here, I used the information on annual series for Argentina to exclude these items from the total consumption previously to 1993, by assuming that the same proportions observed in annual data are verified in all quarters of a given year.

### 3 The Model

In this section, a dynamic general equilibrium model of a small open economy is presented. The model departs from traditional small open economy models with perfect capital mobility in that it allows for the possibility that the economy can choose optimally between defaulting or repaying its external debt. This feature introduces an endogenous borrowing constraint in the tradition of Eaton and Gersovitz (1981) and Kletzer (1984).

Consider a small open economy, where a central planner seeks to maximize the lifetime utility of a representative agent. The agent enjoys utility from a consumption index  $c_t$ , which is a composite from the consumption of tradable ( $c_t^T$ ) and nontradable goods ( $c_t^N$ ). There is no production and the agent receives endowments of nontradable ( $Y^N$ ), assumed constant for simplicity, and tradable goods,  $Y_t^T = Y^T + z_t$ , which is random and fluctuates around the average level,  $Y^T$ , according to a stochastic process for the production shock,  $z_t$ .

International asset/capital markets are incomplete and no contingent contracts are signed.<sup>7</sup> At the beginning of every period  $t$ , the economy inherits a one-period external debt  $d_{t-1}$ , expressed in units of the tradable good, contracted at  $t - 1$  at the exogenous foreign interest rate  $r$ , and realizes the levels of the endowments. Denote  $S(d_{t-1}, z_t) = \{d_{t-1}, z_t\}$  to be the current state of the economy, at time  $t$ . Once the state  $S(d_{t-1}, z_t)$  is known, the central planner decides whether the outstanding debt including interest services,  $(1 + r)d_{t-1}$ , is going to be paid or defaulted. The central planner's decision about the full repayment of the external debt is based on the relative incentives to do so, as follows. The cost of defaulting at time  $t$  is to stay out of the international capital markets from  $t$  onwards, renouncing the possibility of using international borrowing to smooth consumption.<sup>8</sup> Implicitly, we are assuming that default against one lender is taken as a signal by all other international lenders and that they will not only exclude the defaulting country from borrowing again, but will seize its assets if the country eventually tries to invest any assets in another international financial institution. Given the current state, let  $V_t^D$  and  $V_t^R$  be the indirect utility of defaulting at  $t$  (and having to consume the endowments  $Y^N$  and  $Y_t^T$  from this time onwards) or fully repaying the external debt and continuing to be able to borrow abroad. Default at time  $t$  is chosen by the country whenever  $V_t^D > V_t^R$ .

The international capital market consists of lenders who want to receive back the full amount of their loans in all possible states of nature. The idea is to find a borrowing constraint which, at each date and

---

<sup>7</sup>Kehoe and Levine (1993) discuss endogenous borrowing constraints with complete markets. The assumption of incomplete markets seems to fit better the evidence that countries tend to default during recessions. With the insurance given by contingent assets, agents tend to leave the credit contract (that is, to default) during "good times", when they have to make payments as opposed to the "bad times", when they receive the insurance.

<sup>8</sup>The assumption that countries that default will stay out of the international capital markets forever is clearly at odds with the evidence that shows many of defaulting countries being able to borrow again after some renegotiation of their debts. In terms of the model presented in this paper, this assumption means, perhaps, a higher penalty for defaulting countries than what actually occurs. The standard and simple way of dealing with this issue [see Arellano (2004)] is to introduce an exogenous probability of leaving the default state at each period. In order not to introduce unnecessary *ad hoc* features that do not change the underlying economics behind the model, the assumption stays.



state, will induce the country to participate in the asset market instead of defaulting. One could think of the international lenders as a representative international investor, or an outside foreign agency, that have full information about the domestic economy (for instance, its current state and the specification of the borrower/consumer’s preferences) and the borrower’s optimization problem. The only role played by the foreign agents is to set up and enforce the credit limits. Should the sovereign country default on its external debt, the “agency”, or the pool of investors, would exclude it from intertemporal asset trading forever and, as a result, the country would be deprived of the risk sharing opportunities in the future. Aware of potential debt repudiation, in order to prevent default, the foreign agents will impose a borrowing constraint to the small economy, by not lending any amount of funds that makes the planner to chose default over repayment. That is, the external investors will set the credit limit such that the borrower’s expected lifetime utility from participating in the asset market is at least as high as that of staying in financial autarky, where the country consumes its exogenous endowment output.

If  $\bar{d}$  is the maximal amount of funds that the domestic economy can borrow without triggering the strategy of optimal default (that is,  $\bar{d}$  is such that  $V_t^D \leq V_t^R$ ), at every period  $t$ , then the domestic economy is constrained to borrow  $d_t \leq \bar{d}$ . In order to assure repayment in all states of nature, Zhang’s (1997) approach is adopted here by considering the worst case scenario for the foreign lenders to define the critical level of borrowing that triggers default, given the state  $S(d_{t-1}, z_t)$ .

We assume that the life-time utility of the representative agent is given by:

$$V_0 = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (1)$$

where  $u(\cdot)$  is concave, strictly increasing, and twice continuously differentiable;  $\beta \in (0, 1)$  is the subjective discount factor and  $c_t$  is a consumption index, assumed to be a Constant Elasticity of Substitution (CES) aggregator of the consumption of tradables and nontradables, with elasticity of substitution between  $c_t^T$  and  $c_t^N$  given by  $1/(1 + \mu) > 0$ , and weight of tradables in the index equal to  $\omega \in [0, 1]$ :

$$c_t = \left[ \omega (c_t^T)^{-\mu} + (1 - \omega) (c_t^N)^{-\mu} \right]^{-\frac{1}{\mu}} \quad (2)$$

The economy is subject to two resource constraints, one for each type of good. For the nontradable good the constraint means that the economy has to consume the endowment:

$$c_t^N = Y^N \quad (3)$$

Contrarily to Bulow and Rogoff (1989), this paper accepts the notion that default on the external debt precludes a sovereign government not only of borrowing internationally, but also exclude the country from investing its accumulated assets in the international market in the form of bank accounts, treasury bills, stocks, and other state-contingent assets, without the risk of having those assets seized

by international financial institutions or governments. This assumption assures a support for a positive external debt in equilibrium.<sup>9</sup> However, as shown by other empirical studies that use the same type of borrowing constraint considered here [Arellano (2004) and Aguiar and Gopinath (2004)], for reasonable values of the structural parameters on a DGE model applied to a small open economy, the threat of autarky, although capable of producing a positive amount of debt in equilibrium, cannot generate the levels of debt-output ratio observed in actual indebted economies. For this reason, the model will admit an extra-penalty to the defaulting country, which could be motivated by “the common view that after default there is a disruption in the countries’ ability to engage international trade, and this reduces the value of output” [see Cole and Kehoe (2000)]. We assume that, in case of default, there is an output loss of  $(1 - \lambda)$  percent that corresponds to the negative effects that the default state cause in the countries’ international trade.<sup>10</sup> Thus, in case of default, the resource constraint for the nontradable good is:

$$c_t^N = \lambda Y^N \quad (4)$$

For the tradable good, the resource constraint, in case of full repayment, means that the economy keeps the ability to borrow from international lenders and it is given by:

$$c_t^T = Y^T + z_t + d_t - (1 + r) d_{t-1} \quad (5)$$

In case of default, the economy does not have to pay  $(1 + r) d_{t-1}$ , but cannot contract  $d_t$  and must operate in autarky from  $t$  onwards. The resource constraint then tells that the consumption of tradables is to be restricted to the stochastic tradable output minus the default-state output loss:

$$c_t^T = \lambda Y_t^T \quad (6)$$

The process for the shock  $z_t$  is assumed to follow a first-order Markov chain with transition probabilities given by  $f(z_t|z_{t-1})$  and compact support. The finite support for  $z_t$  allows the use of Zhang’s (1997) approach, as mentioned above:

$$z_t \in \Omega_Z = [z_{\min}, z_{\max}] \quad (7)$$

---

<sup>9</sup>Bulow and Rogoff (1989) have shown that “under fairly general conditions, lending to small countries must be supported by the direct sanctions available to creditors, and cannot be supported by a country’s reputation for repayment” [see Bulow and Rogoff (1989), p.43], i.e. the penalty of no further borrowing would not deter repudiation and, consequently, a sovereign could not issue any uncollateralized debt. Their result depends crucially on the controversial assumption that repudiation of debt does not mean that the defaulting country is to be cut off from international capital markets entirely and may keep on participating as a creditor without fearing that its assets would be seized by foreign financial institutions or governments.

However, as Cole and Kehoe (1995, 1998) pointed out, Bulow and Rogoff’s theoretical result has the counterfactual implication that the only explanation of why countries do not default is that there are large direct sanctions for doing so. English (1996) shows historic evidence suggesting that direct sanctions cannot explain why sovereign governments repay their debts.

<sup>10</sup>Chuhan and Sturzenegger (2003) found that the percent contraction in output in Latin America, following the default episodes in the 1990’s, was 2%.

The central planner’s problem is to maximize the objective-function given by equation (1) subject to (2)-(7), a standard No-Ponzi condition and to the following borrowing constraint:

$$d_t \leq \bar{d}$$

where:

$$\bar{d} = \min_{\Omega_Z} \{ \bar{d}_t(z_t) : V_t^R(\bar{d}_t(z_t), z_t) = V_t^D(z_t) \}$$

The constraint described above represents a way of capturing the widespread notion that borrowers face credit limits in reality and, as such, its use in economic models can mimic important features of the real world. Borrowing constraints are typically needed to prevent default and Ponzi schemes (a “natural” borrowing constraint), and to ensure the existence of equilibrium for incomplete markets economies. However, the borrowing constraints used in the literature are often specified arbitrarily outside economic models. The borrowing constraints used in most studies<sup>11</sup> take the form of a lower bound on an investor’s bond holdings, which is a certain percentage of total income that is independent of his individual characteristics and income streams that in reality are important factors in determining the borrowing limit.

Notice that the borrowing constraint defined above depends not only on the countries’s representative agent’s characteristics such as time preference rate, risk aversion and elasticity of substitution between the consumption of tradable and nontradable goods, but also on his exogenous endowment income stream, here completely determined by the shock  $z_t$ . Because the constraint can be interpreted as the borrowing limit such that an investor will not default and live in autarky, Zhang (1997) refers to it as the “no default borrowing constraint.” In terms of this paper, its assumed that emerging economies (given their history and, likely, their experienced default episodes) face this type of borrowing constraint while developed economies do not. Although it is not a feature of the model, one could think of “reputation” as an additional state variable and consider that, at this particular point in time, developed economies have a higher “stock of reputation” than emerging economies, higher enough to signal a very low propensity to default.

One can explore the recursive form of the problem. In terms of notation, henceforth the time subscript  $t$  is dropped from the (indirect) utility functions  $V^D$ ,  $V^R$  and  $V$ , which are going to represent time-invariant value functions. Considering the CES consumption index in (2) and using the resource constraints for the tradable and nontradable goods, one can denote the instantaneous utility function,  $u(c_t) = u(c_t^T, c_t^N)$ , by:

$$u(c_t^T, c_t^N) = u(\lambda(Y^T + z_t) ; \lambda Y^N)$$

---

<sup>11</sup> Examples of models with *ad hoc* borrowing constraints include Aiyagari and Gertler (1991), Telmer (1993), Lucas (1994), and Heaton and Lucas (1996), in the context of using incomplete markets with borrowing constraints in order to resolve the “equity premium puzzle”. In the international macroeconomics literature, examples of the use of *ad hoc* borrowing constraints include Mendoza (2001) and other papers in the “Sudden Stop” literature, as mentioned in the footnote 2

in case of default; and

$$u(c_t^T, c_t^N) = u(Y^T + z_t + d_t - (1+r)d_{t-1}; Y^N)$$

in case of full repayment.

Let the beginning-of-period values of the shock,  $z_t$ , and debt,  $d_{t-1}$ , be in  $\Omega_Z$  and  $D = \{d : d_{\min} \leq d \leq d_{\max}\}$ , respectively. Conditional on the state variables in  $S(d_{t-1}, z_t)$ , and given the Markov process governing the shock, the central planner's problem can be expressed in recursive form as:

$$V^D(z_t) = u(\lambda(Y^T + z_t); \lambda Y^N) + \beta E_z V^D(z_{t+1})$$

in case of default; and as the solution to the following Bellman equation:

$$\begin{aligned} V^R(d_{t-1}, z_t) &= \max_{(d_t)} \{u(Y^T + z_t + d_t - (1+r)d_{t-1}; Y^N) + \beta E_z V(d_t, z_{t+1})\} \\ st : d_t &\leq \bar{d} = \min_{\Omega_Z} \{\bar{d}(z_t) : V^R(\bar{d}(z_t), z_t) = V^D(z_t)\} \\ \text{with } V(d_{t-1}, z_t) &= \max\{V^R(d_{t-1}, z_t), V^D(z_t)\} \end{aligned}$$

in case of full repayment.

The solution of the model consists of three objects: *i*) a state-contingent optimal decision rule for the level of next-period debt<sup>12</sup> that depends on the current realization of the states,  $d(d_{t-1}, z_t)$ ; *ii*) a set of value functions  $V^D(z_t)$ ,  $V^R(d_{t-1}, z_t)$  and  $V(d_{t-1}, z_t)$ ; and *iii*) the level of the borrowing constraint,  $\bar{d}$ . Given the solution, the underlying probability distribution function of the production shock, jointly with the decision rule, determine the transition and limiting distributions of all endogenous variables in the model.

In the empirical application of the model, discussed in the next section, a Constant Relative Risk-Aversion (CRRA) specification for instantaneous utility function:

$$\begin{aligned} u(c_t) &= \frac{c_t^{1-\gamma} - 1}{1-\gamma}, \text{ if } \gamma \neq 1 \\ &= \log(c_t), \text{ if } \gamma = 1 \end{aligned}$$

will be used; where  $\gamma > 0$  is the (reciprocal) of the intertemporal elasticity of substitution on the consumption index (or the risk-aversion parameter).

The model also provides implications for the real exchange rate, as measured by the relative price of nontradable with respect to tradable goods. In the model, the sectorial (shadow) prices are represented by the Lagrange multipliers on the respective resource constraints. At the optimum, there is an implied

---

<sup>12</sup>Obviously, the decision rule for the dynamic path of  $d_t$  implies another,  $c^T(d_{t-1}, z_t)$ , for the consumption of tradable goods.

equation that links the real exchange rate to the  $(c^T/c^N)$  ratio:

$$p_t \equiv \frac{P_t^N}{P_t^T} = \frac{(1-\omega)}{\omega} \left( \frac{c_t^T}{c_t^N} \right)^{(1+\mu)} \quad (8)$$

where  $P_t^N$  and  $P_t^T$  are the Lagrange multipliers associated with the nontradable and tradable resource constraints, respectively.

## 4 Numerical Solution, Calibration and Simulation Results

As it is common in the dynamic general equilibrium literature, the model discussed in this paper does not have an analytical solution. We explore the recursive formulation of the central planner's problem to solve it numerically, using the value function iteration method and discretization of the state-space  $[D \times \Omega_Z]$ , for which, given the finite support  $\Omega_Z$  for the shock, the limits  $d_{\min}$  and  $d_{\max}$  of the set  $D = \{d : d_{\min} \leq d \leq d_{\max}\}$  are appropriately chosen to include the ergodic space.

The algorithm used in the numeric solution is the following. For each iteration  $j$  of the algorithm, given an initial guess for the borrowing constraint,  $\bar{d}^{(j)}$ , the model is solved and the value functions  $V^{D(j)}(z_t)$  and  $V^{R(j)}(d_{t-1}, z_t)$  are computed. During this step, every point in the decision rule  $d^{(j)}(d_{t-1}, z_t)$  such that  $d^{(j)} > \bar{d}^{(j)}$  is replaced by the critic level  $\bar{d}^{(j)}$ . After computing  $V^{D(j)}$  and  $V^{R(j)}$ , an update of the borrowing constraint is obtained using

$$\bar{d}^{(j+1)} = \min_{\Omega_Z} \left\{ \bar{d}(z_t) : V^{R(j)}(\bar{d}(z_t), z_t) = V^{D(j)}(z_t) \right\}$$

The procedure is implemented until convergence with  $\bar{d}^{(j+1)} \approx \bar{d}^{(j)}$ .

The artificial economy is calibrated to match some aspects of the Brazilian economy during the period 1980:1-2001:4, when the net external debt (debt minus international reserves) averaged  $\theta_d = 28.34\%$  and reached a peak of 47.02% of the GDP,<sup>13</sup> which is roughly equivalent to two standard deviations from the mean. It is assumed that Brazil is an economy subject to a borrowing constraint like the one discussed in the previous section and, as such, it could be used as benchmark for the simulation exercise.

In order to calibrate the exogenous sectorial outputs, the procedure used here considered the tradable output share in total GDP observed in Brazil,  $\theta_T = 29.05\%$ , and normalized the (deterministic) steady state values of the tradable output and the relative price of nontradables in terms of tradables to be  $Y_{ss}^T = 100$  and  $p_{ss} = 1$ , respectively. If one sets the average tradable output to be  $Y^T = Y_{ss}^T$ , these figures imply: *i*) that the value of the nontradable output is  $Y^N = 244.21$  and, given a debt-output ratio equal to the average value  $\theta_d$ , *ii*) that the level of debt (in units of tradable goods) at the steady state is  $d_{ss} = 97.56$ . In order to capture the potential movements of the simulated series of external debt, an evenly spaced  $d$ -grid of 800 points was constructed from the interval  $[-100, 700]$ , with negative

<sup>13</sup>Actually, these figures refer to the period 1982:4-2001:4, since quarterly data on Brazilian external debt is not available for the whole period of reference.

values being assets instead of liabilities. Roughly, considering the total output at the steady state ( $Y^T + p_{ss}Y^N = 344.23$ ) as reference, the grid implies debt-output ratios in the range  $[-0.29, 2.03]$ .

For the discretization of the  $z$ -grid, the Markov chain was set to mimic a first-order autoregressive process of the type  $z_t = \rho z_{t-1} + \varepsilon_t$ , with  $\varepsilon_t \sim N(0, \sigma_\varepsilon)$ , using Tauchen's (1986) procedure. The  $z$ -grid has 5 points, evenly spaced in the interval  $[-17.11, 17.11]$  with underlying matrix of transition probabilities given by:

$$\Pi = \begin{bmatrix} 0.3423 & 0.5984 & 0.0591 & 0.0002 & 0.0000 \\ 0.0467 & 0.5669 & 0.3744 & 0.0120 & 0.0000 \\ 0.0016 & 0.1611 & 0.6746 & 0.1611 & 0.0016 \\ 0.0000 & 0.0120 & 0.3744 & 0.5669 & 0.0467 \\ 0.0000 & 0.0002 & 0.0591 & 0.5984 & 0.3423 \end{bmatrix}$$

Table 3 displays the values of the structural parameters used in the calibration exercise. The value for the reciprocal of the intertemporal elasticity substitution (or, equivalently, for the CRRA case, the risk-aversion parameter) was set to  $\gamma = 1.5$ , which is standard.<sup>14</sup> The exogenous interest rate was taken from what the Brazilian government pays in the international capital markets for its sovereign debt, as represented by the Federative Republic of Brazil's *C-Bonds*. Here, the idiosyncratic interest rate  $r$  is considered to be the quarterly equivalent of the average real annual rate on the U.S. Government Bonds (4.% per year, using the inflation rate on the CPI) plus the average spread paid on the *C-Bonds* (803.4 basis points).<sup>15</sup> Following the traditional hypothesis used in the small open economy literature, in order to avoid a unit root in the current account, the subjective discount factor has to satisfy  $\beta(1+r) = 1$  and, thus, was set to  $\beta = 0.9713$ . Its worth mentioning that this value of  $\beta$  is consistent with estimations by Issler and Piqueira (2000) for the Brazilian economy.

The autocorrelation and volatility of the stochastic process of the  $z$  production shock was obtained from an Ordinary Least Squares (OLS) estimation of the HP-detrended output of tradables against its one-period lagged value. Assuming that the output of tradables ( $Y_t^T$ ) has a trend component ( $HPY_t^T$ ) and a business cycle component with zero average (the production shock  $z$ ), the following regression:

$$(Y_t^T - HPY_t^T) = k + \rho(Y_{t-1}^T - HPY_{t-1}^T) + \varepsilon_t$$

was estimated, resulting in  $\rho = 0.65$ . and  $\sigma_\varepsilon = 4.35$ .<sup>16</sup>

The percent output loss in default states,  $(1 - \lambda)$ , was calibrated to approximate the average level of debt-output ratio to the actual value ( $\theta_d = 28.34\%$ ). Notice that the calibrated value  $\lambda = 0.975$ , which

<sup>14</sup>For instance, the value used here is the mid-range value of two very common alternatives,  $\gamma = 1.001$  or  $\gamma = 2$ , used by Greenwood *et al* (1988) and Mendoza (1991), for example. Issler and Piqueira (2000) estimated  $\gamma = 1.7$ , using Brazilian data and the same type utility function used in this paper. The results of the simulation of the model are virtually the same if one uses this value instead of  $\gamma = 1.5$ . Since this paper is not taking a stand on potential differences in preferences among countries, the standard value is used.

<sup>15</sup>For the average foreign real interest rate, the 10-year-maturity U.S. Government Bond was used, whose maturity is comparable to that of the *C-Bonds*. The average spread for the *C-Bonds* refer to the period 1995:1-2001.4, since data was not available before that.

<sup>16</sup>The estimated parameters ( $p$ -values in parenthesis) are  $\hat{k} = 0.123970$  (0.846),  $\hat{\rho} = 0.646766$  (0.000) and  $\hat{\sigma}_\varepsilon = 4.349889$ .

implies output losses of 2.5% during default states, is not very different from the empirical findings by Chuhan and Sturzenegger (2003) mentioned in the footnote 10.

Table 3

Summary of the Calibration Procedure		
Parameter	Values	Target
1. Risk aversion	$\gamma = 1.5000$	Standard
2. Idiosyncratic interest rate	$r = 0.0295$	<i>C-Bond</i> spread over U.S. bonds
3. Subjective discount factor	$\beta = 0.9713$	$\beta(1+r) = 1$
4. average tradable output	$Y^T = 100.00$	normalization
5. constant nontradable output	$Y^N = 244.23$	$\frac{Y^T}{Y^T + p_{ss}Y^N} = \theta_T = 29.05\%$
6. Elasticity of substitution between $c^T$ and $c^N$	$\mu = 1.8750$	$\sigma_y = 2.95\%$
7. Weight of tradables in CES $c$ aggregator	$\omega = 0.0659$	$p_{ss} = \frac{(1-\omega)}{\omega} \left( \frac{Y^T - rd_{ss}}{Y^N} \right)^{(1+\mu)} = 1$
8. Autocorrelation for $z$	$\rho = 0.6468$	OLS estimation
9. Std. dev. of the production shock $z$	$\sigma_\varepsilon = 4.3499$	OLS estimation
10. Output loss in state of default	$\lambda = 0.9750$	$avg \left( \frac{d_t}{Y_t^T + p_t Y^N} \right) \cong \theta_d = 28.34\%$

Perhaps the less straight forward parameters to calibrate were the weight of tradables in the CES consumption aggregator ( $\omega$ ) and the parameter governing the elasticity of substitution between the consumption of tradables and nontradables ( $\mu$ ). Considering the calibration procedure based on a deterministic steady state at which the external debt-output ratio is constant at the average level  $\theta_d$ , the share of tradable output in total output is  $\theta_T$  and the real exchange rate is at the normalized level  $p_{ss} = 1$ , given equation (8), the following system of “steady state” equations must be satisfied:<sup>17</sup>

$$\begin{aligned}
 \theta_T &= \frac{Y_{ss}^T}{Y_{ss}^T + p_{ss}Y^N} \\
 \theta_d &= \frac{d_{ss}}{Y_{ss}^T + p_{ss}Y^N} \\
 c_{ss}^T &= Y_{ss}^T - rd_{ss} \\
 p_{ss} &= \frac{(1-\omega)}{\omega} \left( \frac{c_{ss}^T}{c_{ss}^N} \right)^{(1+\mu)} = 1 \\
 c_{ss}^N &= Y^N
 \end{aligned}$$

Given the above system of “stationary” equations, the parameter  $\omega$  can be expressed as a function

<sup>17</sup>Technically, because of the non-linear nature of the model, which in principle should induce agents to react asymmetrically to positive and negative shocks, a “deterministic steady state” may not be relevant to reflect the long run “average” state of the system. Ideally, in this case, a more precise method of calibration should be carried out through the solution of the whole model for a given set of parameters (all of them) and successive improvements should be made until the target “average” values were obtained. However, this non-linearity did not seem to be important here and the calibration procedure used, based on a deterministic steady state, was able to generate the target averages quite accurately.

of  $\mu$  as follows:

$$\omega = \left\{ \left[ \frac{\left( \frac{1}{\theta_T} - 1 \right)}{\left( 1 - \frac{r\theta_d}{\theta_T} \right)} \right]^{(1+\mu)} + 1 \right\}^{-1}$$

It should be noticed that, in principle, both parameters are important to the volatility of the real exchange rate. However, since the business cycle statistics are usually computed on the log-variables, only  $\mu$  will have an impact on the volatility of (the log of)  $p$ . For instance, by taking the logarithm on both sides of equation (8), it is easy to see that  $VAR(\log p_t) = (1 + \mu)^2 VAR(\log c_t^T)$ , implying that the ratio between the volatilities of (the logs of)  $p_t$  and  $c_t^T$ , as measured by their standard deviations, must be constant and equal to  $(1 + \mu)$ . Because of its effect on the volatility of  $p$ , the parameter  $\mu$  has an influence in the volatilities of both total output,  $Y_t^T + p_t Y^N$ , and total consumption,  $C_t = c_t^T + p_t c^N$ . Among the different possible combinations of values for the two parameters that satisfy the above system of stationary equations,  $\omega = 0.0659$  and  $\mu = 1.875$  (which implies an elasticity of substitution between  $c^T$  and  $c^N$  equal to 0.35) were chosen in order to match the total output volatility  $\sigma_y = 2.95\%$  observed in the data (see Table 1).

Table 4 shows the average results of 500 simulations of a time series of size 88, which is the number of quarterly observations for the 1980:1-2001:4 period. The simulated series were transformed according to the same procedure used in the actual data, as discussed in the previous section. In terms of the model,  $\sigma_c$  represents the volatility of (the log of) total consumption (in units of tradable goods) as given by  $C_t = c_t^T + p_t c^N$ . Notice that the comparison between the models for the constrained and unconstrained (perfect capital mobility) economies shows that the type of borrowing constraint used here has the effect of increasing the relative consumption volatility from 0.554 to 0.644, a 16.3% increase. Considering that the average figure implied by the data from Table 1 is 63.4%, one could conclude that the borrowing constraint used here is capable of accounting for 25.7% of the difference in relative consumption volatility between emerging and developed economies (or 24.8% if one consider the comparison between Brazil and the average of developed economies, in Table 1).

Table 4.

Brazil - Output and Consumption Volatility Statistics			
	$\sigma_y$ (%)	$\sigma_c$ (%)	$\sigma_c/\sigma_y$
Brazil (1980.1 - 2001.4)	2.95	3.90	1.308
Model (constrained)	2.95	1.90	0.644
Model (unconstrained)	2.60	1.44	0.554

Although the model manages to increase the relative consumption volatility, it is not able to reproduce both the actual absolute and relative levels of consumption volatility and cannot account for an



interesting feature of the emerging country data (see Table 1), which is the fact that consumption is consistently more volatile than output. Neumeyer and Perri (2004) attribute this excess volatility of consumption to the dominant role played by interest rate shocks in these economies. In an economy that faces both income and interest rates volatility, consumption will be smoother than income if the transitory production shocks are dominant and the opposite happens if, instead, the interest rate shocks are dominant. In this model, the lack of any shocks that affect consumption in a relative independent way from output, such as interest rate shocks, makes it impossible for consumption to fluctuate more than output. For instance, interest rate shocks affect the intertemporal decisions of consumption/savings and act on the consumption growth rate but have only second order effects on the production side (in a production economy, *ceteris paribus*, the main effect would be inducing a substitution of capital by labor). Since the model is not capable of accounting for the absolute volatility of consumption observed in the data from emerging economies, other sources of consumption volatility, that should play a major role in emerging economies while not being much active in developed economies, are clearly missing here.

The results of one particular simulation is shown in Figures 1 and 2, for the unfiltered and HP-filtered simulated series. Notice that the model is capable of generating a pro-cyclical behavior for the consumption series (both tradable consumption and total consumption) as well as for the real exchange rate, as observed in the actual data from emerging economies [see Arellano (2004)]. Also notice that the debt series in the constrained economy follows a similar path as in the unconstrained one, but at a lower level. This feature implies that the borrowing constraint affects the behavior of the economy even when it is not binding. In terms of the supply of credits, the simple possibility of default means less credit to the small economy at all times. From the demand side, agents that consider the possibility of being credit constrained in the future will save more now (hence, less debt). The borrowing constraint will bind only when the cost of a bad production shock, in terms of reducing consumption today, is high enough to induce the agents to borrow until their limit.

Finally, it should be mentioned that the simulated average of the debt-output ratio for the sample is 28.35% in the constrained economy, virtually identical to the actual average observed in Brazilian data ( $\theta_d = 28.34\%$ ). In addition, the level of the debt limit was such that it corresponds to 80.7% of the simulated average GDP. Notice that this level is well above the maximal level for the debt-output ratio observed in Brazil, in the period 1980:1-2001.4 (47.02%).

#### 4.1 Sensitivity Analysis

Information displayed at Tables 5, 6a, 7a and 8a shows how the model for a constrained economy behaves under different values of the structural parameters. The rows marked with a (\*) refer to the baseline case. The columns in the tables, from the left to the right, provide information on the value of the relevant parameter (column 1), on the volatilities of consumption and output (columns 2 and

3, respectively), their ratio (column 4), the average level of debt as a percentage of the GDP (column 5) and the credit limit (column 6), both in level and as a percentage of the GDP (within parenthesis). The tables also show the frequency at which the constraint binds (column 7) and a measure of the explaining power of the model (column 8). This measure of “success” is given by the proportion of the observed percentage difference in  $\sigma_c/\sigma_y$  from the data of emerging and developed economies (that is, the 63.4% gap between  $\sigma_c/\sigma_y = 1.29$ , in emerging countries, and  $\sigma_c/\sigma_y = 0.79$ , in developed economies) that is accounted for by the percentage difference in the relative consumption volatility obtained from the simulated model for the constrained and unconstrained economies. Tables 6b, 7b and 8b, in the Appendix, show information for the unconstrained case.

Table 5 shows how the model for the constrained economy behaves under different values of the parameter  $\lambda$ . The economic principle at work is based on changes in the cost/benefit of defaulting. Notice that the credit limit  $\bar{d}$  falls (rises) with increases (decreases) in the value of  $\lambda$ . In order to understand why this happens one should recall that a higher (lower) value of  $\lambda$  means that the output losses during default states are less (more) important, which reduces (increases) the penalty for staying out of international capital markets. Thus, the higher the parameter  $\lambda$  is, the more likely the domestic agents are going to default (because it costs less), *ceteris paribus*, and the more likely it will trigger a defensive response from the external creditors who will have to reduce their maximal level of conceded credits to avoid default. On the other hand, as  $\lambda$  decreases, it becomes more costly for the country to default and foreign investors can relax the borrowing constraint without fearing default.

Table 5

Constrained Model: Sensitivity to Changes in $\lambda$							
$\lambda$	$\sigma_y$ (%)	$\sigma_c$ (%)	$\sigma_c/\sigma_y$	<i>avg d</i> (% GDP)	$\bar{d}$ (% GDP)	% bind	“success” (%)
0.9725	2.88	1.80	0.625	30.18	300.5 (88.08)	0.22	20.27
*0.9750	2.95	1.90	0.644	28.35	279.5 (80.67)	0.31	25.71
0.9775	3.03	1.99	0.657	26.14	255.4 (73.51)	0.40	29.32
0.9800	3.11	2.10	0.675	23.10	229.4 (65.73)	0.51	34.59
0.9825	3.19	2.20	0.690	19.41	203.4 (57.92)	0.61	38.69
0.9850	3.29	2.32	0.705	15.43	177.4 (50.13)	0.71	43.11
0.9900	3.54	2.64	0.746	5.78	123.3 (34.15)	0.88	54.68
1.0000	4.83	4.12	0.853	-12.53	9.14 ( 2.42)	1.85	85.23

Note: Column 6 shows the borrowing constraint both in level and, inside the parenthesis, as a percentage of the GDP.

Notice that as  $\lambda$  increases, and the constraint becomes more stringent, both output and consumption become more volatile although the effect is more important on consumption since the ratio  $\sigma_c/\sigma_y$  consistently increases. The intuition behind this result is that a lower credit limit imposes additional difficulties to risk sharing and consumption smoothing, causing the consumption of tradables to be more

volatile. A more volatile  $c_t^T$ , in turn, reflects on a more volatile real exchange rate through equation (8).<sup>18</sup> Since total consumption is defined as  $C_t = c_t^T + p_t c^N$ , the more volatile consumption of tradables increases total consumption volatility directly and indirectly, through its effect on  $p_t$  (the effects cannot cancel each other since  $c_t^T$  and  $p_t$  are positively correlated). The same is not true for total output  $Y_t = Y_t^T + p_t Y^N$ , which only suffers the effect of the more volatile real exchange rate.

Table 5 also shows that a higher  $\lambda$  induces a lower average level of debt-output ratio (which eventually becomes negative for the extreme value  $\lambda = 1.0$ ) and, at the same time, increases the frequency at which the borrowing constraint binds, suggesting that the effect of an increasing  $\lambda$  is more important on reducing the credit limit  $\bar{d}$  than on decreasing the domestic agents' borrowing motivation. One should expect that, as  $\bar{d}$  is reduced, with incomplete markets, risk-averse agents would save more (hold less debt) because the risk of being credit constrained in the future is higher the lower the credit limit is.

Finally, notice that the explaining power of the model would be improved if a higher value of  $\lambda$  were used, although the target values for the output volatility and debt-output ratio would be missed.

Table 6a

Constrained Economy: Sensitivity to Changes in $\gamma$							
$\gamma$	$\sigma_y$ (%)	$\sigma_c$ (%)	$\sigma_c/\sigma_y$	<i>avg d</i> (% GDP)	$\bar{d}$ (% GDP)	% bind	"success" (%)
0.50	2.95	1.90	0.644	28.58	279.5 (80.70)	0.33	25.00
1.00	2.95	1.90	0.644	28.44	279.5 (80.68)	0.32	25.00
*1.50	2.95	1.90	0.644	28.35	279.5 (80.67)	0.31	25.71
2.00	2.95	1.90	0.644	28.26	279.5 (80.66)	0.30	24.44
2.50	2.96	1.90	0.642	28.17	279.5 (80.65)	0.29	23.82
3.00	2.96	1.90	0.642	28.11	279.5 (80.64)	0.29	23.82
4.00	2.96	1.90	0.642	27.87	279.5 (80.61)	0.26	23.82

Note: Column 6 shows the borrowing constraint both in level and, inside the parenthesis, as a percentage of the GDP.

Table 6a reveals that the results obtained for  $\sigma_y$  and  $\sigma_c$  in the benchmark (constrained economy) are relatively robust to changes in the coefficient of risk-aversion,  $\gamma$ . Observe that the changes in the results are relative small and the figures do not differ much from the baseline case. In terms of the "success" of the model in matching the data, for example, no gain is possible from choosing alternative values for  $\gamma$ . There are changes, though. For instance, notice that as  $\gamma$  increases and agents become more risk averse, given that markets are incomplete, they tend to save more or, equivalently, hold lower amounts of debt since they become too scared of being credit constrained in the future. That explains why the average level of debt held by domestic agents falls with  $\gamma$ .

<sup>18</sup>Throughout the values of  $\lambda$  in Table 5, the volatilities of  $c^T$  and  $p$  rise monotonically from 0.7% to 1.7% and from 2.2% to 5%, respectively. This information is not displayed at the tables.

On the other hand, one should also expect that more risk-averse agents would be less inclined to default, *ceteris paribus*, since they tend to care more about risk sharing and the cost of defaulting and being deprived of risk sharing in the future becomes higher. In that case, agents do not want to default unless they hold a large amount of debt and/or are hit by a bad enough production shock. Since the cost of default increases for the country, the external investors may relax the credit limit and still receive back the conceded loans. Conversely, if agents have low risk aversion, then they do not care very much about risk sharing in the future, which means that not paying back the debt becomes relatively attractive, forcing the external investors to make the borrowing constraint more stringent to avoid default. However, for the range of values of  $\gamma$  considered at Table 6a, this effect was not quantitatively important and the level of  $\bar{d}$  turned out to be constant. In terms of  $\bar{d}$  as a percentage of the average GDP, the observed reduction is explained as follows. A lower level of (average) debt induces a higher level of average consumption of tradables, which can be fairly approximated by  $avg(c^T) \approx Y^T - r \cdot avg(d)$  provided that  $avg(d) \approx \theta_d [Y^T + avg(p) Y^N]$  and  $(\mu, \omega)$  satisfy  $avg(p) \approx 1$ , as it is the case. A higher average level for  $c^T$  combined with an inelastic (here, constant) level of  $c^N$ , in turn, means a higher average relative price of nontradable goods,  $p$  (see equation 8). The consequence of this appreciation of the real exchange rate is a higher level of total GDP in units of tradable goods, which explains why the constant level of  $\bar{d}$  falls as a percentage of the average GDP as  $\gamma$  increases. The fact that the borrowing constraint is not very sensitive to changes in  $\gamma$  while the average level of debt decreases explains why the borrowing constraint binds less frequently as  $\gamma$  rises.

Table 7a displays the sensitivity analysis to changes in the weight of the tradable good in the CES consumption aggregator,  $\omega$ . One could think of two opposite effects of  $\omega$  in terms of the incitation to default. Since a higher  $\omega$  increases the marginal utility of the consumption of tradable goods at all times, first, there would be higher instantaneous gain from default because, in that event, the country would be able to consume more of a good (tradables) that has a higher weight on the consumption index. On the other hand, intertemporally, there would be a higher cost of default by the same motive (one could also think that a higher  $\omega$  makes the agent to care more about risk-sharing, since the “insurable” part of his consumption becomes more important for his utility). Again, higher benefits of default induce external agents to reduce the level of maximal credit available to the country and higher costs of default make the constraint less stringent. Thus, the first effect would reduce the level of  $\bar{d}$  while the second effect would increase it. Notice that, since the level of  $\bar{d}$  falls (although it increases as a percentage of the GDP because of a real depreciation that more than proportionally reduces the level of the average GDP in units of tradable goods) as  $\omega$  increases, the quantitative relevance of the instantaneous benefits seems to dominate the intertemporal costs of default.

The effects of the constraint is very clear if one compares the sensitivity of the model to changes in  $\omega$  at the constrained (Table 7a) and unconstrained (Table 7b, in the Appendix) economies. Notice that, at the very low value  $\omega = 0.01$  the two economies are virtually identical, since tradable consumption has

a very small impact on the consumption index and the borrowing constraint is set at a very high level, as discussed in the previous paragraph. The level of  $\bar{d}$  is high enough to imply a very low frequency at which the constraint is binding, what makes the two models very close in behavior. Numerically, in the simulations, this frequency was zero, for two decimal places, although it is likely that a higher enough number of simulations would show some cases in which the constraint binds since theoretically the two models are still different.

Table 7a

Constrained Model: Sensitivity to Changes in $\omega$							
$\omega$	$\sigma_y$ (%)	$\sigma_c$ (%)	$\sigma_c/\sigma_y$	<i>avg d</i> (% GDP)	$\bar{d}$ (% GDP)	% bind	“success” (%)
0.0100	1.88	1.71	0.910	6.86	658.0 (36.38)	0.00	0.00
*0.0659	2.95	1.90	0.644	28.35	279.5 (80.67)	0.31	25.71
0.1000	3.42	1.98	0.579	28.63	215.4 (83.22)	0.56	46.21
0.2500	4.35	1.81	0.416	17.35	133.3 (86.04)	0.90	100.97
0.5000	4.97	1.53	0.308	6.27	104.3 (87.88)	0.98	138.59
0.7500	5.27	1.35	0.256	0.76	94.3 (88.69)	1.02	152.51
0.9900	5.44	1.24	0.228	-2.75	89.2 (89.05)	1.03	162.97

Note: Column 6 shows the borrowing constraint both in level and, inside the parenthesis, as a percentage of the GDP.

However, as  $\omega$  rises, interesting differences show up regarding the constrained and the unconstrained economies. First, notice that the volatility of output departs from the same value (1.88%) and rises in both economies, but it increases more rapidly in the constrained case. The intuition of this result is the following: since  $Y_t = Y_t^T + p_t Y^N$ , the volatility of output depends on the (exogenous) volatility of  $Y_t^T$ , as well as on the (endogenous) volatility of  $p_t$  and on the (also endogenous) covariance between the two,  $cov(Y_t^T, p_t) > 0$ . Now, at the unconstrained economy, the volatility of  $p_t$  (not shown in the tables) is almost insensitive to changes in  $\omega$  (it goes from 1.76% to 1.75% as  $\omega$  changes from 0.01 to 0.99) and the volatility of  $Y_t^T$  is exogenously given. Thus, the only way that  $Y_t$  can become more volatile is through increases in  $cov(Y_t^T, p_t)$ , possibly due to the fact that the proportion of tradables in total consumption and total GDP increases with  $\omega$ . At the constrained economy, on the other hand, on top of the effect described above, the volatility of  $p_t$  rises (from 1.77% to 3.56% as  $\omega$  goes from 0.01 to 0.99) rather than stays constant, which explains the sharper increase in  $\sigma_y$  verified in Table 7a in comparison to Table 7b.

The rising volatility of  $p_t$  at the constrained economy, while constant at the unconstrained economy, is certainly an effect of the borrowing constraint that becomes even more stringent with increases in  $\omega$  and makes tradable consumption smoothing more difficult. Not surprisingly, the same happens with the volatility of  $c_t^T$  (constant at 0.61% at the unconstrained economy and rising from 0.61% to 1.24% at

the constrained economy, as  $\omega$  changes in Table 7a). Recall that, since the same standard procedure for business cycle statistics is being used here, in particular the variables are treated in logarithmic scale, the ratio between the volatilities of  $p_t$  and  $c_t^T$  has to be equal to  $(1 + \mu) = 2.875$ .<sup>19</sup>

A second difference observed in Tables 7a and 7b, for the constrained and unconstrained economies, is that the volatilities of total consumption are identical in both economies for  $\omega = 0.01$ , but similarly to what happens with  $\sigma_y$ , they become different as  $\omega$  rises. At the unconstrained economy,  $\sigma_c$  falls monotonically with increases in  $\omega$ , while at the constrained economy there is an initial phase in which  $\sigma_c$  rises. In the case an unconstrained economy, the monotonic fall in  $\sigma_c$  is purely statistical in nature, a consequence of the reduction of the term  $(1 - \omega) / \omega$ . Note that, since  $C_t = c_t^T + p_t c^N$  and  $p_t$  is given by equation (8), one can write:

$$C_t = c_t^T + \left[ \frac{(1 - \omega)}{\omega} (c^N)^{-\mu} \right] (c_t^T)^{(1+\mu)}$$

and, as  $\omega$  goes from 0 to 1, the term  $(1 - \omega) / \omega$  goes from infinity to zero and the volatility of total consumption converges (falls) to the volatility of tradable consumption, which does not change with  $\omega$ , as discussed above. That is also the reason for the more depreciated real exchange rate (lower  $p_t$ ) that follows from the increase in  $\omega$  (see equation 8). The same effects occur in the constrained case with the important difference that, because the constraint becomes more stringent with a rising value of  $\omega$ , tradable consumption volatility increases sharply. The net effect on  $\sigma_c$  depends on the relative importance of these direct and indirect effects (through increases in tradable consumption volatility) induced by a rise in  $\omega$ . The direct effect makes  $\sigma_c$  to fall while the indirect effect acts in the opposite direction. It seems that the indirect effect dominates for small values of  $\omega$  (up to 0.1 in Table 7a) and, as  $(1 - \omega) / \omega$  converges to zero, for higher values of  $\omega$ , the direct effect becomes more important and forces  $\sigma_c$  down.

In terms of the effects of different values of  $\omega$ , a final difference between the constrained and unconstrained economies is the behavior of the average level of debt-output ratio. Since a higher  $\omega$  makes tradable consumption more important for the CES consumption aggregator index and for his utility, it makes the representative agent to attach more importance to risk sharing at all times. If markets were complete, probably this would not affect his total savings since there would be complete risk sharing and a reallocation of contingent assets would occur without important effects on total savings. However, with no contingent assets, agents more concerned with risk sharing will tend to save more for self-insurance. In fact, in both the constrained and unconstrained economies the average *level* (not shown in Table 7a) of average debt falls.

---

<sup>19</sup>For instance, up to a rounding error effect (the values are presented with only two decimal places):

$$\frac{1.76\%}{0.61\%} \approx \frac{1.75\%}{0.61\%} \approx \frac{1.77\%}{0.61\%} \approx \frac{3.56\%}{1.24\%} \approx 2.875$$

At the unconstrained economy, where there is no risk of a shortage of credits, the average level of debt falls by 11% (124 to 110.6), but at the constrained economy, where the risk of becoming credit constrained is real, and increasing with  $\omega$ , the average level of debt falls by 102.2% (from the same 124 as in the unconstrained economy to -2.75) and the agent becomes a net creditor.

In terms of the debt-output ratios, at the constrained economy, the fall in the level of debt is less than proportional to the fall in the value of the GDP for lower values of  $\omega$  and the debt-output ratio actually rises. But for  $\omega \geq 0.1$  the higher motivation for savings dominates the real depreciation, debt falls quicker than GDP, and the opposite occurs. At the unconstrained economy, since there is no risk of being credit constrained, the fall in debt is smoother and the effects of the real depreciation on total GDP always dominates, which makes the debt-output ratio to grow monotonically with  $\omega$ .

Table 8a displays the sensitivity of the model to changes in the elasticity of substitution between  $c^T$  and  $c^N$ . The most obvious effect of an increase in  $\mu$ ,<sup>20</sup> which means that  $c^T$  and  $c^N$  tend to work more as complements than as substitutes, is a rise in the volatility of  $p_t$  for a given volatility of tradable consumption, according to equation (8). For a given volatility of tradable consumption, a lower elasticity of substitution between  $c^T$  and  $c^N$  implies a lower percentage variation in  $c^N/c^T$  for a given percentage change in  $p$  or, alternatively, that a higher proportional change in  $p$  is required for a given change in the consumption of tradables relative to the consumption of nontradable goods. Notice that, as  $\mu$  rises, both  $\sigma_y$  and  $\sigma_c$  increase as a consequence of the higher volatility of the real exchange rate. At first, for lower values of  $\mu$ , the effect on  $\sigma_c$  is stronger than that on  $\sigma_y$  and  $\sigma_c/\sigma_y$  rises, but the inverse occurs after  $\mu \geq 0.01$ .<sup>21</sup>

As in the case of changes in  $\omega$ , there are two effects caused by variations in  $\mu$ , one instantaneous and another intertemporal. Its the relative importance of how the changing  $\mu$  will affect the two effects that will ultimately determine what happens with the level of the borrowing constraint. For instance, if

---

<sup>20</sup>The results for  $\mu = 1.0$  and  $\mu = -0.25$  are particularly important because they represent a possible alternative for the calibration procedure if one wants to consider values of  $\mu$  close to those implied by the estimates of the elasticity of substitution between  $c^T$  and  $c^N$  used in Arellano (2004) and Mendoza and Uribe (1999).

Arellano (2004) relied on estimation of the elasticity of substitution between tradable and nontradable consumption for Argentina by Gonzales-Rosada and Neumeyer (2003) who found it to be 0.48, implying  $\mu = 1.0833$ . Mendoza and Uribe (1999) used  $\mu = -0.218$ , the same value also used in Mendoza (1995), which implies a elasticity of substitution of 1.28. Needless to say that the existence of empirical studies that provide estimations of  $\mu$  that are lower than the value used in the baseline case is an important caveat for the results of this paper.

<sup>21</sup>Notice that, since:

$$C_t = c_t^T + \left[ \frac{(1-\omega)}{\omega} (c^N)^{-\mu} \right] (c_t^T)^{(1+\mu)}$$

and

$$Y_t = Y_t^T + \left[ \frac{(1-\omega)}{\omega} (c^N)^{-\mu} \right] (c_t^T)^{(1+\mu)}$$

the absolute effects of  $\mu$  are the same in both  $\sigma_c$  and  $\sigma_y$ , given the volatilities of  $c_t^T$  and  $Y_t^T$ . However, the percentage increase depends on the relative share of the volatilities of  $c_t^T$  and  $Y_t^T$ , respectively, on  $\sigma_c$  and  $\sigma_y$ .

the two goods are substitutes (low  $\mu$ ), then risk sharing is relatively less important at all times because, when facing a bad tradable output shock, agents can always substitute away their tradable consumption for nontradable consumption. Thus, the instantaneous gain in terms of a higher tradable consumption in case of default is reduced with reductions in  $\mu$ . However, since this substitution is also possible in the future, the intertemporal cost of default is also reduced. The opposite occurs when  $\mu$  rises: higher instantaneous benefits and, also, higher intertemporal costs of default, since substitutability between the two goods becomes weak and a bad tradable output shock hurts more at all times. Notice, at Table 8a, that the intertemporal effect dominates for lower values ( $\mu \leq 0.5$ ) and, as  $\mu$  increases the borrowing constraint  $\bar{d}$  becomes less stringent. For  $\mu \geq 0.5$ , on the other hand, the benefits of default increase faster than the costs and external investors have to reduce the credit limit to avoid default.

Table 8a

Constrained Model: Sensitivity to Changes in $\mu$							
$\mu$	$\sigma_y$ (%)	$\sigma_c$ (%)	$\sigma_c/\sigma_y$	<i>avg d</i> (% GDP)	$\bar{d}$ (% GDP)	% bind	“success” (%)
-0.750	0.31	0.17	0.548	3.89	641.9 (22.66)	0.00	0.00
-0.250	0.67	0.47	0.701	6.17	641.9 (35.23)	0.00	0.00
0.010	0.86	0.62	0.721	7.81	641.9 (43.93)	0.00	0.00
0.250	1.06	0.75	0.708	9.67	651.9 (54.74)	0.00	0.00
0.500	1.26	0.88	0.698	12.02	670.9 (69.42)	0.00	0.00
1.000	1.73	1.16	0.671	18.18	493.7 (76.35)	0.00	2.32
1.500	2.37	1.53	0.646	25.02	351.6 (78.89)	0.09	10.71
*1.875	2.95	1.90	0.644	28.35	279.5 (80.67)	0.31	25.71
2.000	3.14	2.00	0.637	28.87	260.5 (81.26)	0.37	28.86
3.000	4.39	2.47	0.563	24.35	163.3 (84.91)	0.78	65.63
4.000	5.12	2.37	0.463	14.78	121.3 (87.33)	0.95	103.47
5.000	5.42	2.02	0.373	6.82	102.3 (89.72)	1.02	132.30

Note: Column 6 shows the borrowing constraint both in level and, inside the parenthesis, as a percentage of the GDP.

The borrowing constraint as percentage of the average GDP is monotonically increasing with  $\mu$ , even when the borrowing constraint becomes more stringent. Again, the reason for that is a sharp real depreciation that follows the increase in  $\mu$ , which causes the GDP in units of tradable goods to fall more than proportionally to the fall in  $\bar{d}$ . This real depreciation is a consequence of the fact that nontradable consumption is constant in equilibrium and the two goods tend to become complements, as  $\mu$  increases. With low values of  $\mu$  and higher substitution between the two goods, given that nontradable output and consumption are constant, the relative scarcity of tradable good is reduced, which requires a lower price of tradables relative to nontradables (that is,  $p$  has to rise) and the opposite (i.e real depreciation; a fall in  $p$ ) happens for high values of  $\mu$ .



As the value of  $\mu$  rises, the *level* of the average debt increases initially and falls afterwards (this information is not displayed at Table 8a). For  $\mu \leq 1.0$ , the debt level rises by 6.2% (from 110.7 to 117.6) as  $\mu$  goes from  $-0.75$  to  $1.0$ . For higher than  $1.0$  values of  $\mu$ , the level of debt falls by 93.3% (from 117.6 to 7.9) as  $\mu$  goes from  $1.0$  to  $5.0$ . This result is a consequence of the effects that  $\mu$  has on the borrowing constraint  $\bar{d}$ . While  $\mu$  is still low, and the borrowing constraint becomes less stringent as  $\mu$  rises, agents that are risk-averse and fear being credit constrained will save less because  $\bar{d}$  is too high. Actually, this explains why the constraint does not bind at low values of  $\mu$  and, also, why the constrained and unconstrained economies are virtually the same for values of  $\mu$  that are lower than  $1.0$  (the constraint is so loose that, numerically, the two economies behave almost the same). However, as  $\mu$  increases and the constraint becomes more stringent, the risk of being credit constrained increases and agents will tend to start saving more, reducing their debt.

In terms, of the debt-output ratio, the initial increase is due both to the rise in the average level of debt and to the reduction in the value of total GDP in units of tradables that follows the real depreciation. The fall observed for  $\mu \geq 2.0$  is explained by the fact that the level of debt decreases more than proportionally to the fall in the value of GDP.

## 5 Conclusion

This paper presented empirical evidence of higher relative consumption volatility (to output volatility) experienced by emerging economies compared with developed small open economies. The data indicate that emerging economies have 63.4% more relative consumption volatility than small open developed economies. Using a dynamic general equilibrium model of an endowment, two-goods, small open economy subject to an endogenous borrowing constraint, the paper suggests that the constraint alone, although having a limited explaining power on the relative consumption volatility differential, is able to increase the relative consumption volatility by 9 percentage points (or 16.3%), which corresponds to more than 25% of the observed in the data from emerging (likely to be constrained) and small developed open economies.

The model does relatively well quantitatively in explaining the empirical evidence discussed here and, qualitatively, in a number of dimensions such as the pro-cyclical movements of consumption and real exchange rate, as mentioned in the previous section. However, it does not perform well in other dimensions. For example, it is not able to reproduce actual levels of absolute output and consumption volatilities nor is capable of explaining the fact that consumption is consistently more volatile than output. Also, since there is no investment and production in the model, any positive production shock translates into an amelioration of the current account, since only the consumption smoothing mechanism is at work and the investment motive does not exist. Future extensions of this paper intend to address those questions.

## 6 Appendix

Tables 6b, 7b and 8b display information about the sensitivity analysis at the model for the unconstrained economy.

Table 6b

Unconstrained Model: Sensitivity to Changes in $\gamma$				
$\gamma$	$\sigma_y$ (%)	$\sigma_c$ (%)	$\sigma_c/\sigma_y$	avg $d$ (% $GDP$ )
0.50	2.59	1.44	0.556	34.95
1.00	2.59	1.44	0.556	34.93
*1.50	2.60	1.44	0.554	34.91
2.00	2.60	1.45	0.558	34.89
2.50	2.60	1.45	0.558	34.88
3.00	2.60	1.45	0.558	34.87
4.00	2.60	1.45	0.558	34.83

Table 7b

Unconstrained Model: Sensitivity to Changes in $\omega$				
$\omega$	$\sigma_y$ (%)	$\sigma_c$ (%)	$\sigma_c/\sigma_y$	avg $d$ (% $GDP$ )
0.0100	1.88	1.71	0.910	6.86
*0.0659	2.60	1.44	0.554	34.91
0.1000	2.97	1.33	0.448	46.51
0.2500	4.02	1.02	0.254	75.75
0.5000	4.82	0.79	0.164	95.86
0.7500	5.22	0.68	0.130	105.22
0.9900	5.44	0.61	0.112	110.41

Table 8b

Unconstrained Model: Sensitivity to Changes in $\mu$				
$\mu$	$\sigma_y$ (%)	$\sigma_c$ (%)	$\sigma_c/\sigma_y$	avg $d$ (% $GDP$ )
-0.750	0.31	0.17	0.548	3.89
-0.250	0.66	0.47	0.712	6.17
0.010	0.86	0.62	0.721	7.81
0.250	1.06	0.75	0.708	9.67
0.500	1.26	0.88	0.698	12.02
1.000	1.71	1.13	0.661	18.27
1.500	2.20	1.33	0.605	26.86
*1.875	2.60	1.44	0.554	34.91
2.000	2.73	1.47	0.539	37.86
3.000	3.80	1.51	0.397	63.92
4.000	4.65	1.30	0.280	86.65
5.000	5.13	1.04	0.203	100.37

## 7 References:

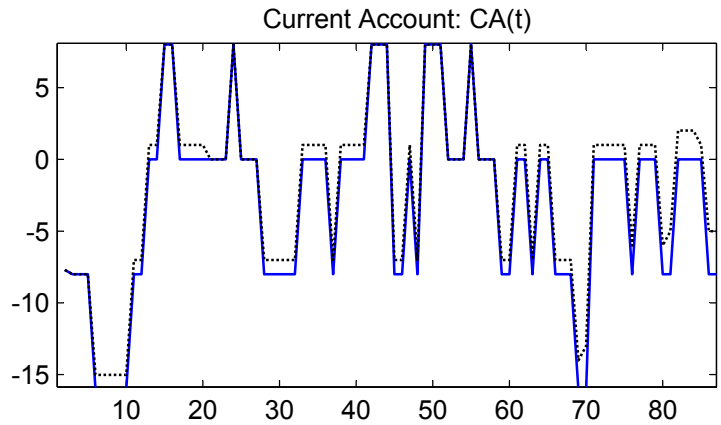
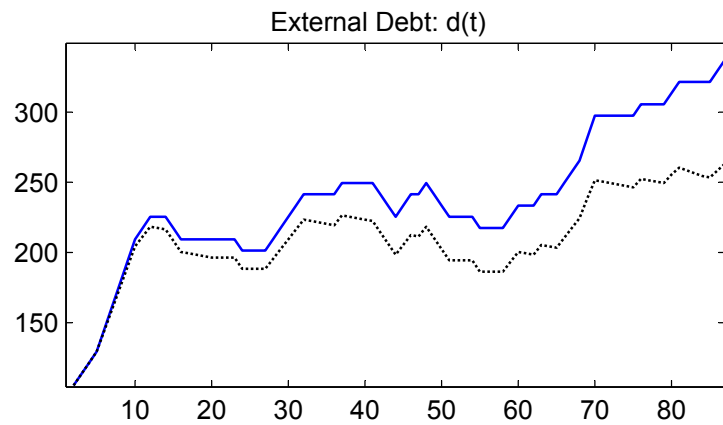
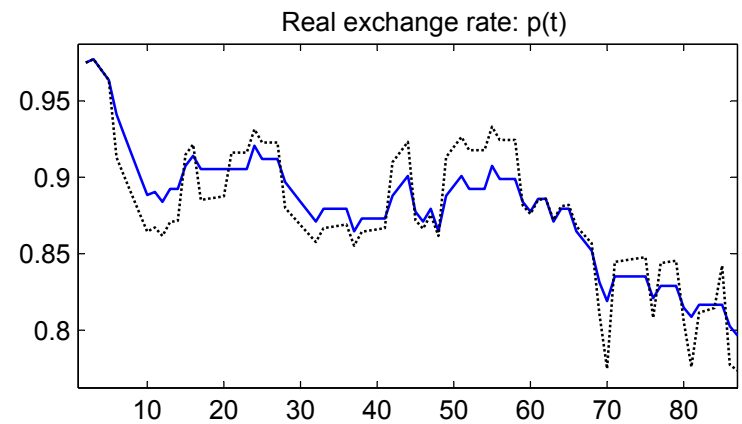
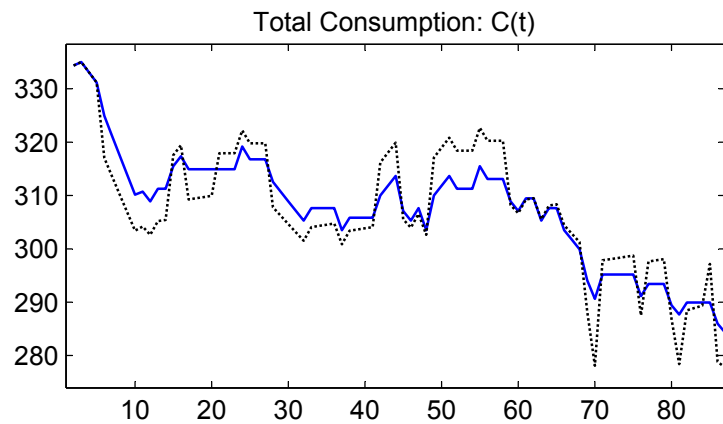
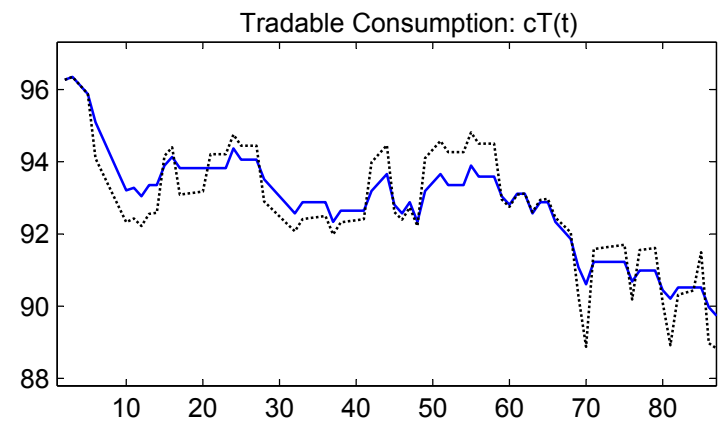
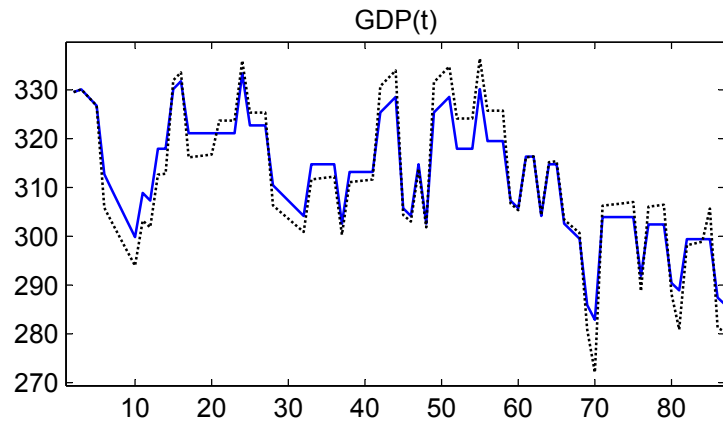
### References

- [1] Aguiar, M. and G. Gopinath, (2004) “Defaultable debt, interest rates and the current account,” mimeo: 1-40.
- [2] Aiyagari, S.R. and M. Gertler, (1991) “Asset returns with transactions costs and uninsured individual risk,” *Journal of Monetary Economics*, 27: 311-331.
- [3] Alvarez, F. and U.J. Jermann, (2000) “Efficiency, equilibrium, and asset pricing with risk of default,” *Econometrica*, 68 (4): 775-797.
- [4] Arellano, C, (2004) “Defaultable risk, the real exchange rate and income fluctuations in emerging economies,” mimeo: 1-43.
- [5] Bulow, J. and K.Rodoff, (1989) “Sovereign debt: is to forgive to forget?,” *American Economic Review*, 79(1): 43-50.
- [6] Calvo, G.A., (1998a) “Capital flow and capital-market crises: the simple economics of sudden stops,” *Journal of Applied Economics* 1: 35-54.
- [7] Calvo, G.A., (1998b) “Explaining sudden stop, growth collapse, and BoP crisis: the case of distortionary output taxes,” *IMF Staff Papers* 50: 1-20.
- [8] Calvo, G.A. and C.M. Reinhart, (1999) “When capital inflows come to sudden stop: consequences and policy options,” mimeo, University of Maryland.
- [9] Calvo, G. A. and C.A. Vegh, (1999) “Inflation stabilization and BOP crises in developing countries”, in: Taylor, J. and M. Woodford, eds., *Handbook of Macroeconomics*. North-Holland, pp. 1531-1614.
- [10] Calvo, G.A., L. Leiderman and C.M. Reinhart, (1993) “Capital inflows and real exchange rate appreciation in Latin America,” *IMF Staff Papers* 40(1): 108-151.
- [11] Chuhan, P. and F. Sturzenegger, (2003) “Default episodes in the 1990’s: what have we learned?,” *working paper*, Universidad Torcuato Di Tella.
- [12] Cole, H.L, J.Dow and W.B. English, (1995) “Default, settlement, and signalling: lending resumption in a reputational model of sovereign debt,” *International Economic Review*, 36(2): 365-385.
- [13] Cole, H.L. and P.J. Kehoe, (1995) “The role of institutions in reputation models of sovereign debt,” *Journal of Monetary Economics*, 35: 45-64.

- [14] Cole, H.L. and P.J. Kehoe, (1998) "Model of sovereign debt: partial versus general reputations," *International Economic Review*, 39(1): 55-70.
- [15] Cooley, T.F. and E.C. Prescott, (1995) "Economic growth and business cycles" in: Cooley, T.F, eds., *Frontiers of business cycle research*. Princeton: Princeton University Press, pp. 1-38.
- [16] Correia, I., J.C. Neves and S. Rebelo, (1995) "Business cycles in a small open economy," *European Economic Review*, 39: 1089-1113.
- [17] Eaton, J. and M. Gersovitz, (1981) "Debt with potential repudiation: theoretical and empirical analysis," *Review of Economic Studies*, 48 (2): 289-309.
- [18] Edwards, S. (2001) "Does the current account matter?" in: S. Edwards and J.A. Frankel, eds., *Preventing crises in emerging markets*. NBER Conference Report. Chicago: the University of Chicago Press, pp. 21-75.
- [19] Eichengreen, B., A. K. Rose and C. Wyplosz, (1995) "Exchange market mayhem: the antecedents and aftermath of speculative attacks," *Economic Policy* 21:251-312.
- [20] English, W., (1996) "Understanding the cost of sovereign default: American state debts in the 1840's," *American Economic Review*, 86: 259-275.
- [21] Frenkel, J.A. and A.K. Rose, (1996) "Currency crises in emerging markets: an empirical treatment," *Board of Governors of the Federal Reserve System* 534: 1-28.
- [22] Gonzales-Rosada, M., and P. A. Neumeyer (2003) "The Elasticity of Substitution in Demand for Non-tradable Goods in Latin America. Case Study: Argentina," *working paper*, Universidad Torquato di Tella.
- [23] Greenwood, J., Z. Hercovitz and G. W. Huffman (1988) "Investment, Capacity Utilization and the Real Business Cycle," *The American Economic Review*, 78 (3): 402-417.
- [24] Grossman, H.I and T. Han, (1999) "Sovereign debt and consumption smoothing," *Journal of Monetary Economics*, 44: 149-158.
- [25] Issler, J.V. and N.S Piqueira, (2000) "Estimating relative risk aversion, the discount rate, and the intertemporal elasticity of substitution in consumption for Brazil using three types of utility function," *Brazilian Review of Econometrics*, 20 (2): 201-239.
- [26] Kaminsky, G.L, S. Lizondo and C. M. Reinhart, (1997) "Leading indicators of currency crises," *IMF Working Paper* 79: 1-43

- [27] Kaminsky, G. L. and C. M. Reinhart, (1999) "The twin crises: the causes of banking and balance-of-payments problems," *The American Economic Review*, 89: 473-500.
- [28] Kehoe, T. and D.K. Levine, (1993) "Debt-constrained asset markets," *Review of Economic Studies*, 50: 868-888.
- [29] Kletzer, K.M., (1984) "Asymmetries of information and LDC borrowing with sovereign risk," *The Economic Journal*, 94 (374): 287-307.
- [30] Lucas. D., (1994) "Asset pricing with undiversifiable income risk and short sales constraints: Deepening the equity premium puzzle," *Journal of Monetary Economics*, 34: 325-341.
- [31] Mendoza, E.G., (1991) "Real business cycles in a small open economy," *The American Economic Review*, 36: 101-137.
- [32] Mendoza, E.G., (1995) "The Terms of Trade, the Real Exchange Rate and Economic Fluctuations," *International Economic Review*, 81(4): 797-818.
- [33] Mendoza, E. G., (2001) "Credit, prices, and crashes: business cycles with a sudden stop," in: S. Edwards and J.A. Frankel, eds., *Preventing crises in emerging markets*. NBER Conference Report. Chicago: the University of Chicago Press, pp. 335-389.
- [34] Mendoza, E.G., and Uribe, M. (1999) "The Business Cycles of Balance-of-Payment Crises: A Revision of a Mundellian Framework," *NBER working paper 7045*: 1-47.
- [35] Neumeyer, P.A. and F. Perri, (2004) "Business cycle in emerging economies: the role of interest rates," *NBER working paper 10387*: 1-57.
- [36] Tauchen, G., (1986) "Finite state Markov-Chain approximations to univariate and vector autoregressions," *Economics Letters*, 20: 177-181.
- [37] Telmer, C.I., (1993) "Asset pricing and incomplete markets," *Journal of Finance*, 48: 1803-1832.
- [38] Zhang, H.H, (1997) "Endogenous borrowing constraints with incomplete markets," *The Journal of Finance*, 52 (5): 2187-2209.

Figure 1: Simulated Series



— Unconstrained  
..... Constrained

Figure 2: Simulated HP-Filtered Series (Cycle Component)

